Relations and Functions

Ex 1.1

Question 1.
Find $A \times B$, $A \times A$ and $B \times A$
(i) $A = \{2,-2,3\}$ and $B = \{1,-4\}$
(ii) $A = B = \{p,q]$
(iii) $A= \{m,n\}$; $B = (\Phi)$
Solution:
(i) $A = \{2,-2,3\}$, $B = \{1,-4\}$
$A \times B = \{(2, 1), (2, -4), (-2, 1), (-2, -4), (3,1) , (3,-4)\}$
$A \times A = \{(2, 2), (2,-2), (2, 3), (-2, 2), (-2, -2), (-2, 3), (3, 2), (3, -2), (3,3) \}$
$B \times A = \{(1, 2), (1, -2), (1, 3), (-4, 2), (-4, -2), (-4,3)\}$

(ii) $A = B = \{(p,q)]$
$A \times B = \{(p, p), \{p, q), (q, p), (q, q)\}$
$A \times A = \{(p, p), (p, q), (q, p), (q, q)\}$
$B \times A = \{(p,p), \{p, q), (q, p), (q, q)\}$

(iii) $A = \{m,n\} \times \Phi$
$A \times B = \{ \}$
$A \times A = \{(m, m), (m, n), (n, m), (n, n)\}$
$B \times A = \{ \}$

Question 2.
Let $A= \{1,2,3\}$ and $B = \{\times \mid x$ is a prime number less than $10\}$. Find $A \times B$ and $B \times A$.
Answer:
$A = \{1,2,3\}$, $B = \{2, 3, 5, 7\}$
$A \times B = \{1,2,3\} \times \{2, 3, 5, 7\}$
$= \{(1, 2) (1, 3) (1, 5) (1, 7) (2, 2)$
$(2, 3) (2, 5) (2, 7)(3, 2) (3, 3) (3, 5) (3, 7)\}$
$B \times A = \{2, 3, 5, 7\} \times \{1,2,3\}$
$= \{(2, 1)(2, 2)(2, 3)(3, 1)(3, 2)(3, 3) (5, 1)(5, 2)(5, 3) (7, 1) (7,2)(7, 3)\}$

Question 3.
If $B \times A = \{(-2, 3),(-2, 4),(0, 3),(0, 4),(3, 3), (3, 4)\}$ find A and B.

Solution:
B × A ={(-2, 3), (-2, 4), (0, 3), (0, 4), (3, 3), (3, 4)}
A = {3, 4), B = { -2, 0, 3}

Question 4.
If A= {5, 6}, B = {4, 5 ,6}, C = {5, 6, 7}, Show that A × A = (B × B) ∩ (C × C)
Answer:
A ={5,6}, B = {4,5,6}, C = {5, 6,7}
A × A = {5, 6} × {5,6}
= {(5, 5) (5, 6) (6, 5) (6, 6)} ….(1)
B × B = {4, 5, 6} × {4, 5, 6}
= {(4, 4)(4, 5)(4, 6)(5, 4)(5, 5) (5, 6) (6, 4)(6, 5) (6, 6)}
C × C = {5,6,7} × {5,6,7}
= {(5, 5)(5, 6)(5, 7)(6, 5)(6, 6) (6, 7)(7, 5)(7, 6) (7, 7)}
(B × B) ∩ (C × C) = {(5, 5)(5, 6)(6, 5)(6, 6)} ….(2)
From (1) and (2) we get
A × A = (B × B) ∩ (C × C)

Question 5.
Given A ={1, 2, 3}, B = {2, 3, 5}, C = {3, 4} and D = {1, 3, 5}, check if (A ∩ C) x (B ∩ D) = (A × B) ∩ (C × D) is true?
Solution:
LHS = {(A∩C) × (B∩D)
A ∩C = {3}
B ∩D = {3, 5}
(A ∩ C) × (B ∩ D) = {(3, 3) (3, 5)} ………….. (1)
RHS = (A × B) ∩ (C × D)
A × B = {(1, 2), (1, 3), (1, 5), (2, 2), (2, 3), (2, 5), (3, 2), (3, 3), (3, 5)}
C × D = {(3, 1), (3, 3), (3, 5), (4, 1), (4, 3), (4, 5)}
(A × B) ∩ (C × D) = {(3, 3), (3, 5)} …(2)
∴ (1) = (2) ∴ It is true.

Question 6.
Let A = {x ∈ W | x < 2},
B = {x ∈ N | 1 < 1 < × < 4} and
C = {3,5}. Verify that
(i) A × (B ∪ C) = (A × B) ∪ (A × C)
(ii) A × (B ∩ C) = (A × B) ∩ (A × C)
(iii) (A ∪ B) × C = (A × C) ∪ (B × C)
Answer:
(i) A = {0, 1}
B = {2,3,4}
C = {3,5}

(i) A × (B ∪ C) = (A × B) ∪ (A × c)
B ∪ C = {2, 3,4} ∪ {3,5}
= {2, 3, 4, 5}
A × (B ∪ C) = {0, 1} × {2, 3, 4, 5}
= {(0, 2) (0, 3) (0, 4) (0, 5) (1, 2) (1, 3)(1, 4)(1, 5)} ….(1)
A × B = {0, 1} × {2,3,4}
= {(0,2) (0,3) (0,4) (1,2) (1,3) (1,4) }
A × C = {0, 1} × {3, 5}
{(0, 3) (0, 5) (1,3) (1,5)}
(A × B) ∪ (A × C) = {(0, 2) (0, 3) (0, 4) (0, 5) (1, 2)(1, 3)(1, 4)(1, 5)} ….(2)
From (1) and (2) we get
A × (B ∪ C) = (A × B) ∪ (A × C)

(ii) A × (B n C) = (A × B) n (A × C)
B ∩ C = {2,3,4} ∩ {3,5}
= {3}
A × (B ∩ C) = {0, 1} × {3}
= {(0,3) (1,3)} ….(1)
A × B = {0,1} × {2,3,4}
= {(0, 2) (0, 3) (0, 4) (1,2) (1,3) (1,4)}
A × C = {0,1} × {3,5}
{(0, 3) (0, 5) (1,3) (1,5)}
(A × B) n (A × C) = {(0, 3) (1, 3)} ….(2)
From (1) and (2) we get
A × (B n C) = (A × B) n (A × C)

(iii) (A ∪ B) × C = (A × C) ∪ (B × C)
A ∪ B = {0, 1} ∪ {2,3,4}
= {0,1, 2, 3, 4}
(A ∪ B) × C = {0, 1,2, 3,4} × {3,5}
= {(0, 3) (0, 5) (1, 3) (1, 5)(2, 3) (2, 5) (3, 3)(3, 5) (4, 3)(4, 5)} ….(1)
A × C = {0, 1} × {3,5}
= {(0,3) (0,5) (1,3) (1,5)}
B × C = {2,3,4} × {3,5}
= {(2,3) (2,5) (3,3) (3,5)(4,3)(4,5)}
(A × C) ∪ (B × C) = {(0, 3) (0, 5) (1, 3) (1, 5) (2, 3)(2, 5) (3, 3) (3, 5) (4, 3) (4, 5)} ….(2)
From (1) and (2) we get
(A ∪ B) × C = (A × C) ∪ (B × C)

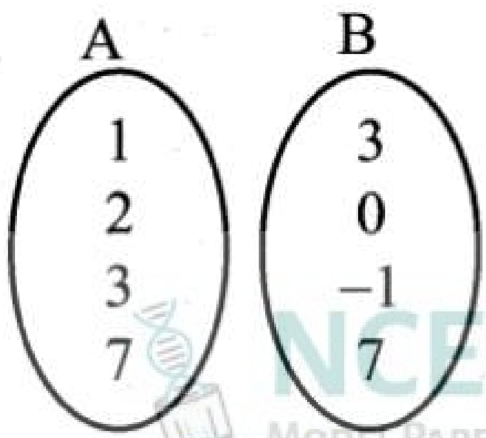
Question 7.
Let A = The set of all natural numbers less than 8, B = The set of all prime numbers less than 8, C = The set of even prime number. Verify that

(i) $(A \cap B) \times c = (A \times C) \cap (B \times C)$
(ii) $A \times (B - C) = (A \times B) - (A \times C)$
$A = \{1, 2, 3, 4, 5, 6, 7\}$
$B = \{2, 3, 5, 7\}$
$C = \{2\}$
Solution:
(i)$(A \cap B) \times C = (A \times c) \cap (B \times C)$
LHS $= (A \cap B) \times C$
$A \cap B = \{2, 3, 5, 7\}$
$(A \cap B) \times C = \{(2, 2), (3, 2), (5, 2), (7, 2)\}$ ………… (1)
RHS $= (A \times C) \cap (B \times C)$
$(A \times C) = \{(1, 2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2), (7, 2)\}$
$(B \times C) = \{2, 2), (3, 2), (5, 2), (7, 2)\}$
$(A \times C) \cap (B \times C) = \{(2, 2), (3, 2), (5, 2), (7, 2)\}$ ……….. (2)
(1) = (2)
∴ LHS = RHS. Hence it is verified.

(ii) $A \times (B - C) = (A \times B) - (A \times C)$
LHS $= A \times (B - C)$
$(B - C) = \{3,5,7\}$
$A \times (B - C) = \{(1, 3), (1, 5), (1, 7), (2, 3), (2, 5), (2, 7), (3, 3), (3, 5), (3, 7), (4, 3), (4, 5), (4, 7), (5, 3), (5, 5), (5, 7), (6, 3), (6, 5), (6, 7), (7, 3), (7, 5), (7, 7)\}$ …………. (1)
RHS $= (A \times B) - (A \times C)$
$(A \times B) = \{(1,2), (1,3), (1,5), (1,7),$
$(2, 2), (2, 3), (2, 5), (2, 7),$
$(3, 2), (3, 3), (3, 5), (3, 7),$
$(4, 2), (4, 3), (4, 5), (4, 7),$
$(5, 2), (5, 3), (5, 5), (5, 7),$
$(6, 2), (6, 3), (6, 5), (6, 7),$
$(7, 2), (7, 3), (7, 5), (7,7)\}$
$(A \times C) = \{(1, 2), (2, 2),(3, 2),(4, 2), (5, 2), (6, 2), (7, 2)\}$
$(A \times B) - (A \times C) = \{(1, 3), (1, 5), (1, 7), (2, 3), (2, 5), (2, 7), (3, 3), (3, 5), (3, 7), (4, 3), (4, 5), (4, 7), (5, 3), (5, 5), (5, 7), (6, 3), (6, 5), (6, 7), (7, 3), (7, 5), (7,7)\}$ ………….. (2)
(1) = (2) ⇒ LHS = RHS.
Hence it is verified.

Ex 1.2

Question 1.
Let A = {1, 2, 3, 7} and B = {3, 0, -1, 7}, which of the following are relation from A to B ?
(i) R_1 = {(2, 1), (7, 1)}
(ii) R_2 = {(-1, 1)}
(iii) R_3 = {(2, -1), (7, 7), (1, 3)}
(iv) R_4 = {(7,-1), (0, 3), (3, 3), (0, 7)}
(i) A = {1, 2, 3, 7}, B = {3, 0,-1, 7}
Solution:
R_1 = {(2,1), (7,1)}



It is not a relation there is no element as 1 in B.
(ii) R_2 = {(-1, 1)}
It is not [∵ -1 ∉ A, 1 ∉ B]
(iii) R_3 = {(2, -1), (7, 7), (1, 3)}
It is a relation.
R_4 = {(7,-1), (0, 3), (3, 3), (0, 7)}
It is also not a relation. [∵ 0 ∉ A]

Question 2.
Let A = {1, 2, 3, 4, ….., 45} and R be the relation defined as "is square of " on A. Write R as a subset of A × A. Also, find the domain and range of R.
Answer:
A = {1,2, 3, 4 . . . . 45}
The relation is defined as "is square of'
R = {(1,1) (2, 4) (3, 9)
(4, 16) (5,25) (6, 36)}
Domain of R = {1, 2, 3, 4, 5, 6}
Range of R = {1, 4, 9, 16, 25, 36}

Question 3.
A Relation R is given by the set {(x, y) /y = x + 3, x ∈ {0, 1, 2, 3, 4, 5}}. Determine its domain and range.
Solution:
x = {0, 1, 2, 3, 4, 5}
y = x + 3

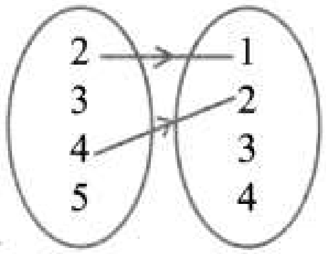
$$\text{i.e. } y = \begin{Bmatrix} (0+3)=3 \\ (1+3)=4 \\ (2+3)=5 \\ (3+3)=6 \\ (4+3)=7 \\ (5+3)=8 \end{Bmatrix}$$

⇒ y = {3, 4, 5, 6, 7, 8}
R = {(x, y)}
= {(0, 3),(1, 4),(2, 5),(3, 6), (4, 7), (5, 8)}
Domain of R = {0, 1, 2, 3, 4, 5}
Range of R = {3, 4, 5, 6, 7, 8}

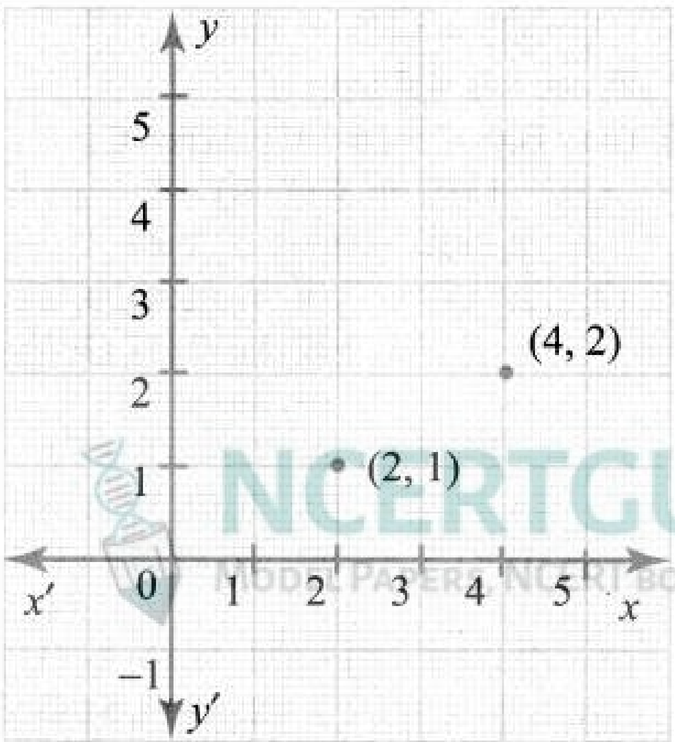
Question 4.
Represent each of the given relation by (a) an arrow diagram, (b) a graph and (c) a set in roster form, wherever possible.
(i) {(x, y)|x = 2y,x ∈ {2, 3, 4, 5},y ∈ {1, 2, 3, 4)
(ii) {(x, y)|y = x + 3, x, y are natural numbers <10}
Solution:
(i){(x, y)|x = 2y, x ∈ {2, 3, 4, 5},y ∈ {1, 2, 3, 4}} R = (x = 2y)
2 = 2 × 1 = 2

4 = 2 × 2 = 4

(a)



(b)



(c) {(2, 1), (4, 2)}

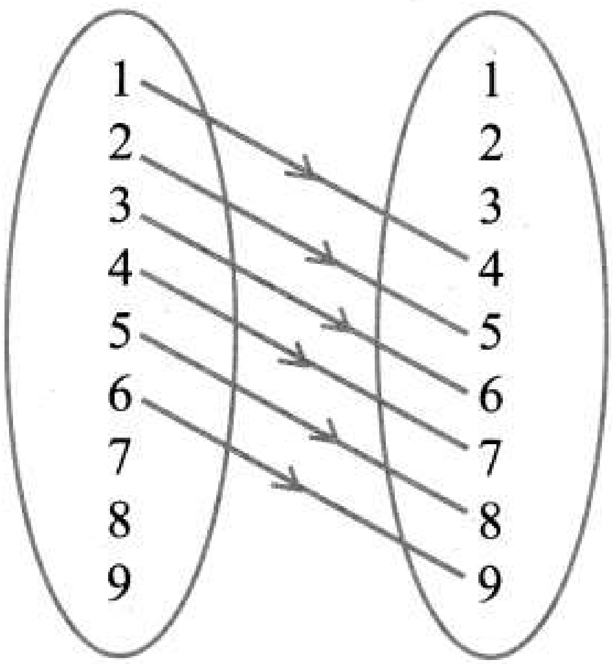
(ii) {(x, y)|y = x + 3, x,+ are natural numbers <10}

x = {1, 2, 3, 4, 5, 6, 7, 8, 9} R = (y = x + 3)

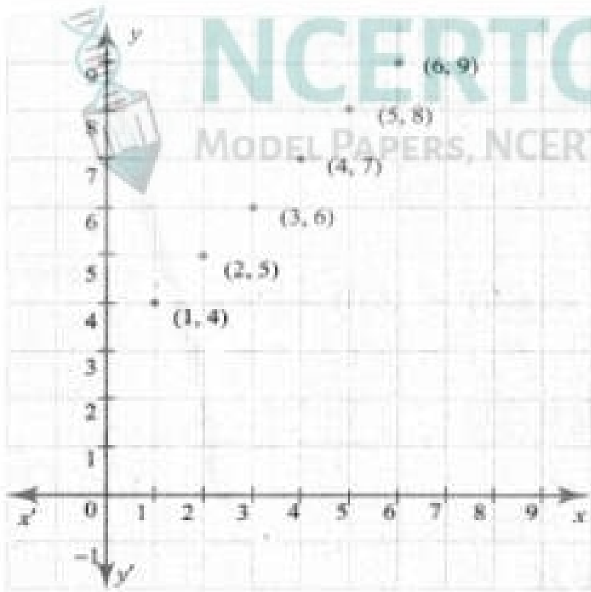
y = {1, 2, 3, 4, 5, 6, 7, 8, 9}

R = {(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)}

(a)



(b)

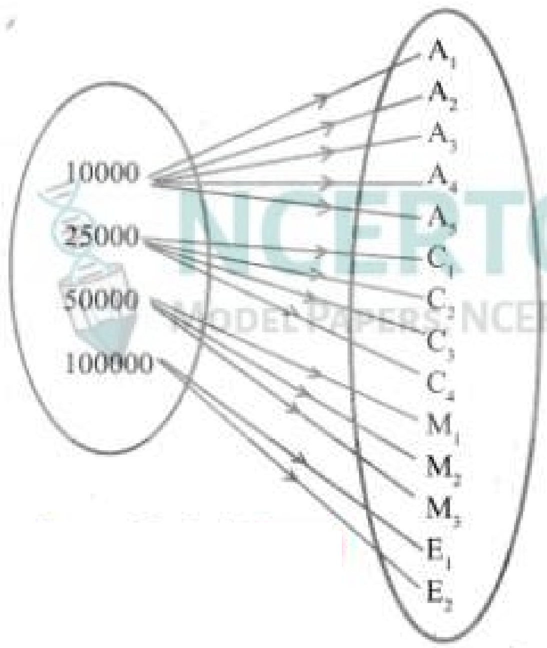


(c) R = {(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)}

Question 5.
A company has four categories of employees given by Assistants (A), Clerks (C), Managers (M) and an Executive Officer (E). The company provide □10,000, □25,000, □50,000 and □1,00,000 as salaries to the people who work in the categories A, C, M and E respectively. If A_1, A_2, A_3, A_4 and As were Assistants; C_1, C_2, C_3, C_4 were Clerks; M_1, M_2, M_3 were managers and E_1,E_2 were Executive officers and if the relation R is defined by xRy, where x is the salary given to

person y, express the relation R through an ordered pair and an arrow diagram.
Solution:
A – Assistants → A_1, A_2, A_3, A_4, A_5
C – Clerks → C_1, C_2, C_3, C_4
D – Managers → M_1, M_2, M_3
E – Executive officer → E_1, E_2
(a) R = {(10,000, A_1), (10,000, A_2), (10,000, A_3),
(10,000, A_4), (10,000, A_5), (25,000, C_1),
(25,000, C_2), (25,000, C_3), (25,000, C_4),
(50,000, M_1), (50,000, M_2), (50,000, M_3),
(1,00,000, E_1), (1,00,000, E_2)}

(b)

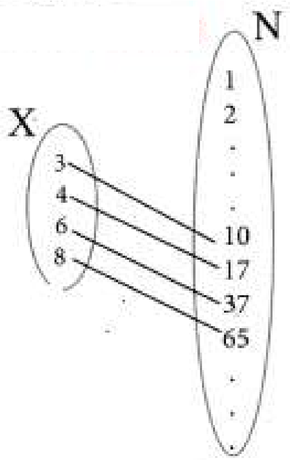


Ex 1.3

Question 1.
Let f = {(x, y)|x, y ∈ N and y = 2x} be a relation on N. Find the domain, co-domain and range. Is this relation a function?
Solution:
F = {(x, y)|x, y ∈ N and y = 2x}
x = {1, 2, 3,…}
y = {1 × 2, 2 × 2, 3 × 2, 4 × 2, 5 × 2 …}
R = {(1, 2), (2, 4), (3, 6), (4, 8), (5, 10),…}
Domain of R = {1, 2, 3, 4,…},
Co-domain = {1, 2, 3…..}
Range of R = {2, 4, 6, 8, 10,…}
Yes, this relation is a function.

Question 2.
Let X = {3, 4, 6, 8}. Determine whether the relation R = {(x, f(x))|x ∈ X, f(x) = x^2 + 1} is a function from X to N ?
Solution:
x = {3,4, 6, 8}
R = ((x, f(x))|x ∈ X, f(x) = X^2 + 1}

f(x) = x^2 + 1
f(3) = 3^2 + 1 = 10
f(4) = 4^2 + 1 = 17
f(6) = 6^2 + 1 = 37
f(8) = 8^2 + 1 = 65

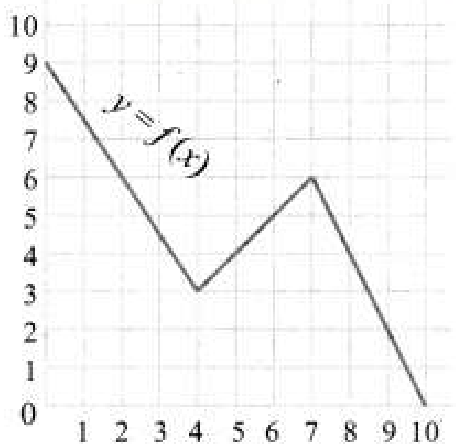


R = {(3, 10), (4, 17), (6, 37), (8, 65)}
R = {(3, 10), (4, 17), (6, 37), (8, 65)}
Yes, R is a function from X to N.

Question 3.
Given the function
f : x → x^2 – 5x + 6, evaluate
(i) f(-1)
(ii) f(2 a)
(iii) f(2)
(iv) f(x – 1)
Answer:

$f(x) = x^2 - 5x + 6$
(i) $f(-1) = (-1)^2 - 5(-1) + 6 = 1 + 5 + 6 = 12$
(ii) $f(2a) = (2a)^2 - 5(2a) + 6 = 4a^2 - 10a + 6$
(iii) $f(2) = 2^2 - 5(2) + 6 = 4 - 10 + 6 = 0$
(iv) $f(x - 1) = (x - 1)^2 - 5(x - 1) + 6$
$= x^2 - 2x + 1 - 5x + 5 + 6$
$= x^2 - 7x + 12$

Question 4.
A graph representing the function f(x) is given in figure it is clear that f(9) = 2.



(i) Find the following values of the function
(a) f(0)
(b) f(7)
(c) f(2)
(d) f(10)
(ii) For what value of x is f (x) = 1?
(iii) Describe the following
(i) Domain
(ii) Range.
(iv) What is the image of 6 under f?
Solution:
From the graph
(a) f(0) = 9
(b) f(7) = 6
(c) f(2) = 6
(d) f(10) = 0
(ii) At x = 9.5, f(x) = 1
(iii) Domain = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
$= \{x \mid 0 < x < 10, x \in R\}$
Range $= \{x \mid 0 < x < 9, x \in R\}$
= {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}
(iv) The image of 6 under f is 5.

Question 5.
Let f(x) = 2x + 5. If x ≠ 0 then find $\frac{f(x+2)-f(2)}{x}$
Solution:

Given f(x) = 2x + 5, x ≠ 0.

$$\frac{f(x+2)-f(2)}{x}$$

$$\begin{aligned} f(x) &= 2x+5 \\ &\Rightarrow f(x+2) \\ &= 2(x+2)+5 \\ &= 2x+4+5 = 2x+9 \\ \Rightarrow f(2) &= 2(2)+5 = 4+5 = 9 \end{aligned}$$

$$\therefore \frac{f(x+2)-f(2)}{x} = \frac{2x+9-9}{x} = \frac{2x}{x} = 2$$

Question 6.
A function fis defined by f(x) = 2x – 3
(i) find $\frac{f(0)+f(1)}{2}$
(ii) find x such that f(x) = 0.
(iii) find x such that f(x) = x.
(iv) find x such that f(x) = f(1 – x).
Solution:
Given f(x) = 2x – 3
(i) find $\frac{f(0)+f(1)}{2}$
f(0) = 2(0) – 3 = -3
f(1) = 2(1) – 3 = -1
$\therefore \frac{f(0)+f(1)}{2} = \frac{-3-1}{2} = \frac{-4}{2}$ = -2

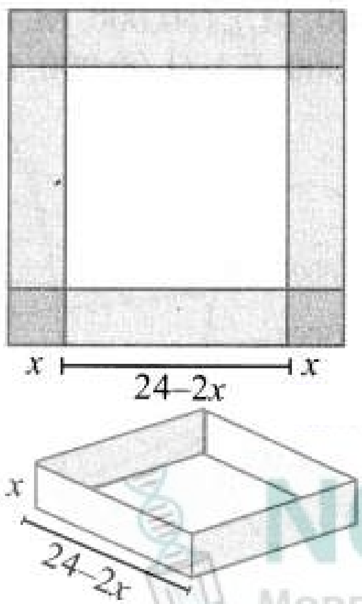
(ii) f(x) = 0
⇒ 2x – 3 = 0
2x = 3
x = $\frac{3}{2}$

(iii) f(x) = x
⇒ 2x – 3 = x ⇒ 2x – x = 3
x = 3

(iv) f(x) = f(1 – x)
2x – 3 = 2(1 – x) – 3
2x – 3 = 2x – 2x – 3
2x + 2x = 2 – 3 + 3
4x = 2

$x = \frac{2}{4}$
$x = \frac{1}{2}$

Question 7.
An open box is to be made from a square piece of material, 24 cm on a side, by cutting equal squares from the corners and turning up the sides as shown in figure. Express the volume V of the box as a function of x.



Solution:
Volume of the box = Volume of the cuboid
= l × b × h cu. units
Here l = 24 – 2x
b = 24 – 2x
h = x
∴ V = (24 – 2x) (24 – 2x) × x
$= (576 – 48x – 48x + 4x^2)x$
$V = 4x^3 – 96x^2 + 576x$

Question 8.
A function f is defined bv f(x) = 3 – 2x . Find x such that f(x2) = (f(x))2.
Solution:
f(x) = 3 – 2x

$f(x^2) = 3 - 2x^2$

$$\begin{aligned}
(f(x))^2 &= (3-2x)^2 = 9 - 12x + 4x^2 \\
f(x^2) &= (f(x))^2 \Rightarrow 3 - 2x^2 = 9 - 12x + 4x^2 \\
6x^2 - 12x + 6 &= 0 \; [\div 6] \\
x^2 - 2x + 1 &= 0 \\
(x-1)(x-1) &= 0 \\
x &= 1, 1
\end{aligned}$$

1

−1 −1

Question 9.
A plane is flying at a speed of 500 km per hour. Express the distance d travelled by the plane as function of time r in hours.
Answer:
Speed of the plane = 500 km/hr
Distance travelled in "t" hours
= 500 × t (distance = speed × time)
= 500 t

Question 10.
The data in the adjacent table depicts the length of a woman's forehand and her corresponding height. Based on this data, a student finds a relationship between the height (y) and the forehand length(x) as y = ax + b, where a, b are constants.

Length 'x' of forehand (in cm)	Height 'y' (in inches)
35	56
45	65
50	69.5
55	74

(i) Check if this relation is a function.
(ii) Find a and b.
(iii) Find the height of a woman whose forehand length is 40 cm.
(iv) Find the length of forehand of a woman if her height is 53.3 inches.
Solution:

(i) Given y = ax + b …………. (1)
The ordered pairs are R = {(35, 56) (45, 65) (50, 69.5) (55, 74)}
∴ Hence this relation is a function.

(ii) Consider any two ordered pairs $\overset{x}{35}, \overset{y}{56}$ $(\overset{x}{45}, \overset{y}{65})$ substituting in (1) we get,

$$65 = 45a + b \quad ....(2)$$
$$56 = 35a + b \quad ....(3)$$

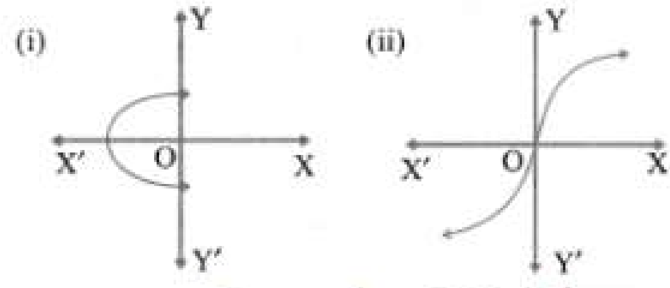
Subtracting, $9 = 10a$

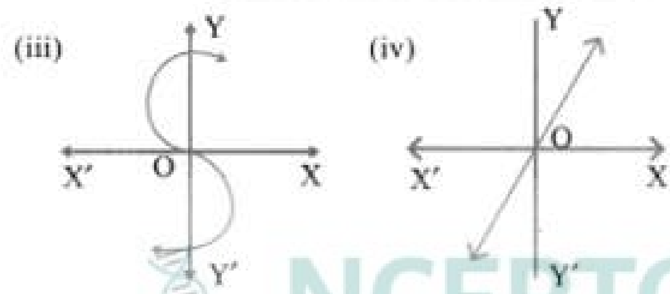
$$\therefore a = \frac{9}{10} = 0.9$$

Substituting a = 0.9 in (2) we get
⇒ 65 = 45(.9) + b
⇒ 65 = 40.5 + b
⇒ b = 65 – 40.5
⇒ b = 24.5
∴ a = 0.9, b = 24.5
∴ y = 0.9x + 24.5
(iii) Given x = 40 , y = ?
∴ (4) → y = 0.9 (40) + 24.5
⇒ y = 36 + 24.5
⇒ y = 60.5 inches
(iv) Given y = 53.3 inches, x = ?
(4) → 53.3 = 0.9x + 24.5
⇒ 53.3 – 24.5 = 0.9x
⇒ 28.8 = 0.9x
⇒ x = $\frac{28.8}{0.9}$ = 32 cm
∴ When y = 53.3 inches, x = 32 cm

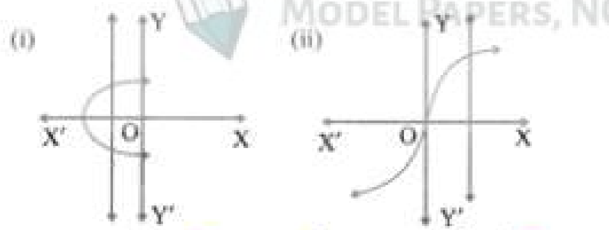
Ex 1.4

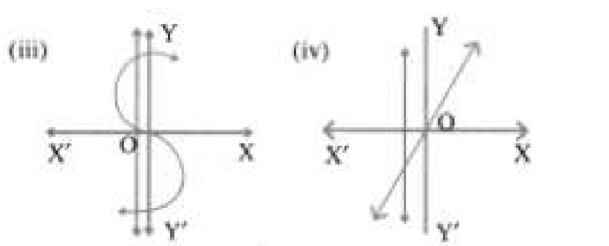
Question 1.
Determine whether the graph given below represent functions. Give reason for your answers concerning each graph.





Solution:





(i) It is not a function. The graph meets the vertical line at more than one points.
(ii) It is a function as the curve meets the vertical line at only one point.
(iii) It is not a function as it meets the vertical line at more than one points.
(iv) It is a function as it meets the vertical line at only one point.

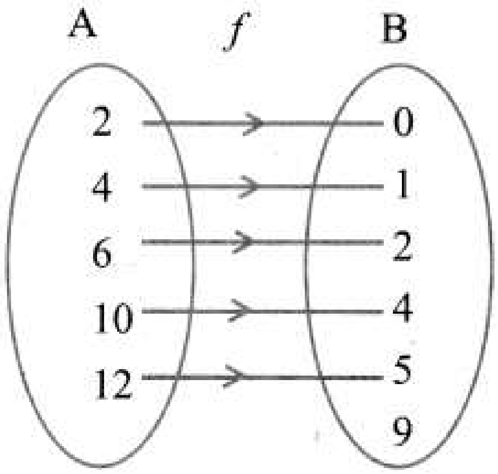
Question 2.
Let f :A → B be a function defined by f(x) = $\frac{x}{2}$ − 1, Where A = {2, 4, 6, 10, 12},
B = {0, 1, 2, 4, 5, 9}. Represent f by
(i) set of ordered pairs;
(ii) a table;
(iii) an arrow diagram;
(iv) a graph
Solution:
f: A → B
A = {2, 4, 6, 10, 12}, B = {0, 1, 2, 4, 5, 9}

$$f(x) = \frac{x}{2} - 1,$$
$$f(2) = \frac{2}{2} - 1 = 0$$
$$f(4) = \frac{4}{2} - 1 = 1$$
$$f(6) = \frac{6}{2} - 1 = 2$$
$$f(10) = \frac{10}{2} - 1 = 4$$
$$f(12) = \frac{12}{2} - 1 = 5$$

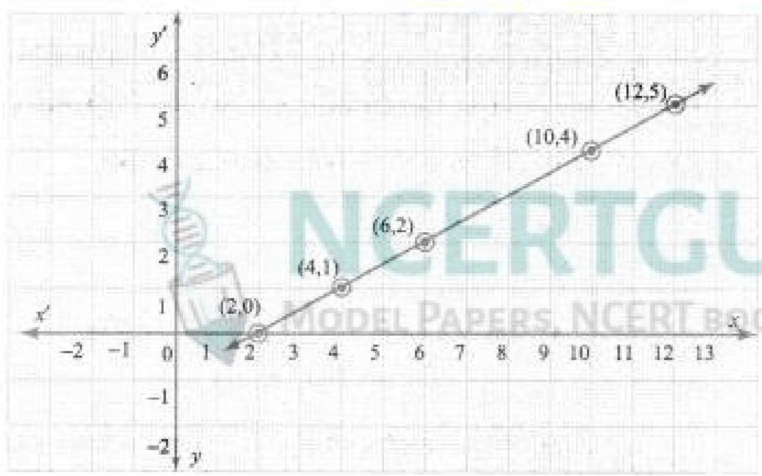
(i) Set of ordered pairs
= {(2, 0), (4, 1), (6, 2), (10, 4), (12, 5)}
(ii) a table

x	2	4	6	10	12
$f(x)$	0	1	2	4	5

(iii) an arrow diagram;

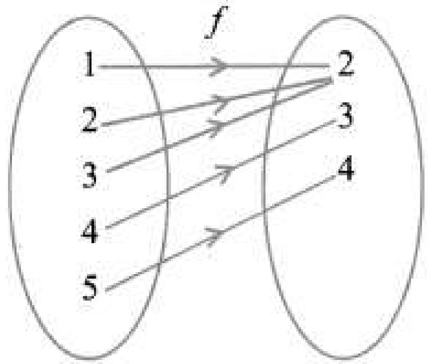


(iv) a graph



Question 3.
Represent the function f = {(1, 2),(2, 2),(3, 2), (4,3), (5,4)} through
(i) an arrow diagram
(ii) a table form
(iii) a graph
Solution:
f = {(1, 2), (2, 2), (3, 2), (4, 3), (5, 4)}

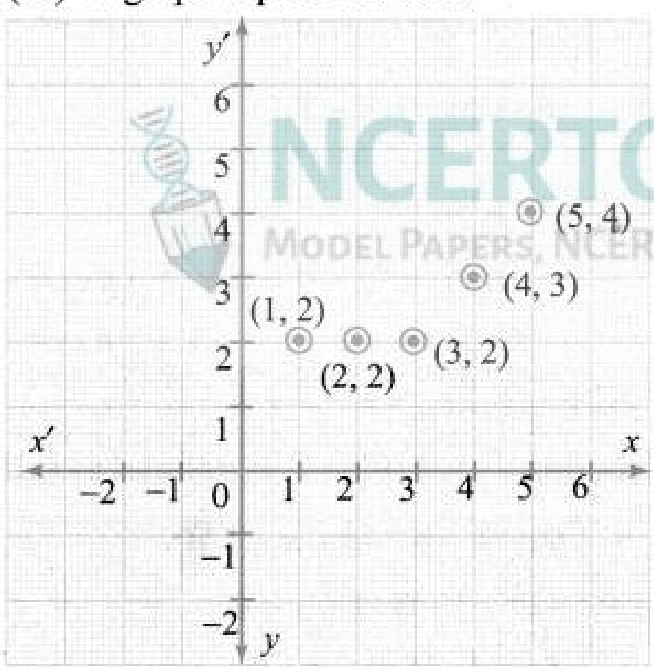
(i) An arrow diagram.



(ii) a table form

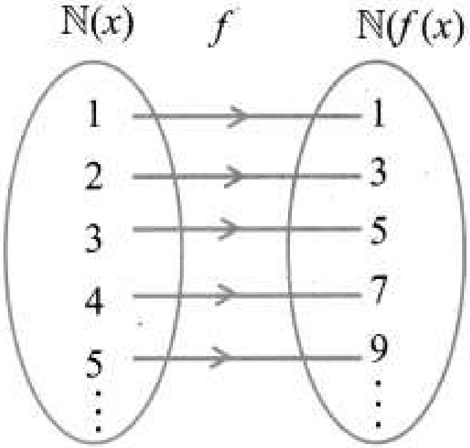
x	1	2	3	4	5
$f(x)$	2	2	2	3	4

(iii) A graph representation.



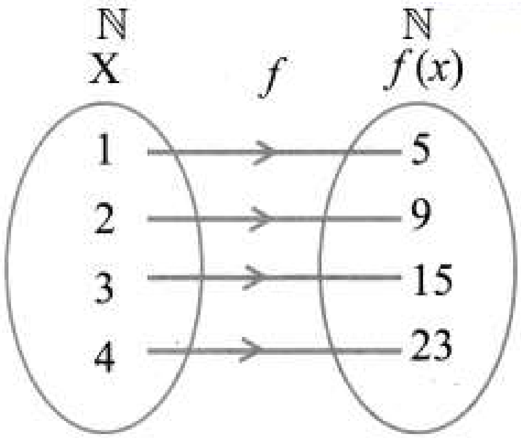
Question 4.
Show that the function f : N → N defined by f{x) = 2x – 1 is one – one but not onto.
Solution:
f: N → N
f(x) = 2x – 1
N = {1, 2, 3, 4, 5,…}
f(1) = 2(1) – 1 = 1
f(2) = 2(2) – 1 = 3

f(3) = 2(3) – 1 = 5
f(4) = 2(4) – 1 = 7
f(5) = 2(5) – 1 = 9



In the figure, for different elements in x, there are different images in f(x).
Hence f : N → N is a one-one function.
A function f: N → N is said to be onto function if the range of f is equal to the co-domain of f
Range = {1, 3, 5, 7, 9,…}
Co-domain = {1, 2, 3,..}
But here the range is not equal to co-domain. Therefore it is one-one but not onto function.

Question 5.
Show that the function f: N → N defined by f (m) = m^2 + m + 3 is one – one function.
Solution:
f: N → N
f(m) = m^2 + m + 3
N = {1, 2, 3, 4, 5…..}m ∈ N
f{m) = m^2 + m + 3
f(1) = 1^2 + 1 + 3 = 5
f(2) = 2^2 + 2 + 3 = 9
f(3) = 3^2 + 3 + 3 = 15
f(4) = 4^2 + 4 + 3 = 23



In the figure, for different elements in the (X) domain, there are different images in f(x). Hence f: N → N is a one to one but not onto function as the range of f is not equal to co-domain.
Hence it is proved.

Question 6.
Let A = {1,2, 3, 4} and B = N .
Let f: A → B be defined by $f(x) = x^3$ then,
(i) find the range of f
(ii) identify the type of function
Answer:
A = {1,2, 3,4}
B = {1,2, 3, 4, 5,….}
$f(x) = x^3$
$f(1) = 1^3 = 1$
$f(2) = 2^3 = 8$
$f(3) = 3^3 = 27$
$f(4) = 4^3 = 64$
(i) Range = {1,8, 27, 64}
(ii) one -one and into function.

Question 7.
In each of the following cases state whether the function is bijective or not. Justify your answer.
(i) f: R → R defined by f(x) = 2x + 1
(ii) f: R → R defined by $f(x) = 3 – 4x^2$
Solution:
(i) f : R → R
f(x) = 2x + 1
f(1) = 2(1) + 1 = 3
f(2) = 2(2) + 1 = 5

f(-1) = 2(-1) + 1 = -1
f(0) = 2(0) + 1 = 1
It is a bijective function. Distinct elements of A have distinct images in B and every element in B has a pre-image in A.
(ii) f: R → R; $f(x) = 3 – 4x^2$
$f(1) = 3 – 4(1^2) = 3 – 4 = -1$
$f(2) = 3 – 4(2^2) = 3 – 16 = -13$
$f(-1) = 3 – 4(-1)^2 = 3 – 4 = -1$
It is not bijective function since it is not one-one

Question 8.
Let A = {-1, 1} and B = {0, 2}. If the function f: A → B defined by f(x) = ax + b is an onto function? Find a and b.
Solution:
A= {-1, 1},B = {0, 2}
f: A → B, f(x) = ax + b
f(-1) = a(-1) + b = -a + b
f(1) = a(1) + b = a + b
Since f(x) is onto, f(-1) = 0
⇒ -a + b = 0 …(1)
& f(1) = 2
⇒ a + b = 2 …(2)
-a + b = 0

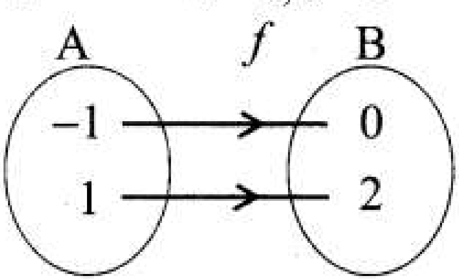
$$\begin{aligned} a + b &= 2 \\ \hline 2b &= 2 \\ b &= 1 \end{aligned}$$

$$\therefore (2) \Rightarrow a + 1 = 2$$

$$a = 2 - 1$$

$$a = 1$$

$$\therefore \quad a = 1, b = 1$$



Question 9.
If the function f is defined by

$$f(x) = \begin{cases} x+2 & \text{if} \quad x > 1 \\ 2 & \text{if} \quad -1 \le x \le 1 \\ x-1 & \text{if} \quad -3 < x < -1 \end{cases}$$ **find the values of**

(i) f(3)
(ii) f(0)
(iii) f(-1.5)
(iv) f(2) + f(-2)
Solution:
(i) f(3) ⇒ f(x) = x + 2 ⇒ 3 + 2 = 5
(ii) f(0) ⇒ 2
(iii) f (- 1.5) = x – 1
= -1.5 – 1 = -2.5
(iv) f(2) + f(-2)
f(2) = 2 + 2 = 4 [∵ f(x) = x + 2]
f(-2) = -2 – 1 = -3 [∵ f(x) = x – 1]
f(2) + f(-2) = 4 – 3 = 1

Question 10.
A function f: [-5,9] → R is defined as follows:

$$f(x) = \begin{cases} 6x+1 & \text{if} \quad -5 \le x < 2 \\ 5x^2-1 & \text{if} \quad 2 \le x < 6 \\ 3x-4 & \text{if} \quad 6 \le x \le 9 \end{cases}$$

Find (i) $f(-3)+f(2)$ **(ii)** $f(7)-f(1)$

(iii) $2f(4)+f(8)$ **(iv)** $\dfrac{2f(-2)-f(6)}{f(4)+f(-2)}$

Solution:
f : [-5, 9] → R
(i) f(-3) + f(2)
f(-3) = 6x + 1 = 6(-3) + 1 = -17
f(2) = 5 × 2 – 1 = $5(2^2)$ – 1 = 19
∴ f(-3) + f(2) = -17 + 19 = 2

(ii) f(7) – f(1)
f(7) = 3x – 4 = 3(7) – 4 = 17
f(1) = 6x + 1 = 6(1) + 1 = 7
f(7) – f(1) = 17 – 7 = 10

(iii) 2f(4) + f(8)
$f(4) = 5x^2 - 1 = 5 \times 4^2 - 1 = 79$
$f(8) = 3x - 4 = 3 \times 8 - 4 = 20$
$\therefore 2f(4) + f(8) = 2 \times 79 + 20 = 178$

(iv) $\dfrac{2f(-2) - f(6)}{f(4) + f(-2)}$

$$f(-2) = 6x + 1 = 6(-2) + 1 = -11$$
$$f(6) = 3x - 4 = 3(6) - 4 = 14$$
$$f(4) = 5x^2 - 1 = 5(4^2) - 1 = 79$$
$$f(-2) = 6x + 1 = 6(-2) + 1 = -11$$
$$\frac{2f(-2) - f(6)}{f(4) + f(-2)} = \frac{2(-11) - 14}{79 + (-11)} = \frac{-22 - 14}{68}$$
$$= \frac{-36}{68} = \frac{-9}{17}$$

Question 11
The distance S an object travels under the influence of gravity in time t seconds is 1 2 given by S(t) = $\frac{1}{3}gt^2 + at + b$, where, (g is the acceleration due to gravity), a, b are constants. Check if the function S(t) is one-one.
Answer:
$S(t) = \frac{1}{2}gt^2 + at + b$
Let the time be 1, 2, 3 …. n seconds
$S(1) = \frac{1}{2}g(1)^2 + a(1) + b$
$= \frac{g}{2} + a + b$
$S(2) = \frac{1}{2}\ g(2)^2 + a(2) + b$
$= \frac{4g}{2} + 2a + b$
$= 2g + 2a + b$
$S(3) = \frac{1}{2}\ g(3)^2 + a(3) + 6$
$= \frac{9}{2}\ g + 3a + b$
For every different value of t, there will be different distance.
∴ It is a one-one function.

Question 12.
The function 't' which maps temperature in Celsius (C) into temperature in Fahrenheit (F) is defined by t(C)= F where F = $\frac{9}{5}$ C + 32 . Find,
(i) t(0)
(ii) t(28)
(iii) t(-10)
(iv) the value of C when t(C) = 212
(v) the temperature when the Celsius value is equal to the Farenheit value.
Solution:
(i) $t(0) = F$

$$F = \frac{9}{5}(C) + 32 = \frac{9}{5}(0) + 32 = 32°F$$

(ii) $$t(28) = F = \frac{9}{5}(28) + 32 = \frac{252}{5} + 32$$
$$= 50.4 + 32 = 82.4°F$$

(iii) $$t(-10) = F = \frac{9}{5}(-10) + 32 = 14°F$$

(iv) $t(C) = 212$

i.e $$\frac{9}{5}(C) + 32 = 212 \Rightarrow \frac{9}{5}C = 212 - 32 = 180$$

$$\frac{9}{5}C = 180 \Rightarrow C = \frac{\overset{20}{\cancel{180}} \times 5}{\cancel{9}} = 100°C$$
$$C = 100°C.$$

(v) when C = F

$$\frac{9}{5}C + 32 = C$$
$$32 = C - \frac{9}{5}C$$
$$32 = C\left(1 - \frac{9}{5}\right)$$
$$32 = C\left(\frac{5-9}{5}\right)$$
$$32 = C\left(\frac{-4}{5}\right)$$
$$C = \overset{8}{\cancel{32}} \times \frac{-5}{\cancel{4}}$$
$$C = -40°$$

Ex 1.5

Question 1
Using the functions f and g given below, find fog and gof. Check whether fog = gof.
(i) $f(x) = x – 6, g(x) = x^2$
(ii) $f(x) = \frac{2}{x}$, $g(x) = 2x^2 – 1$
(iii) $f(x) = \frac{x+6}{3}$ $g(x) = 3 – x$
(iv) $f(x) = 3 + x, g(x) = x – 4$
(v) $f(x) = 4x^2 – 1, g(x) = 1 + x$
Solution:
(i) $f(x) = x – 6, g(x) = x^2$
$fog(x) = f(g(x)) = f(x^2) = x^2 – 6$ …………….. (1)
$gof(x) = g(f(x)) = g(x – 6) = (x – 6)^2$
$= x^2 + 36 – 12x = x^2 – 12x + 36$ ……………… (2)
(1) ≠ (2)
∴ fog(x) ≠ gof(x)

(ii) $f(x) = \frac{2}{x}, g(x) = 2x^2 - 1$

$$fog(x) = f(g(x)) = f(2x^2 - 1) = \frac{2}{2x^2 - 1} \quad ...(1)$$

$$gof(x) = g(f(x)) = g\left(\frac{2}{x}\right) = 2\left(\frac{2}{x}\right)^2 - 1$$

$$= 2\left(\frac{4}{x^2}\right) - 1 = \frac{8}{x^2} - 1 \quad ...(2)$$

(iii) $f(x) = \frac{x+6}{3}$ $g(x) = 3 - x$

$$fog(x) = f(g(x)) = f(3 - x) = \frac{3 - x + 6}{3}$$

$$= \frac{9 - x}{3} \quad ...(1)$$

$$gof(x) = g(f(x)) = g\left(\frac{x+6}{3}\right) = 3 - \frac{x+6}{3}$$

$$= \frac{9 - x - 6}{3} = \frac{3 - x}{3} \quad ...(2)$$

$(1) \neq (2)$

$fog(x) \neq gof(x)$

(iv) $f(x) = 3 + x$, $g(x) = x - 4$

$fog(x) = f(g(x)) = f(x - 4) = 3 + x - 4$

$= x - 1$ ………… (1)

$gof(x) = g(f(x)) = g(3 + x) = 3 + x - 4$

$= x - 1$ ……………… (2)

Here $fog(x) = gof(x)$

(v) $f(x) = 4x^2 - 1$, $g(x) = 1 + x$

$fog(x) = f(g(x)) = f(1 + x) = 4(1 + x)^2 - 1$

$= 4(1 + x^2 + 2x) - 1 = 4 + 4x^2 + 8x - 1$

$= 4x^2 + 8x + 3$ ……………. (1)

gof(x) = g(f(x)) = g($4x^2$ – 1)
= 1 + $4x^2$ – 1 = $4x^2$ …………….. (2)
(1) ≠ (2)
∴ fog(x) ≠ gof(x)

Question 2.
Find the value of k, such that f o g = g o f

(i) f(x) = 3x + 2, g(x) = 6x – k
Answer:
f(x) = 3x + 2 ;g(x) = 6x – k
fog = f[g(x)]
= f (6x – k)
= 3(6x – k) + 2
= 18x – 3K + 2
g0f= g [f(x)]
= g (3x + 2)
= 6(3x + 2) – k
= 18x + 12 – k
But given fog = gof.
18x – 3x + 2 = 18x + 12 – k
-3k + 2 = 12 – k
-3 k + k = 12-2
-2k = 10
k = $\frac{-10}{2}$ = -5
The value of k = -5

(ii) f(x) = 2x – k, g(x) = 4x + 5
Answer:
f(x) = 2x – k ; g(x) = 4x + 5
fog = f[g(x)]
= f(4x + 5)
= 2(4x + 5) – k
= 8x + 10 – k
gof = g [f(x)]
= g(2x – k)
= 4(2x – k) + 5
= 8x – 4k + 5
But fog = gof
8x + 10 – k = 8x – 4k + 5
-k + 4k = 5 – 10
3k = -5

$k = \frac{-5}{3}$
The value of $k = \frac{-5}{3}$

Question 3.
if f(x) = 2x – 1, g(x) = $\frac{x+1}{2}$, show that fog = gof = x
Solution:
f(x) = 2x – 1, g(x) = $\frac{x+1}{2}$, fog = gof = x

$$f(x) = 2x - 1, g(x) = \frac{x+1}{2}, fog = gof = x$$

$$fog(x) = f(g(x)) = f\left(\frac{x+1}{2}\right)$$

$$= 2\left(\frac{x+1}{2}\right) - 1 = x \quad ...(1)$$

$$gof(x) = g(f(x)) = g(2x-1) = \frac{2x-1+1}{2}$$

$$= \frac{2x}{2} = x \quad ...(2)$$

(1) = (2)

$$fog = gof = x$$

Hence proved.

Question 4.
(i) If f (x) = x^2 – 1, g(x) = x – 2 find a, if g o f(a) = 1.
(a) Find k, if f(k) = 2k -1 and
fof (k) = 5.
Answer:
(i) f(x) = x^2 – 1 ; g(x) = x – 2 .
gof = g [f(x)]
= g(x^2 – 1)
= x^2 – 1 – 2
= x^2 – 3
given gof (a) = 1
a^2 – 3 = 1 [But go f(x) = x^2 – 3]
a^2 = 4
a = $\sqrt{4}$ = ± 2
The value of a = ± 2

(ii) $f(k) = 2k - 1$; $fof(k) = 5$
$fof = f[f(k)]$
$= f(2k - 1)$
$= 2(2k - 1) - 1$
$= 4k - 2 - 1$
$= 4k - 3$
$fof(k) = 5$
$4k - 3 = 5$
$4k = 5 + 3$
$4k = 8$
$k = \frac{8}{4} = 2$
The value of k = 2

Question 5.
Let $A, B, C \subset N$ and a function $f: A \rightarrow B$ be defined by $f(x) = 2x + 1$ and $g: B \rightarrow C$ be defined by $g(x) = x^2$. Find the range of fog and gof
Solution:
$f(x) = 2x + 1$
$g(x) = x^2$
$fog(x) = fg(x)) = f(x^2) = 2x^2 + 1$
$gof(x) = g(f(x)) = g(2x + 1) = (2x + 1)^2$
$= 4x^2 + 4x + 1$
Range of fog is
$\{y/y = 2x^2 + 1, x \in N\}$
Range of gof is
$\{y/y = (2x + 1)^2, x \in N\}$.

Question 6.
Let $f(x) = x^2 - 1$. Find (i) fof (ii) fofof
Answer:
$f(x) = x^2 - 1$
(i) $fof = f[f\{x)]$
$= f(x^2 - 1)$
$= (x^2 - 1)^2 - 1$
$= x^4 - 2x^2 + 1 - 1$
$= x^4 - 2x^2$

(ii) $fofof = fof[f(x)]$
$= fof(x^2 - 1)$

$= f(x^2 - 1)^2 - 1$

$= f(x^4 - 2x^2 + 1 - 1)$

$= f(x^4 - 2x^2)$

fofof $= (x^4 - 2x^2)^2 - 1$

Question 7.

If f: R → R and g : R → R are defined by $f(x) = x^5$ and $g(x) = x^4$ then check if f,g are one-one and fog is one-one?

Solution:

$f(x) = x^5$

$g(x) = x^4$

fog = fog(x) = f(g(x)) = $f(x^4)$

$= (x^4)^5 = x^{20}$

f is one-one, g is not one-one.

∵ $g(1) = 1^4 = 1$

$g(-1) = (-1)^4 = 1$

Different elements have same images

fog is not one-one. [∵ fog (1) = fog (-1) = 1]

Question 8.

Consider the functions f(x), g(x), h(x) as given below. Show that (f o g) o h = f o(g o h) in each case.

(i) $f(x) = x - 1$, $g(x) = 3x + 1$ and $h(x) = x^2$

(ii) $f(x) = x^2$, $g(x) = 2x$ and $h(x) = x + 4$

(iii) $f(x) = x - 4$, $g(x) = x^2$ and $h(x) = 3x - 5$

Answer:

(i) $f(x) = x - 1$, $g(x) = 3x + 1$, $h(x) = x^2$

fog (x) = f[g(x)]

$= f(3x + 1)$

$= 3x + 1 - 1$

fog = 3x

(fog) o h(x) = fog [h(x)] ,

= fog (x^2)

$= 3(x^2)$

(fog) oh $= 3x^2$ …..(1)

goh (x) = g[h(x)]

$= g(x^2)$

$= 3(x^2) + 1$

$= 3x^2 + 1$

fo(goh) x = f [goh(x)]

$= f[3x^2 + 1]$

$= 3x^2 + 1 – 1$

$= 3x^2$ ….(2)

From (1) and (2) we get

(fog) oh = fo (goh)

Hence it is verified

(ii) $f(x) = x^2$; g (x) = 2x and h(x) = x + 4

(fog) x = f[g(x)]

= f (2x)

$= (2x)^2$

$= 4x^2$

(fog) oh (x) = fog [h(x)]

= fog (x + 4)

$= 4(x + 4)^2$

$= 4[x^2 + 8x + 16]$

$= 4x^2 + 32x + 64$ …. (1)

goh (x) = g[h(x)]

= g(x + 4)

= 2(x + 4)

= 2x + 8

fo(goh) x = fo [goh(x)]

= f[2x + 8]

$= (2x + 8)^2$

= 4×2 + 32x + 64 …. (2)

From (1) and (2) we get

(fog) oh = fo(goh)

(iii) f(x) = x – 4 ; g (x) = x^2; h(x) = 3x – 5

fog (x) = f[g(x)]

$= f(x^2)$

$= x^2 – 4$

(fog) oh (x) = fog [h(x)]

= fog (3x – 5)

$= (3x – 5)^2 – 4$

$= 9x^2 – 30x + 25 – 4$

$= 9x^2 – 30x + 21$ ….(1)

goh (x) = g[h(x)]

$= g(3x - 5)$
$= (3x - 5)^2$
$= 9x^2 + 25 - 30x$
fo(goh)x = f[goh(x)]
$= f[9x^2 - 30x + 25]$
$= 9x^2 - 30x + 25 - 4$
$= 9x^2 - 30x + 21$ ….(2)
From (1) and (2) we get
(fog) oh = fo(goh)

Question 9.
Let f ={(-1, 3),(0, -1),(2, -9)} be a linear function from Z into Z . Find f(x).
Solution:
f ={(-1, 3), (0, -1), 2, -9)
f(x) = (ax) + b ………… (1)
is the equation of all linear functions.
∴ f(-1) = 3
f(0) = -1
f(2) = -9
f(x) = ax + b
f(-1) = -a + b = 3 …………… (2)
f(0) = b = -1
-a – 1 = 3 [∵ substituting b = – 1 in (2)]
-a = 4
a = -4
The linear function is -4x – 1. [From (1)]

Question 10.
In electrical circuit theory, a circuit C(t) is called a linear circuit if it satisfies the superposition principle given by $C(at_1 + bt_2) = aC(t_1) + bC(t_2)$, where a, b are constants. Show that the circuit C(t) = 31 is linear.
Answer:
Given C(t) = 3t
$C(at_1) = 3at_1$ …. (1)
$C(bt_2) = 3\ bt_2$ …. (2)
Add (1) and (2)
$C(at_1) + C(bt_2) = 3at_1 + 3bt_2$
$C(at_1 + bt_2) = 3at_1 + 3bt_2$
$= Cat_1 + Cbt_2$ [from (1) and (2)]
$\therefore C(at_1 + bt_2) = C(at_1 + bt_2)$

Superposition principle is satisfied.
∴ $C(t) = 3t$ is a linear function.

Ex 1.6

Question 1.
If n(A × B) = 6 and A = {1, 3} then n(B) is
(1) 1
(2) 2
(3) 3
(4) 6
Answer:
(3) 3
Hint:
If n(A × B) = 6
A = {1, 1}, n(A) = 2
n(B) = 3

Question 2.
A = {a, b,p}, B = {2, 3}, C = {p, q, r, s)
then n[(A ∪ C) × B] is ………….
(1) 8
(2) 20
(3) 12
(4) 16
Answer:
(3) 12
Hint: A ∪ C = [a, b, p] ∪ [p, q, r, s]
= [a, b, p, q, r, s]
n (A ∪ C) = 6
n(B) = 2
∴ n [(A ∪ C)] × B] = 6 × 2 = 12

Question 3.
If A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8} then state which of the following statement is true.
(1) (A × C) ⊂ (B × D)
(2) (B × D) ⊂ (A × C)
(3) (A × B) ⊂ (A × D)
(4) (D × A) ⊂ (B × A)
Answer:
(1) (A × C) ⊂ (B × D)]
Hint:
A = {1, 2}, B = {1, 2, 3, 4},
C = {5, 6}, D ={5, 6, 7, 8}
A × C ={(1,5), (1,6), (2, 5), (2, 6)}
B × D = {(1, 5),(1, 6),(1, 7),(1, 8),(2, 5),(2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8)}
∴ (A × C) ⊂ B × D it is true

Question 4.
If there are 1024 relations from a set A = {1, 2, 3, 4, 5} to a set B, then the number of elements in B is ………………….
(1) 3
(2) 2
(3) 4
(4) 8
Answer:
(2) 2
Hint: n(A) = 5
n(A × B) = 10
(consider 1024 as 10)
n(A) × n(B) = 10
5 × n(B) = 10
n(B) = $\frac{10}{5}$ = 2
n(B) = 2

Question 5.
The range of the relation R = {(x, x^2)|x is a prime number less than 13} is
(1) {2, 3, 5, 7}
(2) {2, 3, 5, 7, 11}
(3) {4, 9, 25, 49, 121}
(4) {1, 4, 9, 25, 49, 121}
Answer:
(3) {4, 9, 25, 49, 121}]
Hint:

R = {(x, x^2)/x is a prime number < 13}
The squares of 2, 3, 5, 7, 11 are
{4, 9, 25, 49, 121}

Question 6.
If the ordered pairs (a + 2,4) and (5, 2a + 6) are equal then (a, b) is ………
(1) (2, -2)
(2) (5, 1)
(3) (2, 3)
(4) (3, -2)
Answer:
(4) (3, -2)
Hint:

$a + 2 = 5$	$4 = 2a + b$
$a = 5 - 2$	$4 = 2(3) + b$
$a = 3$	$4 - 6 = b$
	$-2 = b$

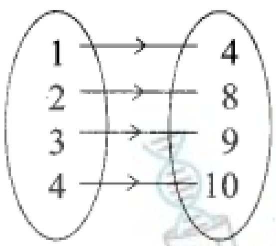
The value of a = 3 and b = -2

Question 7.
Let n(A) = m and n(B) = n then the total number of non-empty relations that can be defined from A to B is
(1) m^n
(2) n^m
(3) $2^{mn} – 1$
(4) 2^{mn}
Answer:
(4) 2^{mn}
Hint:
n(A) = m, n(B) = n
n(A × B) = 2^{mn}

Question 8.
If {(a, 8),(6, b)} represents an identity function, then the value of a and 6 are respectively
(1) (8,6)
(2) (8,8)
(3) (6,8)
(4) (6,6)

Answer:
(1) (8,6)
Hint: f = {{a, 8) (6, 6)}. In an identity function each one is the image of it self.
∴ a = 8, b = 6

Question 9.
Let A = {1, 2, 3, 4} and B = {4, 8, 9, 10}. A function f : A → B given by f = {(1, 4),(2, 8),(3, 9),(4, 10)} is a
(1) Many-one function
(2) Identity function
(3) One-to-one function
(4) Into function
Answer:
(3) One-to one function
Hint:
A = {1, 2, 3, 4), B = {4, 8, 9,10}



Question 10.
If f(x) = $2x^2$ and g (x) = $\frac{1}{3x}$, Then fog is

(1) $\frac{3}{2x^2}$ (2) $\frac{2}{3x^2}$ (3) $\frac{2}{9x^2}$ (4) $\frac{1}{6x^2}$

Answer:
(3) $\frac{2}{9x^2}$
Hint:
f(x) = $2x^2$
g(x) = $\frac{1}{3x}$
fog = f(g(x)) = $f\left(\frac{1}{3x}\right) = 2\left(\frac{1}{3x}\right)^2$
= $2 \times \frac{1}{9x^2} = \frac{2}{9x^2}$

Queston 11.
If f: A → B is a bijective function and if n(B) = 7 , then n(A) is equal to …………..
(1) 7
(2) 49
(3) 1
(4) 14
Answer:
(1) 7
Hint:
n(B) = 7
Since it is a bijective function, the function is one – one and also it is onto.
n(A) = n(B)
∴ n(A) = 7

Question 12.
Let f and g be two functions given by f = {(0, 1), (2, 0), (3, -4), (4, 2), (5, 7)} g = {(0, 2), (1, 0), (2, 4), (-4, 2), (7, 0)} then the range of fog is
(1) {0, 2, 3, 4, 5}
(2) {-4, 1, 0, 2, 7}
(3) {1, 2, 3, 4, 5}
(4) {0, 1, 2}

Answer:
(4) {0, 1, 2}
Hint:
gof = g(f(x))
fog = f(g(x))
= {(0, 2),(1, 0),(2, 4),(-4, 2),(7, 0)}
Range of fog = {0, 1, 2}

Question 13.
Let f (x) = $\sqrt{1+x^2}$ then ………………..
(1) f(xy) = f(x) f(y)
(2) f(xy) ≥ f(x).f(y)
(3) f(xy) ≤ f(x). f(y)
(4) None of these
Answer:
(3) f(xy) ≤ f(x) . f(y)

Question 14.
If g = {(1, 1),(2, 3),(3, 5),(4, 7)} is a function given by g(x) = αx + β then the values of α and β are
(1) (-1, 2)

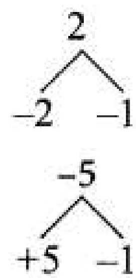
(2) (2, -1)
(3) (-1, -2)
(4) (1, 2)
Answer:
(2) (2,-1)
Hint:
$g(x) = \alpha x + \beta$
$\alpha = 2$
$\beta = -1$
$g(x) = 2x - 1$
$g(1) = 2(1) - 1 = 1$
$g(2) = 2(2) - 1 = 3$
$g(3) = 2(3) - 1 = 5$
$g(4) = 2(4) - 1 = 7$

Question 15.
$f(x) = (x + 1)^3 - (x - 1)^3$ represents a function which is …………….
(1) linear
(2) cubic
(3) reciprocal
(4) quadratic
Answer:
(4) quadratic
Hint: $f(x) = (x + 1)^3 - (x - 1)^3$
[using $a^3 - b^3 = (a - b)^3 + 3ab(a - b)$]
$= (x + 1 - x + 1)^3 + 3(x + 1)(x - 1)$
$(x + 1 - x + 1)$
$= 8 + 3(x^2 - 1)^2$
$= 8 + 6(x^2 - 1)$
$= 8 + 6x^2 - 6$
$= 6x^2 + 2$
It is quadratic polynomial

Unit Exercise 1

Question 1.
If the ordered pairs $(x^2 – 3x, y^2 + 4y)$ and (-2, 5) are equal, then find x and y.
Solution:
$(x^2 – 3x, y^2 + 4y) = (-2, 5)$
$x^2 – 3x = -2$
$x^2 – 3x + 2 = 0$

$$
\begin{aligned}
(x-2)(x-1) &= 0 \\
x &= 2, 1 \\
y^2 + 4y &= 5 \\
y^2 + 4y - 5 &= 0 \\
(y+5)(y-1) &= 0 \\
y &= -5, 1
\end{aligned}
$$



Question 2.
The cartesian product A × A has 9 elements among which (-1, 0) and (0,1) are found.
Find the set A and the remaining elements of A × A.
Answer:
n(A × A) = 9
n(A) = 3
A = {-1,0,1}
A × A = {-1, 0, 1} × {-1, 0, 1}
A × A = {(-1,-1)(-1, 0) (-1, 1)
(0, -1) (0, 0) (0, 1)
(1,-1) (1, 0) (1, 1)}
The remaining elements of A × A =
{(-1, -1) (-1, 1) (0, -1) (0, 0) (1,-1) (1,0) (1,1)}

Question 3.
Given that

$$f(x) = \begin{cases} \sqrt{x-1} & x \geq 1 \\ 4 & x < 1 \end{cases}$$

(i) f(0)
(ii) f(3)
(iii) f(a + 1) in terms of a.(Given that a > 0)
Solution:
(i) f(0) = 4
(ii) f(3) = $\sqrt{3-1} = \sqrt{2}$
(iii) f(a + 1) = $\sqrt{a+1-1} = \sqrt{a}$

Question 4.
Let A = {9,10,11,12,13,14,15,16,17} and let f : A → N be defined by f(n) = the highest prime factor of n ∈ A. Write f as a set of ordered pairs and find the range of f.
Answer:
A= {9, 10, 11, 12, 13, 14, 15, 16, 17}
f: A → N
f(x) = the highest prime factor n ∈ A
f = {(9, 3) (10, 5) (11, 11) (12, 3) (13, 13) (14, 7) (15, 5) (16, 2) (17, 17)}
Range of f = {3, 5, 11, 13, 7, 2, 17}
= {2, 3, 5, 7, 11, 13, 17}

Question 5.
Find the domain of the function f(x) = $\sqrt{1+\sqrt{1-\sqrt{1-x^2}}}$
Solution:
f(x) = $\sqrt{1+\sqrt{1-\sqrt{1-x^2}}}$
Domain of f(x) = {-1, 0, 1}
(x^2 = 1, -1, 0, because $\sqrt{1-x^2}$ should be +ve, or 0)

Question 6.
If f (x) = x^2, g(x) = 3x and h(x) = x – 2, Prove that (f o g)o h = f o(g o h).
Answer:
f(x) = x^2 ; g(x) = 3x and h(x) = x – 2
L.H.S. = (fog) oh
fog = f[g(x)]
= f(3x)
= $(3x)^2 = 9x^2$
(fog) oh = fog[h(x)]

= fog (x – 2)
= $9(x – 2)^2$
= $9[x^2 – 4x + 4]$
= $9x^2 – 36x + 36$ ….(1)
R.H.S. = fo(goh)
goh = g [h(x)]
= g(x – 2)
= 3(x – 2)
= 3x – 6
fo(goh) = fo [goh (x)]
= f(3x – 6)
= $(3x – 6)^2$
= $9x^2 – 36x + 36$ ….(2)
From (1) and (2) we get
L.H.S. = R.H.S.
(fog) oh = fo {goh)

Question 7.
A = {1, 2} and B = {1, 2, 3, 4} , C = {5, 6} and D = {5, 6, 7, 8} . Verify whether A × C is a subset of B × D?
Solution:
A = {1, 2), B = (1, 2, 3, 4)
C = {5, 6}, D = {5, 6, 7, 8)
A × C = {(1, 5), (1, 6), (2, 5), (2, 6)}
B × D = {(1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8)}
(A × C) ⊂ (B × D) It is proved.

Question 8.
If $f(x) = \frac{x-1}{x+1}$, $x \neq 1$ show that $f(f(x)) = -\frac{1}{x}$, Provided $x \neq 0$.
Solution:

$$f(x) = \frac{x-1}{x+1}, x \neq 1$$

$$f(f(x)) = f\left(\frac{x-1}{x+1}\right) = \frac{\left(\frac{x-1}{x+1}\right)-1}{\left(\frac{x-1}{x+1}\right)+1}$$

$$= \frac{\frac{x-1-x-1}{(x+1)}}{\frac{x-1+x+1}{(x+1)}} = \frac{-2}{2x} = \frac{-1}{x}$$

Hence it is proved.

Question 9.

The function/and g are defined by f(x) = 6x + 8; g(x) = $\frac{x-2}{3}$.

(i) Calculate the value of $gg\left(\frac{1}{2}\right)$

(ii) Write an expression for g f(x) in its simplest form.

Solution:

$$f(x) = 6x + 8$$
$$g(x) = \frac{x-2}{3}$$

(i) $$gg(x) = g(g(x))$$
$$= g\left(\frac{x-2}{3}\right) = \frac{\frac{x-2}{3}-2}{3}$$
$$= \frac{x-2-6}{3} \times \frac{1}{3} = \frac{x-8}{9}$$
$$gog\left(\frac{1}{2}\right) = \frac{\frac{1}{2}-8}{9} = \frac{1-16}{2} \times \frac{1}{9}$$
$$= \frac{-15}{18} = \frac{-5}{6}$$

(ii) $$gof(x) = g(f(x)) = g(6x+8)$$
$$= \frac{6x+8-2}{3} = \frac{6x+6}{3}$$
$$= \frac{\cancel{3}(2x+2)}{\cancel{3}}$$
$$= 2x + 2 = 2(x+1)$$

Question 10.
Write the domain of the following real functions

(i) $f(x) = \frac{2x+1}{x-9}$ (ii) $p(x) = \frac{-5}{4x^2+1}$

(iii) $g(x) = \sqrt{x-2}$ (iv) $h(x) = x + 6$

Solution:
(i) f(x) = $\frac{2x+1}{x-9}$
The denominator should not be zero as the function is a real function.
∴ The domain = R – {9}

(ii) p(x) = $\frac{-5}{4x^2+1}$
The domain is R.
(iii) g(x) = $\sqrt{x-2}$
The domain = [2, ∝)
(iv) h(x) = x + 6
The domain is R.

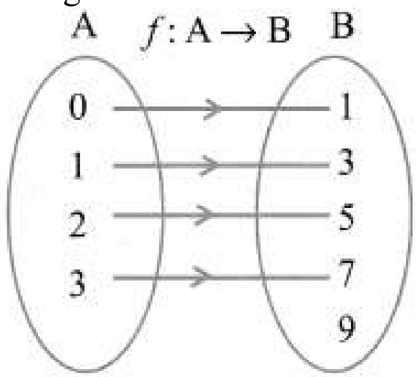
Additional Questions

Question 1.
Let A = {1, 2, 3, 4} and B = {-1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12} Let R = {(1, 3), (2, 6), (3,10), (4, 9)} ⊂ A × B be a relation. Show that R is a function and find its domain, co-domain and the range of R.
Answer:
Domain of R = {1, 2, 3, 4}
Co-domain of R = B = {-1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12}
Range of R= {3, 6, 10, 9}

Question 2.
Let A = {0, 1, 2, 3} and B = {1, 3, 5, 7, 9} be two sets. Let f: A → B be a function given by f(x) = 2x + 1. Represent this function as (i) a set of ordered pairs (ii) a table (iii) an arrow and (iv) a graph.
Solution:
A = {0, 1, 2, 3}, B = {1, 3, 5, 7, 9}
f(x) = 2x + 1
f(0) = 2(0) + 1 = 1
f(1) = 2(1) + 1 = 3
f(2) = 2(2) + 1 = 5
f(3) = 2(3) + 1 = 7
(i) A set of ordered pairs.
f = {(0, 1), (1, 3), (2, 5), (3, 7)}
(ii) A table

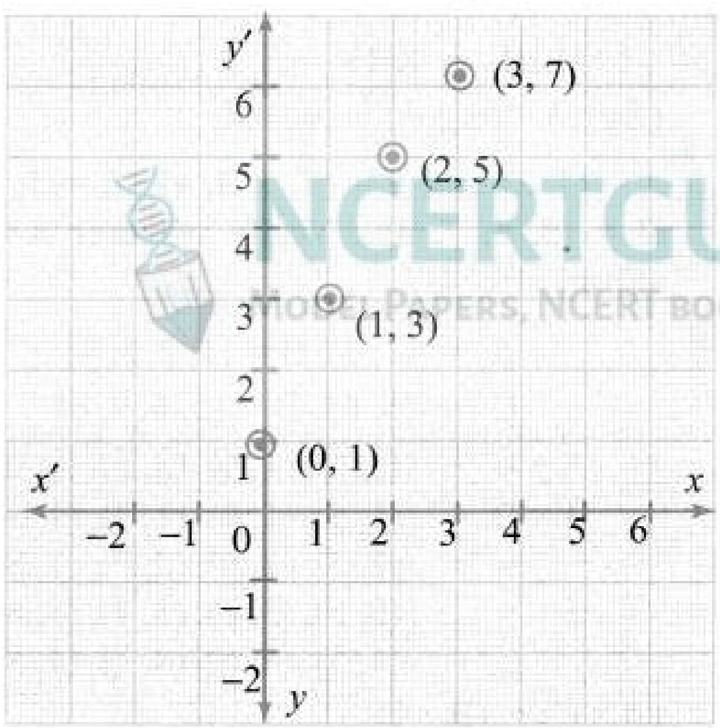
x	0	1	2	3
$f(x)$	1	3	5	7

(iii) An arrow diagram



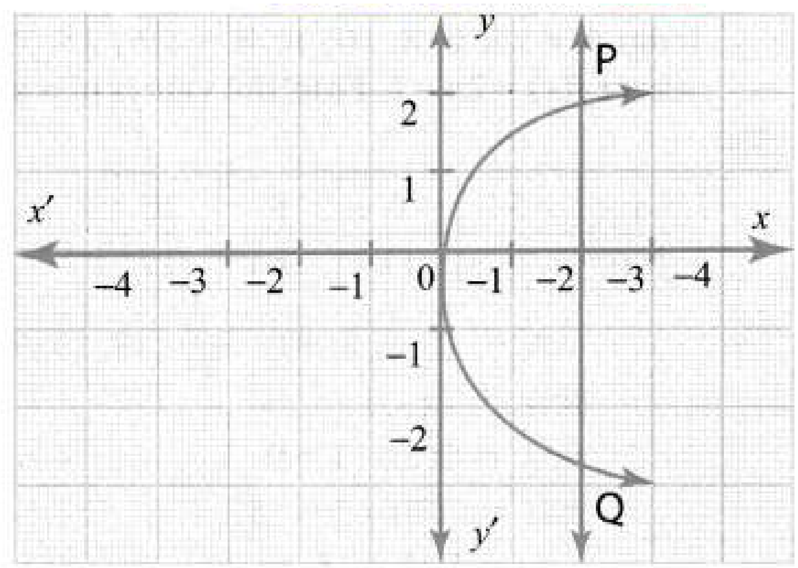
(iv) A Graph $f = \{(x, f(x)/x \in A\}$

$= \{(0, 1), (1, 3), (2, 5), (3, 7)\}$



Question 3.

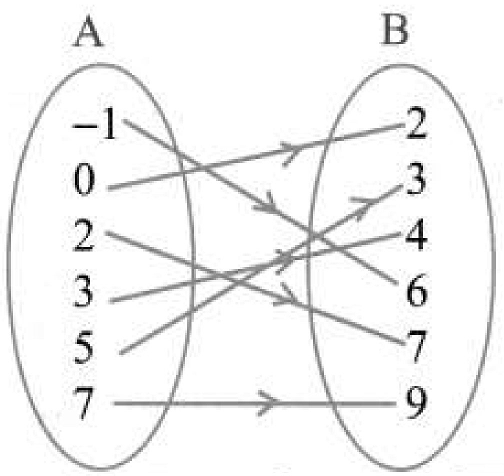
State whether the graph represent a function. Use vertical line test.



Solution:
It is not a function as the vertical line PQ cuts the graph at two points.

Question 4.
Let f = {(2, 7), (3, 4), (7, 9), (-1, 6), (0, 2), (5, 3)} be a function from A = {-1, 0, 2, 3, 5, 7} to B = {2, 3, 4, 6, 7, 9}. Is this (i) an one-one function (ii) an onto function, (iii) both one- one and onto function?

Solution:
It is both one-one and onto function.



All the elements in A have their separate images in B. All the elements in B have their preimage in A. Therefore it is one-one and onto function.

Question 5.
A function f: (-7,6) → R is defined as follows.

$$f(x) = \begin{cases} x^2 + 2x + 1 & -7 \le x < -5 \\ x + 5 & -5 \le x \le 2 \\ x - 1 & 2 < x < 6 \end{cases}$$

Find (i) 2f(-4) + 3 f(2)
(ii) f(-7) – f(-3)
Solution:

$$f(x) = \begin{cases} x^2 + 2x + 1 : -7 \le x < -5 \\ x + 5 \quad : -5 \le x < -2 \\ x - 1 \quad : 2 < x < 6 \end{cases}$$

(i) 2f(-4) + 3f(2)
f(-4) = x + 5 = -4 + 5 = 1
2f(-4) = 2 × 1 = 2
f(2) = x + 5 = 2 + 5 = 7
3f(2) = 3(7) = 21
∴ 2f(-4) + 3f(2) = 2 + 21 = 23

(ii) f(-7) = $x^2 + 2x + 1$
= $(-7)^2 + 2(-7) + 1$
= 49 – 14 + 1 = 36
f(3) = x + 5 = -3 + 5 = 2
f(-7) – f(-3) = 36 – 2 = 34

Question 6.
If A = {2,3, 5} and B = {1, 4} then find
(i) A × B
(ii) B × A
Answer:
A = {2, 3, 5}
B = {1, 4}

(i) A × B = {2,3,5} × {1,4}
= {(2, 1) (2, 4) (3, 1) (3, 4) (5,1) (5, 4)}.

(ii) B × A = {1,4} × {2,3,5}
= {(1,2) (1,3) (1,5) (4, 2) (4, 3) (4, 5)}

Question 7.
Let A = {5, 6, 7, 8};
B = {- 11, 4, 7, -10, -7, – 9, -13} and
f = {(x,y): y = 3 – 2x, x ∈ A, y ∈ B}.
(i) Write down the elements of f.
(ii) What is the co-domain?
(iii) What is the range?
(iv) Identify the type of function.
Answer:
Given, A = {5, 6, 7, 8},
B = {- 11,4, 7,-10,-7,-9,-13}
y = 3 – 2x
ie; f(x) = 3 – 2x
f(5) = 3 – 2 (5) = 3 – 10 = – 7
f(6) = 3 – 2 (6) = 3 – 12 = – 9
f(7) = 3 – 2(7) = 3 – 14 = – 11
f(8) = 3 – 2 (8) = 3 – 16 = – 13
(i) f = {(5, – 7), (6, – 9), (7, – 11), (8, – 13)}
(ii) Co-domain (B)
= {-11,4, 7,-10,-7,-9,-13} i
(iii) Range = {-7, – 9, -11,-13}
(iv) It is one-one function.

Question 8.
A function f: [1, 6] → R is defined as follows:

$$f(x) = \begin{cases} 1+x, & 1 \le x < 2 \\ 2x-1, & 2 \le x < 4 \\ 3x^2-10, & 4 \le x < 6 \end{cases}, x \in \mathbb{R}: 1 \le x < 6)$$

Find the value of (i) f(5)
(ii) f(3)
(iii) f(2) – f(4).
Solution:

$$f(x) = \begin{cases} 1+x & :1 \le x < 2 \\ 2x-1 & :2 \le x < 4 \\ 3x^2-10 & :4 \le x < 6 \end{cases}$$

(i) f(5) = $3x^2$ – 10
= 3 (5^2) – 10 = 75 – 10 = 65

(ii) f(3) = 2x – 1
= 2(3) – 1 = 6 – 1 = 5
(ii) f(2) – f(4)
f(2) = 2x – 1
= 2(2) – 1 = 3
f(4) = $3x^2$ – 10
= $3(4^2)$ – 10 = 38
∴ f(2) – f(4) = 3 – 38 = 35

Question 9.
The following table represents a function from A = {5, 6, 8, 10} to B = {19, 15, 9, 11}, where f(x) = 2x – 1. Find the values of a and b.
Solution:

x	5	6	8	10
$f(x)$	a	11	b	19

A = {5, 6, 8, 10}, B = {19, 15, 9, 11}
f(x) = 2x – 1
f(5) = 2(5) – 1 = 9
f(8) = 2(8) – 1 = 15
∴ a = 9, b = 15

Question 10.
If R = {(a, -2), (-5, 6), (8, c), (d, -1)} represents the identity function, find the values of a,b,c and d.
Solution:
R = {(a, -2), (-5, b), (8, c), (d, -1)} represents the identity function.
a = -2, b = -5, c = 8, d = -1.