Trigonometry

Ex 6.1

Question 1.
Prove the following identities.
(i) $\cot \theta + \tan \theta = \sec \theta \operatorname{cosec} \theta$
(ii) $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$
Solution:

(i) L.H.S $= \dfrac{\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{\cos\theta}$

Hint: $1 + \tan^2\theta = \sec^2\theta$
$\tan^2\theta = \sec^2\theta - 1$

$$= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta}$$

$$= \frac{1}{\cos\theta} \cdot \frac{1}{\sin\theta} = \sec\theta \operatorname{cosec}\theta = \text{R.H.S}$$

(ii) L.H.S $= \tan^2\theta\,(\tan^2\theta + 1)$

$$= \tan^2\theta(\sec^2\theta)$$
$$= (\sec^2\theta - 1)(\sec^2\theta)$$
$$= \sec^4\theta - \sec^2\theta = \text{R.H.S}$$

Question 2.
Prove the following identities

(i) $\dfrac{1-\tan^2\theta}{\cot^2\theta - 1} = \tan^2\theta$

(ii) $\dfrac{\cos\theta}{1+\sin\theta} = \sec\theta - \tan\theta$

Solution:

$$\text{L.H.S} = \frac{1-\tan^2\theta}{\cot^2\theta-1} = \frac{1-(\sec^2\theta-1)}{(\operatorname{cosec}^2\theta-1)-1}$$

Hint: $1+\cot^2\theta = \operatorname{cosec}^2\theta$

$$= \frac{\frac{1-\sin^2\theta}{\cos^2\theta}}{\frac{\cos^2\theta}{\sin^2\theta}-1} = \frac{\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}}{\frac{\cos^2\theta-\sin^2\theta}{\sin^2\theta}}$$

$$= \frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}\times\frac{\sin^2\theta}{\cos^2\theta-\sin^2\theta}$$

Hint:
$\sin^2\theta + \cos^2\theta = 1$
$\cos^2\theta = 1 - \sin^2\theta$
$2\cos^2\theta = 2 - 2\sin^2\theta$

$$= \frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta = \text{R.H.S}$$

(ii) $\text{L.H.S} = \dfrac{\cos\theta}{1+\sin\theta}$

(Multiplying Numerator and denominator by $(1-\sin\theta)$)

$$= \frac{\cos\theta}{1+\sin\theta}\times\frac{1-\sin\theta}{1-\sin\theta} = \frac{\cos\theta-\cos\theta\sin\theta}{1-\sin^2\theta}$$

$$= \frac{\cos\theta-\cos\theta\sin\theta}{\cos^2\theta} = \frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}$$

$$= \sec\theta - \tan\theta = \text{R.H.S}$$

Question 3.
Prove the following identities

(i) $\sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}} = \sec\theta + \tan\theta$

(ii) $\sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}} = 2\sec\theta$

Solution:

(i) $\text{L.H.S} = \sqrt{\frac{1+\sin\theta}{1-\sin\theta} \times \frac{1+\sin\theta}{1+\sin\theta}}$

$$= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}} = \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}$$

$$= \frac{1+\sin\theta}{\cos\theta} = \sec\theta + \tan\theta = \text{R.H.S}$$

(ii) $\text{L.H.S} = \sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \sqrt{\frac{1+\sin\theta}{1-\sin\theta}} \times \frac{\sqrt{1+\sin\theta}}{1+\sin\theta}$

$$= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}} = \frac{1+\sin\theta}{\cos\theta} = \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= \sec\theta + \tan\theta \quad \ldots (1)$$

$$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} \times \frac{\sqrt{1-\sin\theta}}{1-\sin\theta}$$

$$= \frac{1-\sin\theta}{\sqrt{1-\sin^2\theta}}$$

$$= \frac{1-\sin\theta}{\cos\theta} = \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$$

$$= \sec\theta - \tan\theta \quad \ldots (2)$$

$(1) + (2) \Rightarrow \sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$

$= \sec\theta + \tan\theta + \sec\theta - \tan\theta$

$= 2\sec\theta = \text{R.H.S}$ Hence proved

Question 4.
Prove the following identities
(i) $\sec^6\theta = \tan^6\theta + 3\tan^2\theta \sec^2\theta + 1$
(ii) $(\sin\theta + \sec\theta)^2 + (\cos\theta + \text{cosec}\theta)^2$
$= 1 + (\sec\theta + \text{cosec}\theta)^2$
(i) L.H.S $= \sec^6\theta = (\sec^2\theta)^3 = (1 + \tan^2\theta)^3 = (\tan^2\theta + 1)^3$
$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$= (\tan^2\theta)^3 + 3(\tan^2\theta)^2 \times 1 + 3 \times \tan^2\theta \times 1^2 + 1$
$= \tan^6\theta + 3\tan^2\theta \times (\sec^2\theta - 1) + 3\tan^2\theta + 1$
$= \tan^6\theta + 3\tan^2\theta\sec^2\theta - 3\tan^2\theta + 3\tan^2\theta + 1$
$= \tan^6\theta + 3\tan^2\theta \sec^2\theta + 1$ = R.H.S

(ii) L.H.S = $(\sin\theta + \sec\theta)^2 + (\cos\theta + \text{cosec}\theta)^2$
$= \sin^2\theta + 2\sin\theta \sec\theta + \sec^2\theta + \cos^2\theta + 2\cos\theta\, \text{cosec}\theta + \text{cosec}^2\theta$

$$1 + \sec^2\theta + \text{cosec}^2\theta + 2\sin\theta \sec\theta + 2\cos\theta\, \text{cosec}\theta$$

$$1 + \sec^2\theta + \text{cosec}^2\theta + 2\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$$

$$1 + \sec^2\theta + \text{cosec}^2\theta + 2\left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}\right)$$

$$1 + \sec^2\theta + \text{cosec}^2\theta + 2\times\left(\frac{1}{\sin\theta\cos\theta}\right)$$

$$= 1 + (\sec^2\theta + \text{cosec}^2\theta + 2\sec\theta\, \text{cosec}\theta)$$

$$= 1 + (\sec\theta + \text{cosec}\theta)^2 = \text{R.H.S}$$

Question 5.
Prove the following identities

(i) $\sec^4\theta\,(1 - \sin^4\theta) - 2\tan^2\theta = 1$

(ii) $\dfrac{\cot\theta - \cos\theta}{\cot\theta + \cos\theta} = \dfrac{\text{cosec}\,\theta - 1}{\text{cosec}\,\theta + 1}$

Solution:

(i) L.H.S $=\sec^4\theta\,(1-\sin^4\theta)-2\tan^2\theta$

$$\frac{1-\sin^4\theta}{\cos^4\theta}-2\tan^2\theta$$

$$=\sec^4\theta-\tan^4\theta-2\tan^2\theta$$

$$=\frac{1}{\cos^4\theta}-\frac{\sin^4\theta}{\cos^4\theta}-\frac{2\sin^2\theta}{\cos^2\theta}$$

$$=\frac{1-\sin^4\theta-2\sin^2\theta\cos^2\theta}{\cos^4\theta}$$

$$=1+\frac{\cos^4\theta-\cos^4\theta-\sin^4\theta-2\sin^2\theta\cos^2\theta}{\cos^4\theta}$$

$$=\frac{1+\cos^4\theta-[\sin^4\theta+2\sin^2\theta\cos^2\theta+\cos^4\theta]}{\cos^4\theta}$$

$$=\frac{1+\cos^4\theta-(\sin^2\theta+\cos^2\theta)^2}{\cos^4\theta}$$

$$=\frac{\cancel{1}+\cos^4\theta-\cancel{1}}{\cos^4\theta}$$

$$=\frac{\cos^4\theta}{\cos^4\theta}=1=\text{R.H.S}$$

$$\frac{\cot\theta-\cos\theta}{\cot\theta+\cos\theta}=\frac{\frac{\cos\theta}{\sin\theta}-\cos\theta}{\frac{\cos\theta}{\sin\theta}+\cos\theta}$$

$$=\frac{\cancel{\cos\theta}\left(\frac{1}{\sin\theta}-1\right)}{\cancel{\cos\theta}\left(\frac{1}{\sin\theta}+1\right)}$$

$$=\frac{\operatorname{cosec}\theta-1}{\operatorname{cosec}\theta+1}=\text{R.H.S}$$

Question 6.
Prove the following identities

(i) $$\frac{\sin A - \sin B}{\cos A + \cos B} + \frac{\cos A - \cos B}{\sin A + \sin B} = 0$$

(ii) $$\frac{\sin^3 A + \cos^3 A}{\sin A + \cos A} + \frac{\sin^3 A - \cos^3 A}{\sin A - \cos A} = 2$$

Solution:

(i) LHS $= \dfrac{\sin A - \sin B}{\cos A + \cos B} + \dfrac{\cos A - \cos B}{\sin A + \sin B}$

$$= \frac{(\sin A - \sin B)(\sin A + \sin B) + (\cos A + \cos B)(\cos A - \cos B)}{(\cos A + \cos B)(\sin A + \sin B)}$$

$$= \frac{\sin^2 A - \sin^2 B + \cos^2 A - \cos^2 B}{(\cos A + \cos B)(\sin A + \sin B)}$$

$$= \frac{(\sin^2 A + \cos^2 A) - (\sin^2 B + \cos^2 B)}{(\cos A + \cos B)(\sin A + \sin B)}$$

$$= \frac{1-1}{(\cos A + \cos B)(\sin A + \sin B)} = 0$$

= R.H.S

(ii) $\dfrac{\sin^3 A + \cos^3 A}{\sin A + \cos A} + \dfrac{\sin^3 A - \cos^3 A}{\sin A - \cos A} = 2$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$\therefore a^3 + b^3 = (a+b)^3 - 3a^2b - 3ab^2$$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$\therefore a^3 - b^3 = (a-b)^3 + 3a^2b - 3ab^2$$

$$\sin^3 A + \cos^3 A = (\sin A + \cos A)^3 - 3\sin^2 A \cos A - 3\sin A \cos^2 A$$

$$(\sin A + \cos A)^3 - 3\sin A \cos A$$

$$(\sin A + \cos A) \quad ..... (1)$$

$$\sin^3 A - \cos^3 A = (\sin A - \cos A)^3 + 3\sin A \cos A$$
$$(\sin A - \cos A) \quad ..... (2)$$

Substituting (1) and (2) in LHS, we get

$$\text{L.H.S} = (\sin A + \cos A)^2 - 3\cancel{\sin A}\cos A + (\sin A - \cos A)^2 + 3\cancel{\sin A}\cos A$$

$$= \underbrace{\sin^2 A + \cos^2 A}_{1} + 2\sin A \cos A$$

$$+ \underbrace{\sin^2 A + \cos^2 A}_{1} - 2\sin A \cancel{\cos A}$$

$$= 1 + 1 = 2 = \text{R.H.S}$$

Question 7.

(i) If $\sin\theta + \cos\theta = \sqrt{3}$, then prove that $\tan\theta + \cot\theta = 1$.

(ii) If $\sqrt{3}\sin\theta - \cos\theta = 0$, then show that $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$.

Solution:

(i) Given $\sin\theta + \cos\theta = \sqrt{3}$

Squaring both sides we get

$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$

$\therefore \sin\theta \cos\theta = 1$ ..... (1)

use (1) to prove, that

$\tan\theta + \cot\theta = 1$ as follows

$$\text{L.H.S} = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$$

$$= \frac{1}{\cos\theta\sin\theta} = \frac{1}{1} = 1$$

= R.H.S.

(ii) $\sqrt{3}\sin\theta - \cos\theta = 0 \Rightarrow \sqrt{3}\sin\theta = \cos\theta$

$$\therefore \quad \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^\circ$$

$$\text{L.H.S} = \tan 3\theta$$

$$= \tan(3 \times 30) = \tan 90 = \infty$$

$$\text{R.H.S} = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$$

Substituting $\tan\theta = \frac{1}{\sqrt{3}}$ we get

$$= \frac{3 \times \frac{1}{\sqrt{3}} - \left(\frac{1}{\sqrt{3}}\right)^3}{1 - \cancel{3} \times \frac{1}{\cancel{3}}} = \frac{\sqrt{3} - \frac{1}{3\sqrt{3}}}{1 - 1} = \infty$$

$$\therefore \quad \text{L.H.S} = \text{R.H.S}$$

Question 8.

(i) If $\frac{\cos\alpha}{\cos\beta} = m$ and $\frac{\cos\alpha}{\sin\beta} = n$ then prove that $(m^2 + n^2)\cos^2\beta = n^2$

(ii) If $\cot\theta + \tan\theta = x$ and $\sec\theta - \cos\theta = y$ then prove that $(x^2 y)^{\frac{2}{3}} - (xy^2)^{\frac{2}{3}} = 1$

Solution:

(i) LHS:

$$(m^2 + n^2)\cos^2\beta = \left(\frac{\cos^2\alpha}{\cos^2\beta} + \frac{\cos^2\alpha}{\sin^2\beta}\right) \times \cos^2\beta$$

$$= \left(\frac{\sin^2\beta\cos^2\alpha + \cos^2\alpha\,\cos^2\beta}{\cancel{\cos^2\beta}\,\sin^2\beta}\right) \times \cancel{\cos^2\beta}$$

$$= \cos^2\alpha + \frac{\cos^2\alpha(1 - \sin^2\beta)}{\sin^2\beta}$$

$$= \cancel{\cos^2\alpha} + \frac{\cos^2\alpha}{\sin^2\beta} - \cancel{\cos^2\alpha}$$

$$= \left(\frac{\cos\alpha}{\sin\beta}\right)^2 = n^2 = \text{R.H.S}$$

(ii) $x = \cot\theta + \tan\theta = \dfrac{\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{\cos\theta}$

$$= \frac{\cos^2\theta + \sin^2\theta}{\cos\theta\sin\theta} = \frac{1}{\sin\theta\cos\theta}$$

$$y = \sec\theta - \cos\theta = \frac{1}{\cos\theta} - \cos\theta$$

$$= \frac{1-\cos^2\theta}{\cos\theta}$$

$$= \frac{\sin^2\theta}{\cos\theta}$$

$$\therefore x^2y = \frac{1}{\cancel{\sin^2\theta}\cos^2\theta} \times \frac{\cancel{\sin^2\theta}}{\cos\theta}$$

$$= \frac{1}{\cos^3\theta}$$

$$xy^2 = \frac{1}{\sin\theta\cos\theta} \times \frac{\sin^4\theta}{\cos^2\theta}$$

$$= \frac{\sin^3\theta}{\cos^3\theta}$$

$$(x^2y)^{\frac{2}{3}} = \left(\frac{1}{\cos^3\theta}\right)^{\frac{2}{3}} = \frac{1}{\cos^2\theta}$$

$$(xy^2)^{\frac{2}{3}} = \left(\frac{\sin^3\theta}{\cos^3\theta}\right)^{\frac{2}{3}} = \frac{\sin^2\theta}{\cos^2\theta}$$

$$\text{LHS} = \frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta} = \frac{1-\sin^2\theta}{\cos^2\theta}$$

$$= \frac{\cos^2\theta}{\cos^2\theta} = 1 = \text{R.H.S}$$

Question 9.

(i) If sinθ + cosθ = p and secθ + cosecθ = q then prove that $q(p^2 - 1) = 2p$
(ii) If $\sin\theta(1 + \sin^2\theta) = \cos^2\theta$, then prove that $\cos^6\theta - 4\cos^4\theta + 8\cos^2\theta = 4$
Solution:

(i)
$$\begin{aligned} p &= \sin\theta + \cos\theta \\ p^2 &= \underbrace{\sin^2\theta + \cos^2\theta}_{1} + 2\sin\theta\cos\theta \\ p^2 - 1 &= 2\sin\theta\cos\theta \\ q &= \sec\theta + \operatorname{cosec}\theta = \frac{1}{\cos\theta} + \frac{1}{\sin\theta} \\ &= \frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta} \end{aligned}$$

$$\therefore \text{ L.H.S } q(p^2 - 1) = \frac{\sin\theta + \cos\theta}{\cancel{\sin\theta\cos\theta}} \times 2\,\cancel{\sin\theta\cos\theta}$$
$$= 2(\sin\theta + \cos\theta)$$
$$= 2p = \text{R.H.S}$$

(ii) Given $\sin\theta(1 + \sin^2\theta) = \cos^2\theta$
Sustitute $\sin^2\theta = 1 - co^2\theta$ and take cos θ = c
squaring (1) on bothsides we get
$\sin^2\theta(1 + \sin^2\theta)^2 = \cos^4\theta$
$(1 - c^2)(1 + 1 - c^2) = c^4$
$(1 - c^2)(2 - c^2)^2 = c^4$
$(1 - c^2)(4 + c^4 - 4c^2) = c^4$
$4 + c^4 - 4c^2 - 4c^2 - c^6 + 4c^4 = c^4$
$-c^6 + 4c^4 - 8c^2 = -4$
$c^6 - 4c^4 + 8c^2 = -4$
ie $\cos 6\theta - 4\cos 4\theta + 8\cos^2\theta = 4$ = RHS
∴ Hence proved

Question 10.

If $\frac{\cos\theta}{1 + \sin\theta} = \frac{1}{a}$ then prove that $\frac{a^2 - 1}{a^2 + 1} = \sin\theta$

Solution:

$$a^2 = \frac{(1+\sin\theta)^2}{\cos^2\theta} = \frac{1+\sin^2\theta+2\sin\theta}{\cos^2\theta}$$

$$\therefore a^2-1 = \frac{\sin^2\theta+2\sin\theta+1-\cos^2\theta}{\cos^2\theta}$$

$$= \frac{\sin^2\theta+2\sin\theta+\sin^2\theta}{\cos^2\theta}$$

$$= \frac{2\sin^2\theta+2\sin\theta}{\cos^2\theta}$$

$$a^2+1 = \frac{\sin^2\theta+2\sin\theta+1+\cos^2\theta}{\cos^2\theta}$$

$$= \frac{1+2\sin\theta+1}{\cos^2\theta} = \frac{2+2\sin\theta}{\cos^2\theta}$$

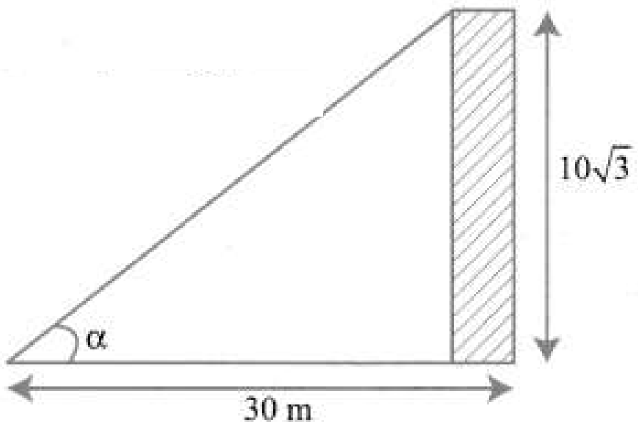
$$\therefore \text{L.H.S } \frac{a^2-1}{a^2+1} = \frac{2\sin^2\theta+2\sin\theta}{2\sin\theta+2}$$

$$= \frac{\cancel{2}\sin\theta(\cancel{\sin\theta+1})}{\cancel{2}(\cancel{\sin\theta+1})}$$

$$= \sin\theta = \text{R.H.S}$$

Ex 6.2

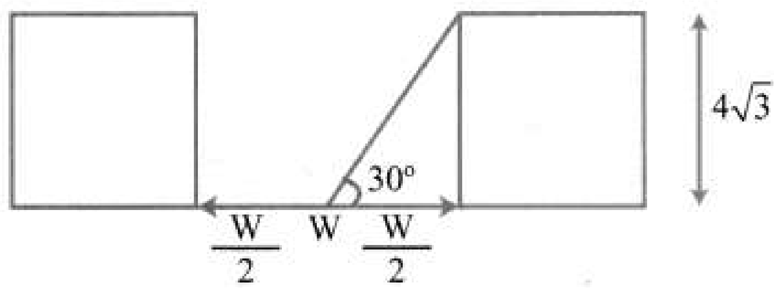
Question 1.
Find the angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of a tower of height $10\sqrt{3}$ m.
Solution:



$$\tan\alpha = \frac{\text{opp. side}}{\text{adj. side}} = \frac{10\sqrt{3}}{30} = \frac{1}{\sqrt{3}}$$

$$\therefore \quad \alpha = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ$$

Question 2.
A road is flanked on either side by continuous rows of houses of height $4\sqrt{3}$ m with no space in between them. A pedestrian is standing on the median of the road facing a row house. The angle of elevation from the pedestrian to the top of the house is 30°. Find the width of the road.
Solution:

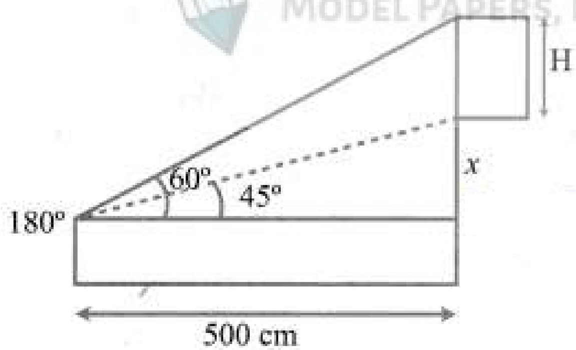


$$\tan 30^\circ = \frac{4\sqrt{3}}{\frac{w}{2}} = \frac{1}{\sqrt{3}}$$

$$\frac{w}{2} = 4\sqrt{3} \times \sqrt{3}$$

$$w = 12 \times 2 = 24 \text{ m}$$

Question 3.
To a man standing outside his house, the angles of elevation of the top and bottom of a window are 60° and 45° respectively. If the height of the man is 180 cm and if he is 5 m away from the wall, what is the height of the window? ($\sqrt{3} = 1.732$)
Solution:



Let 'H' be the fit of the window. Given that elevation of top of the window is 60°.

$$\tan 60^\circ = \frac{H + x}{500} = \sqrt{3}$$

$$H + x = 500\sqrt{3}$$

Given that elevation of bottom of the window is 45°.

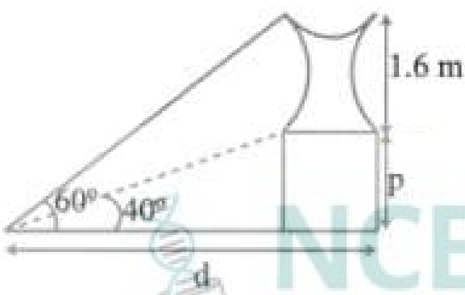
$$\therefore \tan 45° = \frac{x}{500} = 1 \Rightarrow x = 500$$

$$\therefore \quad H = 500\sqrt{3} - 500 = 866 - 500$$

$$= 366 \text{ cm} = 3.66 \text{ m}$$

∴ Height of the window = 3.66 m

Question 4.
A statue 1.6 m tall stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 40°. Find the height of the pedestal. (tan 40° = 0.8391, $\sqrt{3}$ = 1.732)
Solution:



Let ‘p’ be the fit of the pedestal and d be the distance of statue from point of cabs, on the ground.

Given the elevation of top of the statue from p[f] on ground is 60°.

$$\therefore \tan 60^\circ = \frac{1\cdot 6+p}{d} = \sqrt{3}$$

Also given the elevation of top of the pedestal from point on ground is 40°.

$$\tan 40^\circ = \frac{p}{d} = 0.8391$$

$$p = 0.8391\,d$$

$$1\cdot 6+p = \sqrt{3}d$$

$$1\cdot 6+p = 1.732\,d$$

$$1\,.\,6 + 0.8391d = 1.732d$$

$$\therefore \quad 0.8929d = 1.6$$

$$\therefore \quad d = 1.79$$

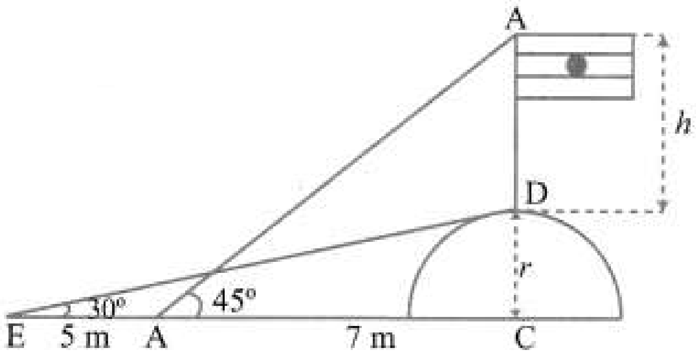
$$\therefore \text{height of pedestal} = p = 0.839 \times d$$

$$= 0.839 \times 1.79$$

$$= 1.503 \text{ m}$$

Question 5.
A flag pole 'h' metres is on the top of the hemispherical dome of radius V metres. A man is standing 7 m away from the dome. Seeing the top of the pole at an angle 45° and moving 5 m away from the dome and seeing the bottom of the pole at an angle 30°. Find
(i) the height of the pole
(ii) radius of the dome.

Solution:



$$\text{In } \Delta ABC, \tan 45^\circ = \frac{h+r}{r+7} = 1$$

$$h+\cancel{r} = \cancel{r}+7$$

$$\Rightarrow \qquad h = 7 \text{ m}$$

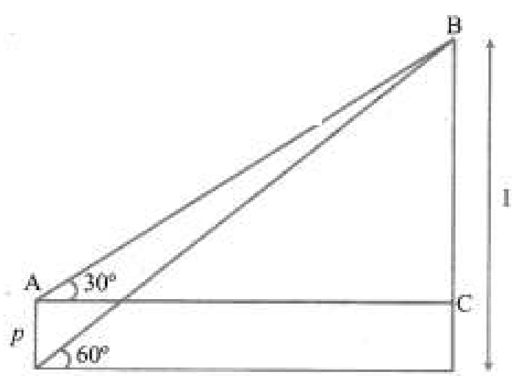
$$\text{in } \Delta EDC, \tan 30^\circ = \frac{r}{5+7+r} = \frac{1}{\sqrt{3}}$$

$$\sqrt{3}r = 12 + r$$

$$\therefore \qquad 0.732r = 12$$

$$r = 16.39 \text{ m}$$

Question 6.
The top of a 15 m high tower makes an angle of elevation of 60° with the bottom of an electronic pole and angle of elevation of 30° with the top of the pole. What is the height of the electric pole?
Solution:



Let BD be tower of height = 15 m
AE be pole of height = ‘p’

In Δ EBD, $\tan 60^\circ = \dfrac{15}{x} = \sqrt{3}$

$\therefore \quad x = 5\sqrt{3}$

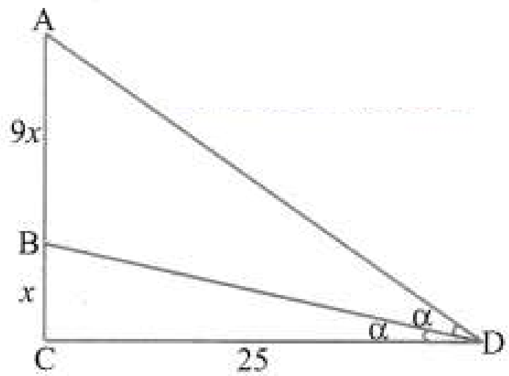
In Δ ABC,

$$\tan 30^\circ = \frac{BC}{AC} = \frac{15-p}{5\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$\therefore 15 - p = 5$

$p = 10$ m

Question 7.
A vertical pole fixed to the ground is divided in the ratio 1 : 9 by a mark on it with lower part shorter than the upper part. If the two parts subtend equal angles at a place on the ground, 25 m away from the base of the pole, what is the height of the pole?
Solution:



Let AC be the pole and let point 'B' divide it in the ratio $\cancel{x} : 9\cancel{x} = 1 : 9$.

Let 'D' be the point 25 m.

$$\tan\alpha = \frac{x}{25} \qquad \tan 2\alpha = \frac{10x}{25}$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$$

$$\frac{\cancel{10x}^{5}}{\cancel{25}} = \frac{2\times\frac{x}{25}}{1-\frac{x^2}{625}}$$

$$5-\frac{5x^2}{625} = 1 \Rightarrow \frac{5x^2}{625} = 4 \Rightarrow x^2 = \frac{4\times\cancel{625}^{125}}{\cancel{5}} = 500$$

$$x = \sqrt{500} = 10\sqrt{5}$$

$$\text{Height of pole} = 10x$$

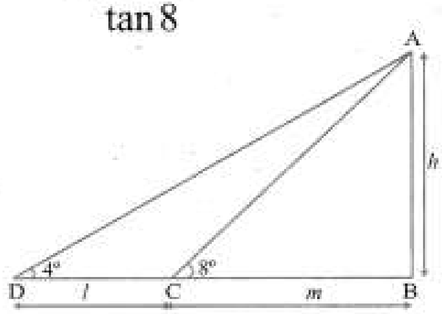
$$x = 100\sqrt{5}\ \text{m}$$

Question 8.
A traveler approaches a mountain on highway. He measures the angle of elevation to the peak at each milestone. At two consecutive milestones the angles measured are 4° and 8°. What is the height of the peak if the distance between consecutive milestones is 1 mile, (tan 4° = 0.0699, tan 8° = 0.1405).
Solution:

Let AB denote the height of the peak and be 'h'.
In ΔABC,

$$\tan 8^\circ = \frac{AB}{BC} = \frac{h}{m}$$

$$m = \frac{h}{\tan 8} \qquad \text{..... (1)}$$



In ΔABD,

$$\tan 4^\circ = \frac{AB}{BD} = \frac{h}{m+1}$$

$$m+1 = \frac{h}{\tan 4} \qquad \text{..... (2)}$$

From (1) and (2)

$$\therefore \quad \frac{h}{\tan 8}+1 = \frac{h}{\tan 4}$$

$$1 = \frac{h}{\tan 4} - \frac{h}{\tan 8}$$

$$h\left[\frac{\tan 8 - \tan 4}{\tan 4 \tan 8}\right] = 1$$

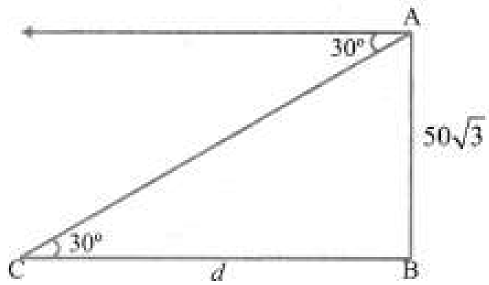
$$h = \frac{\tan 4 \times \tan 8}{\tan 8 - \tan 4}$$

$$= \frac{.0699 \times .1405}{.1405 - .0699}$$

$$= \frac{.00982}{.0706} = 0.14 \text{ mile (approx)}$$

Ex 6.3

Question 1.
From the top of a rock $50\sqrt{3}$ m high, the angle of depression of a car on the ground is observed to be 30°. Find the distance of the car from the rock.
Solution:



In the figure

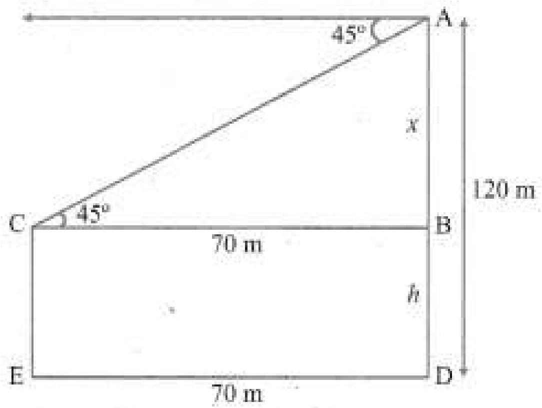
$$\tan 30 = \frac{AB}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{50\sqrt{3}}{d}$$

$$d = 50\ \sqrt{3}\times\sqrt{3}$$

$$= 50 \times 3 = 150 \text{ m.}$$

Question 2.
The horizontal distance between two buildings is 70 m. The angle of depression of the top of the first building when seen from the top of the second building is 45°. If the height of the second building is 120 m, find the height of the first building.
Solution:



In the figure, $\tan \angle C = \tan 45° = 1.$

$$1 = \frac{AB}{70} = \frac{x}{70}$$

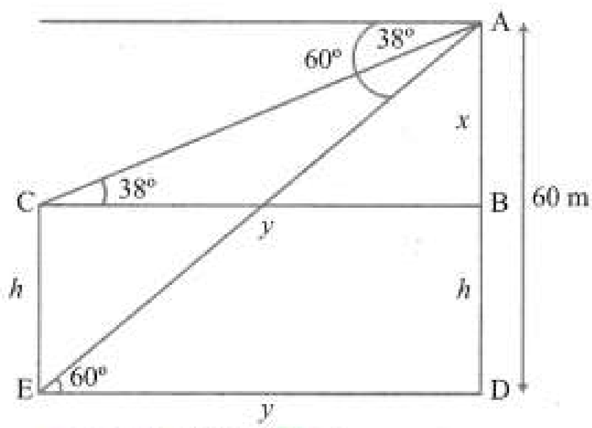
$$x = 70 \text{ m.}$$

$$\therefore \quad BD = 120 - AB$$

$$h = 120 - 70 = 50 \text{ m}.$$

∴ The height of the first building is 50m.

Question 3.
From the top of the tower 60 m high the angles of depression of the top and bottom of a vertical lamp post are observed to be 38° and 60° respectively. Find the height of the lamp post, (tan 38° = 0.7813, $\sqrt{3}$ = 1.732)
Solution:



From the figure,

$$\tan \angle E = \tan 60°$$

$$\sqrt{3} = \frac{60}{y}$$

$$\sqrt{3}\, y = 60$$

$$y = \frac{60 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$$

$$y = \frac{60\sqrt{3}}{3} = 20\sqrt{3} \quad \ldots(1)$$

$$\tan \angle C = \frac{x}{y}$$

$$0.7813 = \frac{x}{20\sqrt{3}}$$

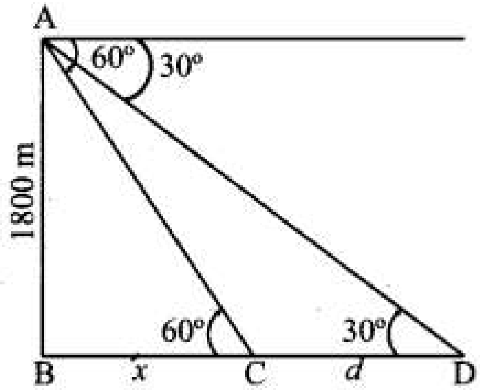
$$x = 20\sqrt{3} \times 0.7813$$

$$= 20 \times 1.732 \times 0.7813$$

$$= 27.064$$

∴ The height of the lamp post = CE
CE = BD = 60 – 27.064 = 32.93 m.

Question 4.
An aeroplane at an altitude of 1800 m finds that two boats are sailing towards it in the same direction. The angles of depression of the boats as observed from the aeroplane are 60° and 30° respectively. Find the distance between the two boats. ($\sqrt{3}$ = 1.732)
Solution:



In the figure

$$\tan C = \tan 60° = \sqrt{3} \quad \text{... (1)}$$

$$\tan D = \tan 30° = \frac{1}{\sqrt{3}} \quad \text{... (2)}$$

(1), (2) gives, $\tan 60° = \sqrt{3}$,

$$\frac{1800}{x} = \sqrt{3}$$

$$\sqrt{3}\,x = 1800$$

$$x = \frac{1800 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$$

$$= \frac{1800\sqrt{3}}{3} = 600\sqrt{3} \quad \text{...(3)}$$

$$\tan 30° = \frac{1}{\sqrt{3}} = \frac{1800}{d+x}$$

$$d + x = 1800\sqrt{3}$$

$$d + 600\sqrt{3} = 1800\sqrt{3}$$

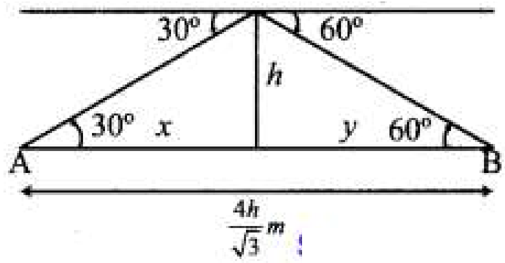
$$d = 1800\sqrt{3} - 600\sqrt{3}$$

$$= 1200\sqrt{3} .$$

Distance between the boats = $1200\sqrt{3}$ m
= 2078.4 m

Question 5.
From the top of a lighthouse, the angle of depression of two ships on the opposite sides of it are observed to be 30° and 60°. If the height of the lighthouse is h meters and the line joining the ships

passes through the foot of the lighthouse, show that the distance between the ships is $\frac{4h}{\sqrt{3}}$ m.
Solution:



$$\tan 30° = \frac{h}{x} = \frac{1}{\sqrt{3}}$$

$$x = \sqrt{3}\,h$$

$$\tan 60° = \frac{h}{y} = \sqrt{3}$$

$$\sqrt{3}\,y = h$$

$$y = \frac{h}{\sqrt{3}}$$

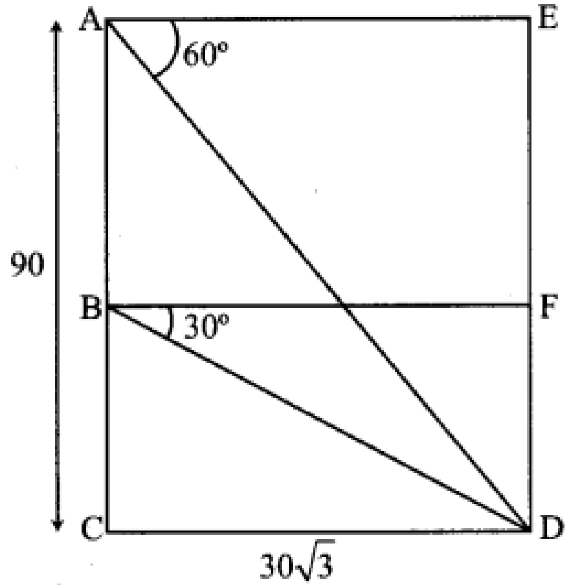
$\therefore$ The distance between the ships A & B $= x + y$

$$= \sqrt{3}\,h + \frac{h}{\sqrt{3}}$$

$$= \frac{3h+h}{\sqrt{3}} = \frac{4h}{\sqrt{3}} \text{ m.}$$

It is proved.

Question 6.
A lift in a building of height 90 feet with transparent glass walls is descending from the top of the building. At the top of the building, the angle of depression to a fountain in the garden is 60°. Two minutes later, the angle of depression reduces to 30°. If the fountain is $30\sqrt{3}$ feet from the entrance of the lift, find the speed of the lift which is descending.
Solution:



$\therefore \quad AB = \text{speed} \times 2 \qquad (D = st)$

In ΔAED, $\tan 60^\circ = \dfrac{DE}{AE}$

$$= \frac{DE}{30\sqrt{3}} = \sqrt{3} \Rightarrow DE = 90 \text{ ft.}$$

In ΔBFD, $\tan 30^\circ = \dfrac{DF}{BF} = \dfrac{1}{\sqrt{3}}$

$$\frac{DF}{30\sqrt{3}} = \frac{1}{\sqrt{3}} \Rightarrow DF = 30 \text{ ft}$$

$\therefore EF = DE - DF = 90 - 30 = 60$ ft.

$\therefore AB = EF = 60$ ft

$$\text{Speed} = \frac{\text{Distance}}{\text{time}} = \frac{60}{2} \text{ ft/min}$$

$$= 30 \text{ ft/min}$$

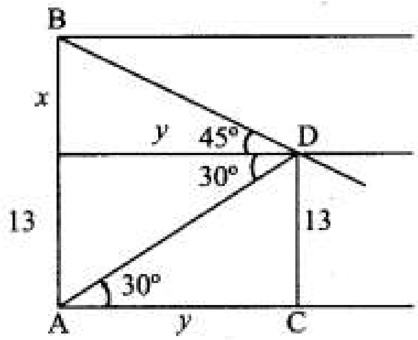
$$= \frac{30 \times .305}{60} m/\text{sec}$$

$$= .15 \text{ m/sec } [\because 1 \text{ foot} = .305 \text{ m}]$$

Ex 6.4

Question 1.
From the top of a tree of height 13 m the angle of elevation and depression of the top and bottom of another tree are 45° and 30° respectively. Find the height of the second tree. ($\sqrt{3}$ = 1.732)

Solution:



In the figure, AB is the 2nd tree.

$$\tan 45^\circ = \frac{x}{y} = 1$$

$$x = y$$

$$\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{13}{y}$$

$$y = 13\sqrt{3} = x.$$

$\therefore$ The height of second tree is $13 + x$

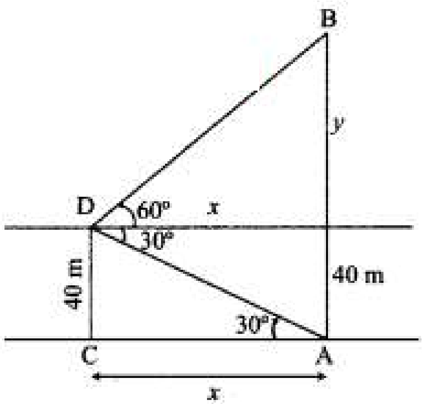
$$= 13 + 13\sqrt{3}$$

$$= 13(1 + \sqrt{3})$$

$$= 13(1 + 1.732) = 13 \times 2.732$$

$$= 35.52 \text{ m.}$$

Question 2.
A man is standing on the deck of a ship, which is 40 m above water level. He observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of the hill as 30°. Calculate the distance of the hill from the ship and the height of the hill. ($\sqrt{3}$ = 1.732)
Solution:



$$\tan 60^\circ = \frac{y}{x} = \sqrt{3}$$

$$y = \sqrt{3}\,x \qquad ..... (1)$$

$$\tan 30^\circ = \frac{40}{x} = \frac{1}{\sqrt{3}}$$

$$x = 40\sqrt{3}$$

$$\therefore \quad y = \sqrt{3}\,x$$

$$= \sqrt{3} \times 40\sqrt{3} = 120 \text{ m}$$

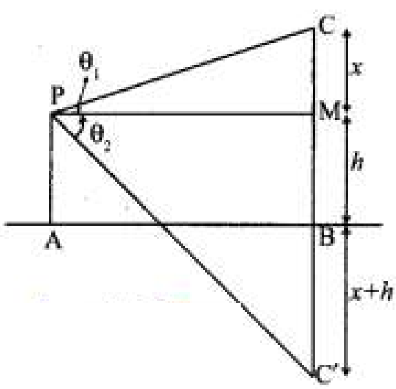
∴ The height of the hill = 120 + 40 = 160 m

The distance of the hill from the ship is AC = x = $40\sqrt{3}$ m = 69.28 m

Question 3.

If the angle of elevation of a cloud from a point 'h' metres above a lake is θ_1 and the angle of depression of its reflection in the lake is θ_2. Prove that the height that the cloud is located from the ground is $\frac{h(\tan\theta_1+\tan\theta_2)}{\tan\theta_2-\tan\theta_1}$

Solution:



Let AB be the surface of the lake and let p be the point of observation such that AP = h meters.

Let C be the position of the cloud and C' be its reflection in the lake. Then CB = C'B.
Let PM be $\perp^r$ from P on CB
Then $\angle CPM = \theta_1$, and $\angle MPC = \theta_2$
Let CM = x.
Then CB = CM + MB = CM + PA
= x + h

In ΔCPM, we have, $\tan\theta_1 = \dfrac{CM}{PM}$

$$\Rightarrow \tan\theta_1 = \frac{x}{AB}$$

$$\Rightarrow AB = x\cot\theta_1 \qquad \text{..... (1)}$$

$$(\because PM = AB)$$

In $\Delta PMC'$, we have

$$\tan\theta_2 = \frac{C'M}{PM}$$

$$\Rightarrow \tan\theta_2 = \frac{x+2h}{AB}$$

$$(\because C'M = C'B + BM = x + h + h)$$

$$\Rightarrow AB = (x + 2h)\cot\theta_2 \qquad \text{..... (2)}$$

From (1) & (2), we have

$$x\cot\theta_1 = (x + 2h)\cot\theta_2$$

On equating the values of AB.

$$\Rightarrow x(\cot\theta_1 - \cot\theta_2) = 2h\cot\theta_2$$

$$\Rightarrow x\left(\frac{1}{\tan\theta_1} - \frac{1}{\tan\theta_2}\right) = \frac{2h}{\tan\theta_2}$$

$$\Rightarrow x\left(\frac{\tan\theta_2 - \tan\theta_1}{\tan\theta_1 \cdot \tan\cancel{\theta_2}}\right) = \frac{2h}{\tan\cancel{\theta_2}}$$

$$\Rightarrow \qquad x = \frac{2h\tan\theta_1}{\tan\theta_2 - \tan\theta_1}$$

Hence the height CB of the cloud is given by

$CB = x + h$

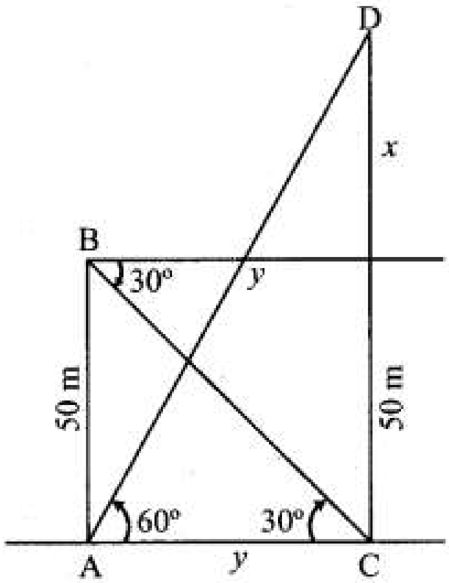
$$\Rightarrow \qquad CB = \frac{2h\tan\theta_1}{\tan\theta_2 - \tan\theta_1} + h$$

$$\Rightarrow \qquad CB = \frac{2h\tan\theta_1 + h(\tan\theta_2 - \tan\theta_1)}{\tan\theta_2 - \tan\theta_1}$$

$$CB = \frac{h(\tan\theta_1 + \tan\theta_2)}{\tan\theta_2 - \tan\theta_1}.$$

Hence proved

Question 4.
The angle of elevation of the top of a cell phone tower from the foot of a high apartment is 60° and the angle of depression of the foot of the tower from the top of the apartment is 30°. If the height of the apartment is 50 m, find the height of the cell phone tower. According to radiations control norms, the minimum height of a cell phone tower should be 120 m. State if the height of the above mentioned cell phone tower meets the radiation norms.
Solution:



In the figure AB is the building CD is the cell phone tower.

$$\tan 60^\circ = \frac{50+x}{y} = \sqrt{3}$$

$$50 + x = y\sqrt{3}$$

$$x = y\sqrt{3} - 50 \quad \text{...(1)}$$

$$\tan 30^\circ = \frac{50}{y} = \frac{1}{\sqrt{3}}$$

$$y = 50\sqrt{3} \quad \text{...(2)}$$

Substitute $y = 50\sqrt{3}$ in (1)

$$x = 50\sqrt{3} \times \sqrt{3} - 50$$

$$= 150 - 50 = 100 \text{ m}$$

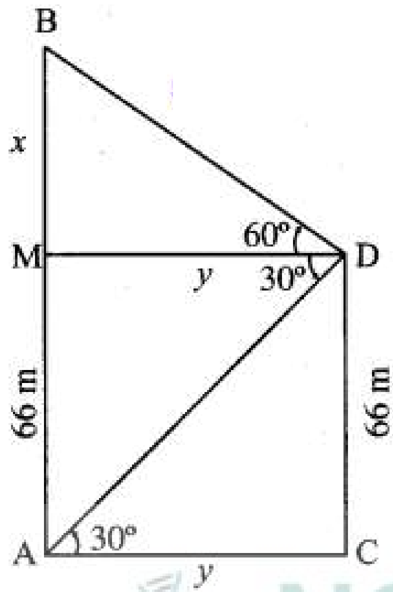
$\therefore$ The height of tower $= 50 + x = 50 + 100$

$= 150$ m.

Since 150m > 120m, yes the height of the above mentioned tower meet the radiation norms.

Question 5.
The angles of elevation and depression of the top and bottom of a lamp post from the top of a 66 m

high apartment are 600 and 30° respectively. Find
(i) The height of the lamp post.
(ii) The difference between height of the lamp post and the apartment.
(iii) The distance between the lamp post and the apartment. ($\sqrt{3}$ = 1.732)
Solution:



Let AB – Lamp post; CD – Apartment

In the figure

$$\tan 60° = \frac{x}{y} = \sqrt{3}$$

$$x = \sqrt{3}\,y$$

$$\tan 30^\circ = \frac{66}{y} = \frac{1}{\sqrt{3}}$$

$$y = 66\sqrt{3}$$

$$\therefore \quad x = \sqrt{3} \times 66\sqrt{3}$$

$$(\because y = 66\sqrt{3})$$

$$= 66 \times 3 = 198 \text{ m}$$

(i) The height of the Lamp post is $= 66 + x$

$$= 66 + 198$$

$$AB = 264 \text{ m}$$

(ii) The difference between the height of the Lamp post and the apartment is

$$BM = AB - AM = 264 - 66$$

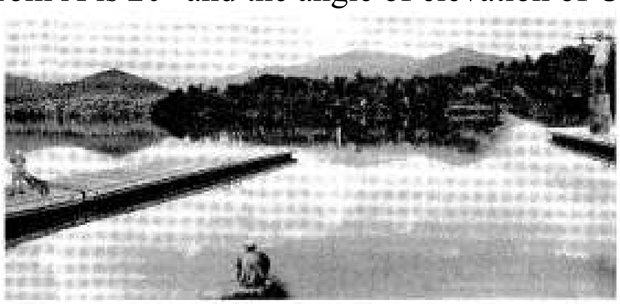
$$= 198 \text{ m} \quad (\because CD = AM)$$

(iii) The distance between the Lamp post and the apartment

$$y = 66\sqrt{3}$$

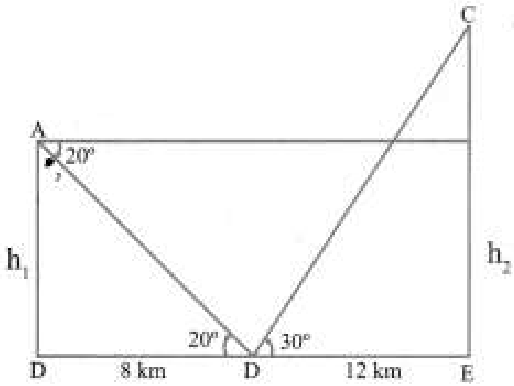
= 114.312 m

Question 6.
Three villagers A, B and C can see each T other across a valley. The horizontal distance between A and B is 8 km and the horizontal distance between B and C is 12 km. The angle of depression of B from A is 20° and the angle of elevation of C from B is 30° . Calculate :



(i) the vertical height between A and B.
(ii) the vertical height between B and C. ($\tan 20^\circ = 0.3640$, $\sqrt{3} = 1.732$)

Solution:



(i) Vertical height between A and B.

$$\tan 20^\circ = \frac{AD}{8}$$

$$AD = 8 \tan 20^\circ.$$

$$\simeq 2.91 \simeq 3 \text{ km}.$$

(ii) The vertical height between B and C.

$$\tan 30^\circ = \frac{CE}{BE}$$

$$CE = \tan 30\ BE.$$

$$= \frac{1}{\sqrt{3}} \times 12 = \frac{12 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$$

$$= 4\sqrt{3}$$

$$= 4 \times 1.732$$

$$= 6.93 \text{ km}$$

Ex 6.5

Multiple choice questions.
Question 1.
The value of $\sin^2\theta + \frac{1}{1+\tan^2\theta}$ is equal to
(1) $\tan^2\theta$
(2) 1
(3) $\cot^2\theta$
(4) 0
Solution:
(2) 1
Hint:

$$\sin^2\theta + \frac{1}{1+\frac{\sin^2\theta}{\cos^2\theta}}$$

$$= \sin^2\theta + \frac{1}{\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}} = \sin^2\theta + \frac{\cos^2\theta}{1}$$

$$= 1$$

Question 2.
$\tan\theta \operatorname{cosec}^2\theta - \tan\theta$ is equal to
(1) $\sec\theta$
(2) $\cot^2\theta$
(3) $\sin\theta$
(4) $\cot\theta$

Solution:
(4) $\cot\theta$
Hint:

$$\frac{\cancel{\sin\theta}}{\cos\theta}\times\frac{1}{\cancel{\sin^2\theta}}-\frac{\sin\theta}{\cos\theta}$$

$$=\frac{1}{\sin\theta\cos\theta}-\frac{\sin\theta}{\cos\theta}$$

$$=\frac{1-\sin^2\theta}{\sin\theta\cos\theta}=\frac{\cancel{\cos^2\theta}}{\sin\theta\cancel{\cos\theta}}=\cot\theta$$

Question 3.
If $(\sin\alpha + \operatorname{cosec}\alpha)^2 + (\cos\alpha + \sec\alpha)^2 = k + \tan^2\alpha + \cot^2\alpha$, then the value of k is equal to
(1) 9
(2) 7
(3) 5
(4) 3
Solution:
(2) 7
Hint:

$(\sin\alpha + \operatorname{cosec}\alpha)^2 + (\cos\alpha + \sec\alpha)^2 = k + \tan^2\alpha + \cot^2\alpha$
$\sin^2\alpha + \operatorname{cosec}^2\alpha + 2\sin\alpha\operatorname{cosec}\alpha + \cos^2\alpha + \sec^2\alpha + 2\cos\alpha\sec\alpha = k + \tan^2\alpha + \cot^2\alpha$
$\sin^2\alpha + \cos^2\alpha + \operatorname{cosec}^2\alpha + \sec^2\alpha + 2\sin\alpha \times \frac{1}{\sin\alpha} + 2\cos\alpha \times \frac{1}{\cos\alpha} = k + \tan^2\alpha + \cot^2\alpha$
$1 + 1 + \cot^2\alpha + 1 + \tan^2\alpha + 2 + 2 = k + \tan^2\alpha + \cot^2\alpha$
$7 + \cot^2\alpha + \tan^2\alpha = k + \tan^2\alpha + \cot^2\alpha$
$\therefore k = 7$

Question 4.
If $\sin\theta + \cos\theta = a$ and $\sec\theta + \operatorname{cosec}\theta = b$, then the value of $b(a^2 - 1)$ is equal to
(1) 2a
(2) 3a
(3) 0
(4) 2ab
Solution:
(1) 2a
$a = \sin\theta + \cos\theta$

b = sec θ + cosec θ

$$b(a^2 - 1) = \sec\theta + \text{cosec}\,\theta\,[(\sin\theta + \cos\theta)^2 - 1]$$

$$= \sec\theta + \text{cosec}\,\theta$$

$$[\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1]$$

$$= (\sec\theta + \text{cosec}\,\theta)[2\sin\theta\cos\theta]$$

$$= 2\sin\theta\ \cancel{\cos\theta}\cdot\frac{1}{\cancel{\cos\theta}} + 2\,\cancel{\sin\theta}\,\cos\theta \times \cancel{\sin\theta}$$

$$= 2\sin\theta + 2\cos\theta$$

$$= 2(\sin\theta + \cos\theta)$$

$$= 2a$$

Question 5.

If 3x = sec θ and $\frac{5}{x}$ = tan θ, then $x^2 - \frac{1}{x^2}$ is

(1) 25

(2) $\frac{1}{25}$

(3) 5

(4) 1

Solution:

$$5x = \sec\theta, \quad \frac{5}{x} = \tan\theta$$

$$x^2 - \frac{1}{x^2} = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$$

$$= \left(\frac{\sec\theta}{5} + \frac{\tan\theta}{5}\right)\left(\frac{\sec\theta}{5} - \frac{\tan\theta}{5}\right)$$

$$= \left(\frac{\sec\theta + \tan\theta}{5}\right)\left(\frac{\sec\theta - \tan\theta}{5}\right)$$

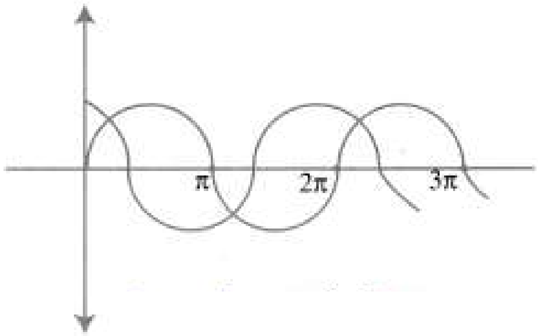
$$= \frac{\sec^2\theta - \tan^2\theta}{25} = \frac{\sec^2\theta - (\sec^2\theta - 1)}{25}$$

$$= \frac{\sec^2\theta - \sec^2\theta + 1}{25}$$

$$= \frac{1}{25}$$

Question 6.
If $\sin\theta = \cos\theta$, then $2\tan^2\theta + \sin^2\theta - 1$ is equal to
(1) $\frac{-3}{2}$
(2) $\frac{3}{2}$
(3) $\frac{2}{3}$
(4) $\frac{-2}{3}$
Solution:
(2) $\frac{3}{2}$

Hint:



$$\text{If } \sin\theta = \cos\theta$$

$$\Rightarrow \quad \theta = \frac{\pi}{4} \text{ or } 45°$$

$$2\tan^2\theta + \sin^2\theta - 1 = 2(1) + \frac{1}{2} - 1$$

$$= \frac{3}{2}$$

Question 7.
If x = a tan θ and y = b sec θ then

(1) $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$ (2) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

(3) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (4) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$

Solution:
(1) $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$

Hint:

$$x = a\tan\theta \Rightarrow \frac{x}{a} = \tan\theta$$

$$y = b\sec\theta \Rightarrow \frac{y}{b} = \sec\theta$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \tan^2\theta - \sec^2\theta$$

$$= \cancel{\sec^2\theta} - 1 - \cancel{\sec^2\theta}$$

$$\Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

$$\Rightarrow \frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$$

Question 8.
(1 + tan θ + sec θ) (1 + cot θ – cosec θ) is equal to
(1) 0
(2) 1
(3) 2
(4) -1
Solution:
(3) 2

Hint:

$$(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta)$$

$$= \left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)\left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)$$

$$= \left(\frac{\cos\theta + \sin\theta + 1}{\cos\theta}\right)\left(\frac{\sin\theta + \cos\theta - 1}{\sin\theta}\right)$$

$$= \frac{((\cos\theta + \sin\theta) + 1)((\sin\theta + \cos\theta) - 1)}{\sin\theta\cos\theta}$$

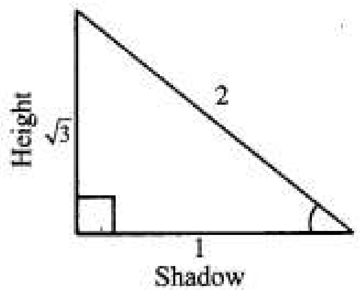
$$= \frac{(\cos\theta + \sin\theta)^2 - 1^2}{\sin\theta\cos\theta}$$

$$= \frac{\cos^2\theta + 2\cos\theta\sin\theta + \sin^2\theta - 1}{\sin\theta\cos\theta}$$

$$= \frac{2\cos\theta\sin\theta}{\cos\theta\sin\theta} = 2$$

Question 9.
a $\cot\theta$ + b $\operatorname{cosec}\theta$ = p and b $\cot\theta$ + a $\operatorname{cosec}\theta$ = q then $p^2 - q^2$ is equal to …………
(1) $a^2 - b^2$
(2) $b^2 - a^2$
(3) $a^2 + b^2$
(4) $b - a$
Answer:
(2) $b^2 - a^2$
Hint:
$p^2 - q^2 = (p + q)(p - q)$
$= (a\cot\theta + b\operatorname{cosec}\theta + b\cot\theta + a\operatorname{cosec}\theta)(a\cot\theta + b\operatorname{cosec}\theta - b\cot\theta - a\operatorname{cosec}\theta)$
$= [\cot\theta(a + b) + \operatorname{cosec}\theta(a + b)][\cot\theta(a - b) + \operatorname{cosec}\theta(b - a)]$
$= (a + b)[\cot\theta + \operatorname{cosec}\theta](a - b)[\operatorname{cosec}\theta(a - b)]$
$= (a + b)[\cot\theta + \operatorname{cosec}\theta](a - b)[\cot\theta - \operatorname{cosec}\theta]$
$= (a + b)(a - b)(\cot^2\theta - \operatorname{cosec}^2\theta)$
$= (a^2 - b^2)(-1) = -(a^2 - b^2)$
$p^2 - q^2 = b^2 - a^2.$

Question 10.
If the ratio of the height of a tower and the length of its shadow is $\sqrt{3}$: 1, then the angle of elevation of the sun has measure.
(1) 45°
(2) 30°
(3) 90°
(4) 60°
Solution:
(4) 60°
Hint:



$$\tan 60° = \frac{\sqrt{3}}{1}$$

Question 11.
The electric pole subtends an angle of 30° at a point on the same level as its foot. At a second point 'b' metres above the first, the depression of the foot of the tower is 60°. The height of the tower (in metres) is equal to

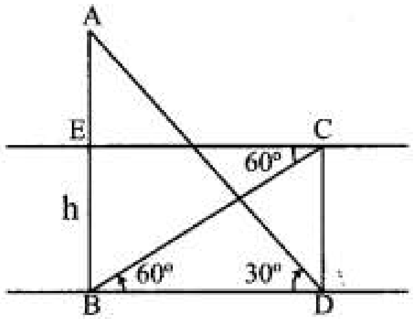
(1) $\sqrt{3}\,b$ (2) $\frac{b}{3}$

(3) $\frac{b}{2}$ (4) $\frac{b}{\sqrt{3}}$

Solution:
(2) $\frac{b}{3}$
Hint:

AB – Pole

In Δ BEC



$$\tan 60° = \frac{BE}{EC} = \sqrt{3}$$

$$\Rightarrow \frac{b}{EC} = \sqrt{3}$$

$$EC = \frac{b}{\sqrt{3}}$$

$$EC = BD = \frac{b}{\sqrt{3}}$$

$$\text{In } \Delta ABD, \tan 30° = \frac{AB}{BD} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{h}{\frac{b}{\sqrt{3}}} = \frac{1}{\sqrt{3}}$$

$$h = \frac{\frac{b}{\sqrt{3}}}{\sqrt{3}} = \frac{b}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{b}{3}$$

Question 12.
A tower is 60 m height. Its shadow is x metres shorter when the sun's altitude is 45° than when it has been 30°, then x is equal to
(1) 41.92 m
(2) 43.92 m
(3) 43 m
(4) 45.6 m°
Solution:
(2) 43.92 m

Hint:

$$\tan 45° = 1 = \frac{60}{y}$$

$$y = 60 \text{ m}$$

$$\tan 30° = \frac{60}{x+y} = \frac{1}{\sqrt{3}}$$

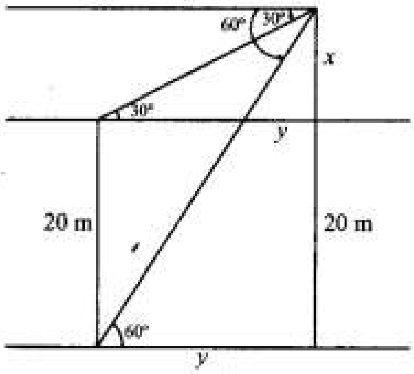
$$60 + x = 60\sqrt{3}$$

$$x = 60\sqrt{3} - 60$$

$$x = 60(\sqrt{3} - 1)$$

$$= 43.92 \text{ m}$$

Question 13.
The angle of depression of the top and bottom of 20 m tall building from the top of a multistoried building are 30° and 60° respectively. The height of the multistoried building and the distance between two buildings (in metres) is
(1) 20, $10\sqrt{3}$
(2) 30, $5\sqrt{3}$
(3) 20, 10
(4) 30, $10\sqrt{3}$
Solution:
(4) 30, $10\sqrt{3}$
Hint:



$$\tan 30° = \frac{x}{y} = \frac{1}{\sqrt{3}}$$

$$\sqrt{3}x = y$$

$$x = \frac{y}{\sqrt{3}} \qquad \text{..... (1)}$$

$$\tan 60° = \frac{20+x}{y} = \sqrt{3}$$

$$20 + x = y\sqrt{3}$$

$$x = y\sqrt{3} - 20 \qquad \text{..... (2)}$$

$$(1) = (2) \Rightarrow \frac{y}{\sqrt{3}} = y\sqrt{3} - 20$$

$$y = y\sqrt{3}\sqrt{3} - 20\sqrt{3}$$

$$y = 3y - 20\sqrt{3}$$

$$\cancel{2}y = {}^{10}\cancel{20}\sqrt{3}$$

$$= 17.32$$

$$x = \frac{17.32\times100}{17.332\times100} = \frac{17320}{1732}$$

$$= 10 \text{ m}$$

∴ Height of tower = 20 + 10 = 30 m

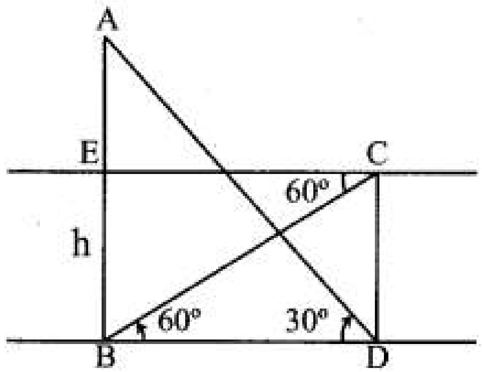
distance = 17.32 m = $10\sqrt{3}$

Question 14.
Two persons are standing 'x' metres apart from each other and the height of the first person is double that of the other. If from the middle point of the line joining their feet an observer finds the angular elevations of their tops to be complementary, then the height of the shorter person (in metres) is

(1) $\sqrt{2}x$ (2) $\frac{x}{2\sqrt{2}}$

(3) $\frac{x}{\sqrt{2}}$ (4) $2x$

Solution:
(2) $\frac{x}{2\sqrt{2}}$
Hint:



In ΔAEB, $\tan\theta = \dfrac{2h}{\frac{x}{2}} = \dfrac{4h}{x}$ ..... (1)

In ΔCED, $\tan(90-\theta) = \dfrac{\frac{h}{x}}{2} = \dfrac{2h}{x}$

$\tan(90-\theta) = \cot\theta = \dfrac{2h}{x}$, $\tan\theta = \dfrac{x}{2h}$ ..... (2)

Equation (1) and (2)

$$\frac{4h}{x} = \frac{x}{2h} \Rightarrow 8h^2 = x^2$$

$$\therefore \quad h^2 = \frac{x^2}{8} \Rightarrow \quad h = \frac{x}{\sqrt{8}}$$

$$h = \frac{x}{2\sqrt{2}}$$

Question 15.
The angle of elevation of a cloud from a point h metres above a lake is β. The angle of depression of its reflection in the lake is 45°. The height of location of the cloud from the lake is

(1) $\frac{h(1+\tan\beta)}{1-\tan\beta}$ (2) $\frac{h(1-\tan\beta)}{1+\tan\beta}$

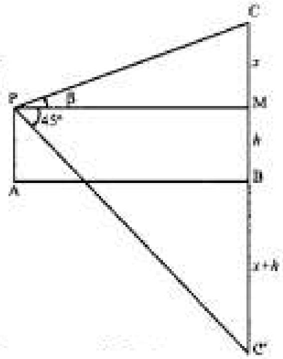
(3) $h\tan(45°-\beta)$ (4) none of these

Solution:

(1) $\frac{h(1+\tan\beta)}{1-\tan\beta}$

Hint:

In CPM, $\tan\beta = \dfrac{x}{AM} = \dfrac{x}{AB}$



$\Rightarrow \quad AB = x\cot\beta.$

In $\Delta PMC'$

$$\tan 45° = \frac{x+2h}{PM}$$

$$= \frac{x+2h}{AB}$$

$$AB = (x+2h)\cot 45° \qquad ...(2)$$

From (1) & (2)

$$\Rightarrow \quad x\cot\beta = (x+2h)\cot 45°$$

$$\Rightarrow x\left(\frac{1}{\tan\beta} - \frac{1}{\tan 45°}\right) = \frac{2h}{\tan 45°}$$

$$\Rightarrow x\left(\frac{\tan 45 - \tan\beta}{\tan\beta\tan 45}\right) = \frac{2h}{\tan 45°}$$

$$\Rightarrow \qquad x = \frac{2h\tan\beta}{1-\tan\beta}$$

$$CB = x+h = \frac{2h\tan\beta}{1-\tan\beta} + h$$

$$= \frac{2h\tan\beta}{1-\tan\beta} + h(1-\tan\beta)$$

$$= \frac{h+h\tan\beta}{1-\tan\beta}$$

$$= \frac{h(1+\tan\beta)}{1-\tan\beta} = 0$$

Unit Exercise 6

Question 1.
Prove that

(i) $\cot^2 A\left(\frac{\sec A - 1}{1+\sin A}\right) + \sec^2 A\left(\frac{\sin A - 1}{1+\sec A}\right) = 0$

(ii) $\frac{\tan^2 \theta - 1}{\tan^2 \theta + 1} = 1 - 2\cos^2\theta$

Solution:

(i) L.H.S. $= \cot^2 A\left(\frac{\sec A - 1}{1+\sin A}\right) + \sec^2 A\left(\frac{\sin A - 1}{1+\sec A}\right)$

$\Rightarrow$ L.H.S=

$$\frac{\cot^2 A(\sec A - 1)(\sec A + 1) + \sec^2 A \cdot (\sin A - 1)(1+\sin A)}{(1+\sin A)(1+\sec A)}$$

$$= \frac{\cot^2 A\left(\sec^2 A - 1\right) + \sec^2 A\left(\sin^2 A - 1\right)}{(1+\sin A)(1+\sec A)}$$

$$= \frac{\cot^2 A \tan^2 A + \sec^2 A\left(\sin^2 A - 1\right)}{(1+\sin A)(1+\sec A)}$$

$$= \frac{\cot^2 A \cdot \tan^2 A - \sec^2 A \cdot \left(1 - \sin^2 A\right)}{(1 + \sin A)(1 + \sec A)}$$

$$= \frac{\cot^2 A \ \tan^2 A - \sec^2 A \cdot \cos^2 A}{(1 + \sin A)(1 + \sec A)}$$

$$= \frac{1 - 1}{(1 + \sin A)(1 - \sec A)} = 0 = \text{RHS.}$$

(ii) $\text{L.H.S} = \dfrac{\tan^2\theta - 1}{\tan^2\theta + 1} = \dfrac{\dfrac{\sin^2\theta}{\cos^2\theta} - 1}{\dfrac{\sin^2\theta}{\cos^2\theta} + 1}$

$$= \frac{\dfrac{\sin^2\theta - \cos^2\theta}{\cancel{\cos^2\theta}}}{\dfrac{\sin^2\theta + \cos^2\theta}{\cancel{\cos^2\theta}}}$$

$$= \frac{\sin^2\theta - \cos^2\theta}{1}$$

$= 1 - \cos^2\theta - \cos^2\theta \quad (\because \sin^2\theta = 1 - \cos^2\theta)$
$= 1 - 2\cos^2\theta = \text{RHS.}$

Question 2.

Prove that $\left(\dfrac{1 + \sin\theta - \cos\theta}{1 + \sin\theta + \cos\theta}\right)^2 = \dfrac{1 - \cos\theta}{1 + \cos\theta}$

Solution:

$$\text{LHS} = \left(\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}\right)^2$$

$$= \frac{(1+\sin\theta)^2+\cos^2\theta-2(1+\sin\theta)(\cos\theta)}{(1+\sin\theta)^2+\cos^2\theta+2(1+\sin\theta)(\cos\theta)}$$

$$= \frac{1+2\sin\theta+\sin^2\theta+\cos^2\theta-2\cos\theta-2\sin\theta\cos\theta}{1+\sin^2\theta+2\sin\theta+\cos^2\theta+2\cos\theta+2\sin\theta\cos\theta}$$

$$= \frac{2+2\sin\theta-2\cos\theta-2\sin\theta\cos\theta}{2+2\sin\theta+2\cos\theta+2\sin\theta\cos\theta}$$

$$= \frac{1+\sin\theta-\cos\theta-\sin\theta\cos\theta}{1+\sin\theta+\cos\theta+\sin\theta\cos\theta} \quad (\because \text{dividing by 2})$$

$$= \frac{(1+\sin\theta)-\cos\theta(1+\sin\theta)}{(1+\sin\theta)+\cos\theta(1+\sin\theta)}$$

$$= \frac{(1-\cos\theta)\cancel{(1+\sin\theta)}}{(1+\cos\theta)\cancel{(1+\sin\theta)}} = \frac{1-\cos\theta}{1+\cos\theta} = \text{RHS.}$$

Hence proved

Question 3.
If $x \sin^3\theta + y \cos^3\theta = \sin\theta\cos\theta$ and $x \sin\theta = y \cos\theta$, then prove that $x^2 + y^2 = 1$.
Solution:
$x \sin^3\theta + y \cos^3\theta = \sin\theta \cos\theta$; $x \sin\theta\ y \cos\theta$.
$x (\sin\theta) [\sin^2\theta + \cos^2\theta] = \sin\theta \cos\theta$

$$(\text{as } x \sin\theta = y\cos\theta)$$

$$\therefore \quad x\,\cancel{\sin\theta} = \cancel{\sin\theta}\,\cos\theta$$

$$\Rightarrow \quad x = \cos\theta$$

$$y = \sin\theta$$

$$\therefore \quad x^2+y^2 = \sin^2\theta + \sin^2\theta = 1.$$

Question 4.
If a cos θ – b sin θ = c, then prove that (a sin θ + b cos θ) = $\pm\sqrt{a^2+b^2-c^2}$
Solution:

If $a \cos\theta - b \sin\theta = c$

$a \sin\theta + b \cos\theta = \pm \sqrt{a^2 + b^2 - c^2}$

L.H.S. we have

$(a \cos\theta - b \sin\theta)^2 + (a \sin\theta + b \cos\theta)^2$

$= a^2 \cos^2\theta + b^2 \sin^2\theta - 2ab \cancel{\cos\theta} \sin\theta$

$\quad + a^2 \sin^2\theta + b^2 \cos^2\theta + 2ab \cancel{\cos\theta} \sin\theta$

$= a^2 (\cos^2\theta + \sin^2\theta) + b^2 (\sin^2\theta + \cos^2\theta)$

$= a^2 + b^2$

$\therefore c^2 + (a \sin\theta + b \cos\theta)^2 = a^2 + b^2$

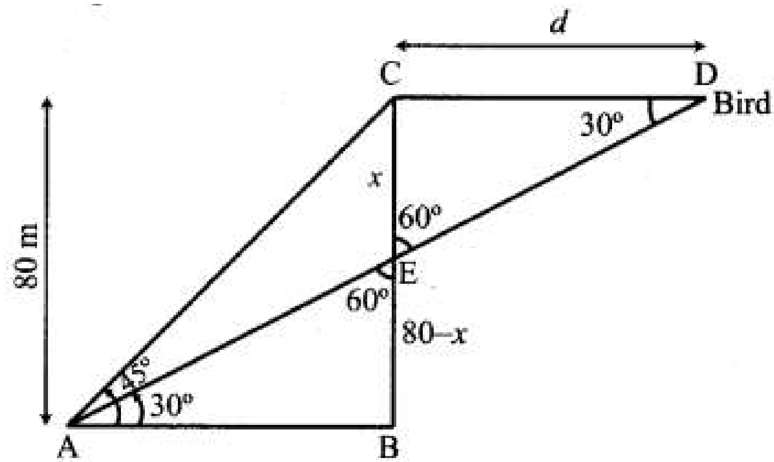
$(\because a \cos\theta - b \cos\theta = c)$

$\Rightarrow (a \sin\theta + b \cos\theta)^2 = a^2 + b^2 - c^2.$

$a \sin\theta + b \cos\theta = \pm \sqrt{a^2 + b^2 - c^2}$.

Hence Proved.

Question 5.
A bird is sitting on the top of a 80 m high tree. From a point on the ground, the angle of elevation of the bird is 45°. The bird flies away horizontally in such away that it remained at a constant height from the ground. After 2 seconds, the angle of elevation of the bird from the same point is 30°. Determine the speed at which the bird flies. ($\sqrt{3}$ = 1.732).
Solution:
Let s be the speed of the bird. In 2 seconds, the bird goes from C to D, it covers a distance 'd'



$\therefore d = s \times t = 2s.$

In ΔABC, $\tan 45° = \dfrac{BC}{AB} = 1.$

$\therefore \quad BC = AB,$

$\therefore \quad AB = 80$ m.

In ΔABE, $\tan 30° = \dfrac{BE}{AB} = \dfrac{80-x}{80} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow 80\sqrt{3} - \sqrt{3}x = 80$

$$-\sqrt{3}x = 80 - 80\sqrt{3}$$
$$\sqrt{3}x = 80(\sqrt{3} - 1)$$
$$\therefore \quad x = \frac{80(\sqrt{3}-1)}{\sqrt{3}}$$
$$\text{In } \Delta \text{ CDE, } \tan 30° = \frac{CE}{CD} = \frac{x}{d}.$$
$$\frac{1}{\cancel{\sqrt{3}}} = \frac{x}{d} = \frac{80(\sqrt{3}-1)}{\cancel{\sqrt{3}}\, d}$$
$$\Rightarrow \quad d = 80(\sqrt{3} - 1)$$
$$= 80(0.732)$$
$$= 58.56 \text{ m}$$
$$\therefore \quad \text{speed} = \frac{\text{distance}}{\text{time}}$$
$$= \frac{58.56 \text{ m}}{2 \text{ seconds}}$$
$$= 29.28 \text{ m/s.}$$

Question 6.
An aeroplane is flying parallel to the Earth's surface at a speed of 175 m/sec and at a height of 600 m. The angle of elevation of the aeroplane from a point on the Earth's surface is 37° at a given point. After what period of time does the angle of elevation increase to 53°? (tan 53° = 1.3270, tan 37° = 0.7536)
Solution:
Let Plane's initial position be A. Plane's final position = D Plane travels from A → D.

Let Plane's initial position be A. Plane's final position = D Plane travels from A → D.

In ΔABC,

$$\tan 37° = \frac{AC}{BC} = BC = \frac{AC}{\tan 37^{\circ}} = \frac{600}{\tan 37^{\circ}}$$

$$BC = BE + EC$$

$$EC = AD = \text{speed} \times \text{time}, \text{speed} = 175 \text{ m/sec}$$

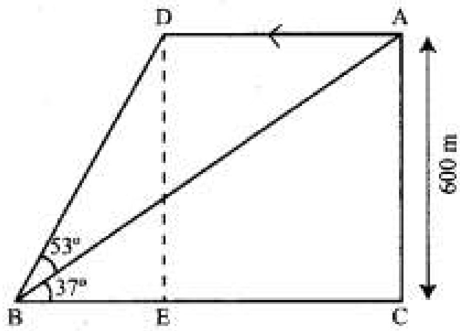
$$\therefore EC = 175 \text{ m/sec.}$$

$$BE = BC - EC = \frac{600}{\tan 37^{\circ}} - 175t \quad \dots (1)$$

In Δ BED,

$$\tan 53° = \frac{DE}{\tan 53^{\circ}}$$

$$BE = \frac{DE}{\tan 53^{\circ}} = \frac{600}{\tan 53^{\circ}} \quad \dots (2)$$



Equating (1) and (2)

$$\frac{600}{\tan 37^\circ} - 175\,t = \frac{600}{\tan 53^\circ}$$

$$\therefore \qquad 175t = 600\left(\frac{1}{\tan 37} - \frac{1}{\tan 53}\right)$$

$$= 600 \times \frac{\tan 53 - \tan 37}{\tan 37 \cdot \tan 53^\circ}$$

$$\tan 37^\circ = \tan(90 - 53^\circ) = \cot 53^\circ.$$

$$\therefore \qquad 175t = \frac{600(\tan 53 - \tan 37)}{1}$$

$$= 600\,(1.327 - 0.753)$$

$$175\,t = 600 \times 0.574$$

$$t = 1.96$$

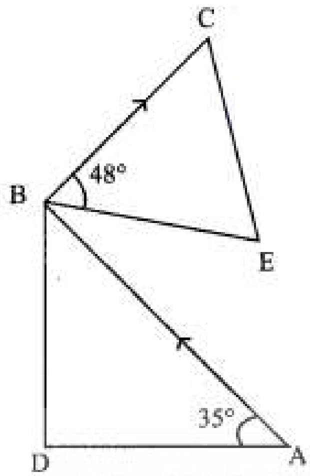
Question 7.
A bird is flying from A towards B at an angle of 35°, a point 30 km away from A. At B it changes its course of flight and heads towards C on a bearing of 48° and distance 32 km away.
(i) How far is B to the North of A?
(ii) How far is B to the West of A?
(iii) How far is C to the North of B?
(iv) How far is C to the East of B?
(sin 55° = 0.8192, cos 55° = 0.5736,
sin 42° = 0.6691, cos 42° = 0.7431)
Solution:

(i) In Δ ABD,

$$\frac{BD}{AB} = \sin 35°$$

$$\therefore BD = AB \sin 35° = 30 \sin 35°$$

$$= 30 \times 0.8192 = 24.58 \text{ km (approx.)}$$



(ii) $$\frac{AD}{AB} = \cos 35°$$

$$\therefore AD = AB \cos 35° = 30 \cos 35°$$

$$= 30 \times 0.5736 = 17.21 \text{ km (approx.)}$$

(iii) $$\frac{CE}{BC} = \sin 48°$$

$$\therefore CE = BC \sin 48° = 32 \sin 48°$$

$$= 32 \times 0.6691 = 21.41 \text{ km (approx.)}$$

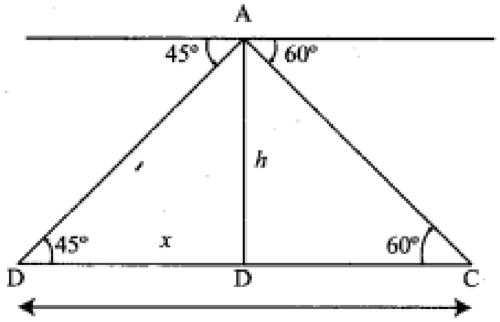
(iv) $$\frac{BE}{BC} = \cos 48°$$

$$\therefore BE = BC \cos 48° = 32 \cos 48°$$

$$= 32 \times 0.7431 = 23.78 \text{ km (approx.)}$$

Question 8.
Two ships are sailing in the sea on either side of the lighthouse. The angles of depression of two ships as observed from the top of the lighthouse are 60° and 45° respectively. If the distance between the ships is $200\left(\frac{\sqrt{3}+1}{\sqrt{3}}\right)$ metres, find the height of the lighthouse.

Solution:
From the figure AB – height of the light house = h CD – Distance between the ships



$$\tan 45° = \frac{h}{x} = 1$$

$$\Rightarrow h = x \qquad \ldots(1)$$

$$\tan 60° = \frac{h}{200\left(\frac{\sqrt{3}+1}{\sqrt{3}}\right) - x} = \sqrt{3}$$

$$h = \sqrt{3}\left(\frac{200(\sqrt{3}+1)}{\sqrt{3}} - x\right)$$

$$= \frac{\cancel{\sqrt{3}}\left(200(\sqrt{3}+1) - \sqrt{3}x\right)}{\cancel{\sqrt{3}}}$$

$$h = 200\sqrt{3} + 200 - \sqrt{3}\,x$$

$$h = 200\sqrt{3} + 200 - \sqrt{3}\,h \quad (\because h = x)$$

$$h + \sqrt{3}\,h = 200(\sqrt{3} + 1)$$

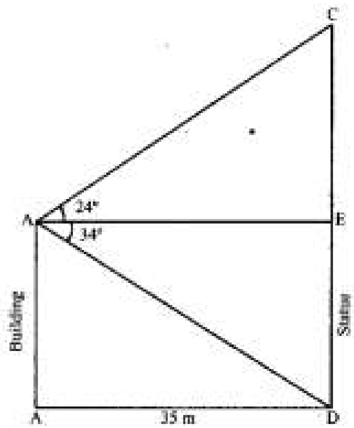
$$h\,\cancel{(1+\sqrt{3})} = 200\,\cancel{(\sqrt{3}+1)}$$

$$h = 200\ m.$$

∴ The height of the light house is 200 metres.

Question 9.
A building and a statue are in opposite side of a street from each other 35 m apart. From a point on the roof of building the angle of elevation of the top of statue is 24° and the angle of depression of base of the statue is 34°. Find the height of the statue.

(tan 24° = 0.4452, tan 34° = 0.6745)
Solution:



Let AB be the building & CD be statue.

In ΔACE,

$$\tan 24° = \frac{CE}{AE}$$

$$\therefore \quad CE = AE \tan 24°$$

$$= 35 \tan 24°$$

In ΔAED, $\tan 34° = \frac{DE}{AE}$

$$\therefore \quad DE = AE \tan 34°$$

$$= 35 \tan 34°$$

$\therefore$ Height of statue = CE + ED

$$= 35 \tan 24° + 35 \tan 34°$$

$$= 35 (\tan 24° + \tan 34°)=$$

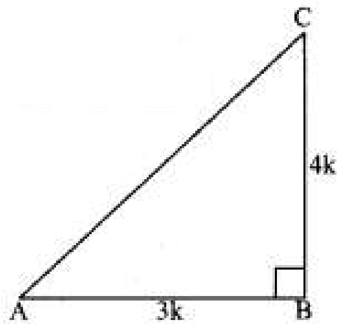
$$= 35 (0.4452 + 0.6745)$$

$$= 35 \times 1.1197$$

$$= 39.19 \text{ metres.}$$

Additional Questions

Question 1.
Given tan A = $\frac{4}{3}$, find the other trigonometric ratios of the angle A.
Solution:
Let us first draw a right ΔABC.
Now, we know that tan A = $\frac{BC}{AB} = \frac{4}{3}$
Therefore, if BC = 4k, then AB = 3k, where k is a positive number.



Now, by using the pythagoras theorem, we have
$AC^2 = AB^2 + BC^2$
$= (4k)^2 + (3k)^2 = 25\ k^2$
AC = 5k
So,

Now, we can write all the trigonometric ratios using their definitions.

$$\sin A = \frac{BC}{AC} = \frac{4k}{5k} = \frac{4}{5}$$

$$\cos A = \frac{AB}{AC} = \frac{3k}{5k} = \frac{3}{5}$$

$$\text{Therefore, } \cot A = \frac{1}{\tan A} = \frac{3}{4}$$

$$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{5}{4}, \text{ and}$$

$$\sec A = \frac{1}{\cos A} = \frac{5}{3}$$

Question 2.

Prove that $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1}{\sec\theta - \tan\theta}$, **using the identity** $\sec^2\theta = 1 + \tan^2\theta$.

Solution:
Since we will apply the identity involving sec θ and tan θ, let us first convert the LHS (of the identity we need to prove) in terms of sec θ and tan θ by dividing numerator and denominator by cos θ.

$$\text{LHS} = \frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta}$$

$$= \frac{(\tan\theta + \sec\theta) - 1}{(\tan\theta - \sec\theta) + 1}$$

$$= \frac{\{(\tan\theta + \sec\theta) - 1\}(\tan\theta - \sec\theta)}{\{(\tan\theta - \sec\theta) + 1\}(\tan\theta - \sec\theta)}$$

$$= \frac{(\tan^2\theta - \sec^2\theta) - (\tan\theta - \sec\theta)}{(\tan\theta - \sec\theta + 1)(\tan\theta - \sec\theta)}$$

$$= \frac{-1 - \tan\theta + \sec\theta}{(\tan\theta - \sec\theta + 1)(\tan\theta - \sec\theta)}$$

$$= \frac{-1}{\tan\theta - \sec\theta}$$

$$= \frac{1}{\sec\theta - \tan\theta}$$

Which is the RHS of the identity, we are required to prove.

Question 3.
Prove that sec A (1 – sin A) (sec A + tan A) = 1.
Solution:

LHS = sec A(1 – sin A)(sec A + tan A)

$$= \left[\frac{1}{\cos A}\right](1-\sin A)\left[\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right]$$

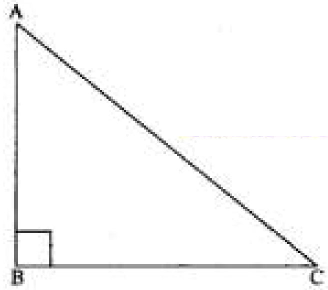
$$= \frac{(1-\sin A)(1+\sin A)}{\cos^2 A}$$

$$= \frac{1-\sin^2 A}{\cos^2 A}$$

$$= \frac{\cos^2 A}{\cos^2 A} = 1 = \text{RHS}$$

Question 4.
In a right triangle ABC, right-angled at B, if tan A = 1, then verify that 2 sin A cos A = 1.
Solution:
In ABC, tan A = $\frac{BC}{AB}$

Let AB = BC = k, where k is a positive number.



Now, $AC = \sqrt{AB^2 + BC^2}$

$= \sqrt{(k)^2 + (k)^2} = k\sqrt{2}$

Therefore,

$$\sin A = \frac{BC}{AC} = \frac{1}{\sqrt{2}} \text{ and}$$

$$\cos A = \frac{AB}{AC} = \frac{1}{\sqrt{2}}$$

So, $2 \sin A \cos A = 2\left[\frac{1}{\sqrt{2}}\right]\left[\frac{1}{\sqrt{2}}\right] = 1$, which is the required value.

Question 5.
If $\sin (A - B) = \frac{1}{2}$, $\cos (A + B) = \frac{1}{2}$, $0° < A + B \leq 90°$, $A > B$, find A and BC
Solution:

Since, $\sin(A - B) = \frac{1}{2}$, $\therefore A - B = 30^\circ$ ..... (1)

Also, since $\cos(A + B) = \frac{1}{2}$,

$\therefore \quad A + B = 60^\circ$ ..... (2)

Solving (1) and (2)

$$A - B + A + B = 30^\circ + 60^\circ$$
$$2A = 90^\circ$$
$$A = 45^\circ$$

We get,
$A = 45^\circ$ and $B = 15^\circ$

Question 6.
Express the ratios cos A, tan A and sec A in terms of sin A.
Solution:
Since
$\cos^2 A + \sin^2 A = 1$, therefore,
$\cos^2 A = 1 - \sin^2 A$
i.e., $\cos A = \pm\sqrt{1 - \sin^2 A}$
This gives $\cos A = \sqrt{1 - \sin^2 A}$

Hence, $\tan A = \frac{\sin A}{\cos A} = \frac{\sin A}{\sqrt{1 - \sin^2 A}}$

and $\sec A = \frac{1}{\cos A} = \frac{1}{\sqrt{1 - \sin^2 A}}$

Question 7.
Evaluate $\frac{\tan 65^\circ}{\cot 25^\circ}$
Solution:
We know:
$\cot A = \tan(90^\circ - A)$

So,

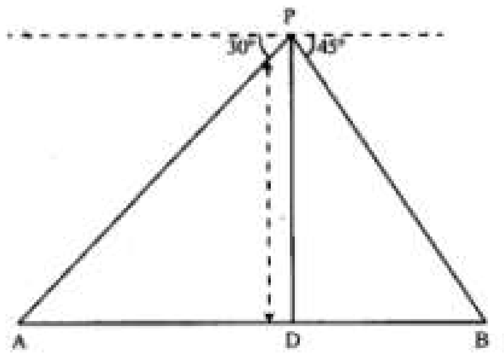
$$\cot 25^\circ = \tan(90^\circ - 25^\circ) = \tan 65^\circ$$

$$\frac{\tan 65^\circ}{\cot 25^\circ} = \frac{\tan 65^\circ}{\tan 65^\circ} = 1$$

Question 8.
Since sin 3A = cos(A – 26°), where 3A is an acute angle, find the value at A.
Solution:
We are given that sin 3A = cos (A – 26°) ….. (1)
Since sin 3A = cos(90° – 3A) we can write (1) as cos(90° – 3A) = cos(A – 26°)
Since 90° – 3A and A – 26° are both acute angles.
90° – 3A = A – 26°
which gives A = 29°

Question 9.
Express cot 85° + cos 75° in terms of trigonometric ratios of angles between 0° and 45°.
Solution:
cot 85° +cos 75°
= cot(90° – 5°) + cos(90° – 15°)
= tan 5° + sin 15°

Question 10.
From a point on a bridge across a river, the angles of depression of the banks on opposite sides at the river are 30° and 45°, respectively. If the bridge is at a height at 3 m from the banks, find the width at the river.
Solution:
A and B represent points on the bank on opposite sides at the river, so that AB is the width of the river. P is a point on the bridge at a height of 3m i.e., DP = 3 m. We are interested to determine the width at the river which is the length at the side AB of the ΔAPB.



$$\text{Now, } AB = AD + DB$$

In right ΔAPD,

$$\angle A = 30^\circ$$

So, $\tan 30^\circ = \dfrac{PD}{AD}$

i.e., $\dfrac{1}{\sqrt{3}} = \dfrac{3}{AD}$ (or) $AD = 3\sqrt{3}$ m

Also, in right ΔPBD,
$B = 45^\circ$
So, $BD = PD = 3$ m
Now, $AB = BD + AD$
$= 3 + 3\sqrt{3} = 3(1 + \sqrt{3})$m
Therefore, the width at the river is $3(\sqrt{3} + 1)$