Statistics and Probability

Ex 8.1

Question 1.
Find the range and coefficient of range of the following data.
(i) 63, 89, 98, 125, 79, 108, 117, 68
(ii) 43.5, 13.6, 18.9, 38.4, 61.4, 29.8
Solution:
Range R = L – S.

$$\text{Co-efficient of range} = \frac{L-S}{L+S}$$

L – Largest value,
S – Smallest value.

(i) 63, 89, 98, 125, 79, 108, 117, 68.
Here L = 125
S = 63
∴ R = L – S = 125 – 63 = 62

$$\text{Co-efficient of range} = \frac{L-S}{L+S}$$

$$= \frac{125-63}{125+63}$$

$$= \frac{62}{188} = 0.33$$

Question 2.
If the range and the smallest value of a set of data are 36.8 and 13.4 respectively, then find the largest value.
Answer:

Range = 36.8
Smallest value (S) = 13.4
Range = L – S
36.8 = L – 13.4
L = 36.8 + 13.4 = 50.2
Largest value = 50.2

Question 3.
Calculate the range of the following data.

Income	400-450	450-500	500-550	550-600	600-650
Number of workers	8	12	30	21	6

Solution:
Here the largest value = 650
The smallest value =400
∴ Range = L – S = 650 – 400
= 250

Question 4.
A teacher asked the students to complete 60 pages of a record notebook. Eight students have completed only 32, 35, 37, 30, 33, 36, 35 and 37 pages. Find the standard deviation of the pages yet to be completed by them.

Solution:

$$\frac{\Sigma x}{n} = \frac{275}{8} = 34.3$$

x	$d = x - \bar{x}$	d^2
32	–2.3	5.29
35	0.7	0.49
37	2.7	7.29
30	–4.3	18.49
33	–1.3	1.69
36	1.7	2.89
35	0.7	0.49
37	2.7	7.29
$\Sigma x = 275$		$\Sigma d^2 = 43.92$

Standard deviation:

$$\sigma = \sqrt{\frac{\Sigma d^2}{n}}$$

$$= \sqrt{\frac{43.92}{8}}$$

$$= 2.34$$

Question 5.
Find the variance and standard deviation of the wages of 9 workers given below:
 310, 290, 320, 280, 300, 290, 320, 310, 280.
Solution:

$$\overline{x} = \frac{\Sigma x}{n} = \frac{2700}{9} = 300$$

x	$d = x - \overline{x}$	d^2
310	10	100
290	–10	100
320	20	400
280	–20	400
300	0	0
290	–10	100
320	20	400
310	10	100
280	–20	400
$\Sigma x = 2700$	0	2000

$$\text{Variance} = \frac{\Sigma d^2}{n} = \frac{2000}{9} = 222.22$$

Standard deviation

$$\sigma = \sqrt{\frac{\Sigma d^2}{n}} = \sqrt{222.22} \cong 14.91$$

Question 6.
A wall clock strikes the bell once at 1 o'clock, 2 times at 2 o'clock, 3 times at 3 o'clock and so on. How many times will it strike in a particular day? Find the standard deviation of the number of strikes the bell make a day.
Solution:

x	$d = x - \bar{x}$	d^2
2	–11	121
4	–9	81
6	–7	49
8	–5	25
10	–3	9
12	–1	1
14	1	1
16	3	9
18	5	25
20	7	49
22	9	81
24	11	121
156		572

$$\bar{x} = \frac{\Sigma x}{n} = 13$$

$$\sigma = \sqrt{\frac{\Sigma d^2}{n}} = \sqrt{\frac{572}{12}} = \sqrt{47.66} \cong 6.9$$

Question 7.
Find the standard deviation of the first 21 natural numbers.
Solution:

$$\bar{x} = \frac{\sum x}{n} = \frac{231}{21} = 11$$

x	$d = x - \bar{x}$	d^2
1	–10	100
2	–9	81
3	–8	64
4	–7	49
5	–6	36
6	–5	25
7	–4	16
8	–3	9
9	–2	4
10	–1	1
11	0	0
12	1	1
13	2	4

14	3	9
15	4	16
16	5	25
17	6	36
18	7	49
19	8	64
20	9	81
21	10	100
231		770

Standard deviation

$$\sigma = \sqrt{\frac{\Sigma d^2}{n}}$$

$$= \sqrt{\frac{770}{21}}$$

$$= \sqrt{36.66}$$

$$\cong 6.05$$

Question 8.
If the standard deviation of a data is 4.5 and if each value of the data is decreased by 5, then find the new standard deviation.
Answer:
The standard deviation of the data = 4.5
Each data is decreased by 5
The new standard deviation = 4.5

Question 9.
If the standard deviation of a data is 3.6 and each value of the data is divided by 3, then find the new variance and new standard deviation.
Solution:
If the standard deviation of a data is 3.6, and each of the data is divided by 3 then the new standard deviation is also divided by 3.
∴ The new standard deviation = $\frac{3.6}{3}$
= 1.2

The new variance = (standard deviation)^2
$= \sigma^2 = 1.2^2 = 1.44$

Question 10.
The rainfall recorded in various places of five districts in a week are given below.

Rainfall (in mm)	45	50	55	60	65	70
Number of places	5	13	4	9	5	4

Find its standard deviation.
Solution:

Rainfall x_i (mm)	No. of places f_i	$f_i x_i$	$d = x - \bar{x}$	d_i^2	$f_i d_i^2$
45	5	225	–11	121	605
50	13	650	–6	36	468
55	4	220	–1	1	4
60	9	540	4	16	144
65	5	325	9	81	405
70	4	280	14	196	784
	N = 40	$\Sigma x f_i = 2240$		$\Sigma f_i d_i^2 = 2410$	

$$\text{mean, } \bar{x} = \frac{\Sigma x_i f_i}{N} = \frac{2240}{40} = 56$$

$\therefore$ Standard deviation

$$\sigma = \sqrt{\frac{\Sigma f_i d_i^2}{N}} = \sqrt{\frac{2410}{40}} = \sqrt{60.25} = 7.76$$

Question 11.
In a study about viral fever, the number of people affected in a town were noted as

Age in years	0 - 10	10-20	20-30	30-40	40-50	50-60	60-70
Number of people affected	3	5	16	18	12	7	4

Find its standard deviation.
Solution:
Let the assumed mean A = 35, C = 10

Age (X)	No. of people affected f_i	Mid (value) x_i	$d_i = x_i - A$	$d_i = \frac{x_i - A}{C}$	$f_i x_i$	d_i^2	$f_i d_i^2$
0–10	3	5	–30	–3	–9	9	27
10–20	5	15	–20	–2	–10	4	20
20–30	16	25	–10	–1	–16	1	16
30–40	18	35	0	0	0	0	0
40–50	12	45	10	1	12	1	12
50–60	7	55	20	2	14	4	28
60–70	4	65	30	3	12	9	36
	N = 65				$\Sigma x f_i = 3$		139

Standard deviation $\sigma = c \times \sqrt{\frac{\Sigma f_i d^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2}$

$= 10 \times \sqrt{\frac{139}{65} - \left(\frac{3}{65}\right)^2}$

$= 10 \times \sqrt{2.138 - (0.046)^2}$

$= 10 \times \sqrt{2.138 - 0.002116}$

$= 10 \times \sqrt{2.136}$

$= 10 \times 1.46$

$= 14.6$

Question 12.
The measurements of the diameters (in cms) of the plates prepared in a factory are given below. Find its standard deviation.

Diameter (cm)	21-24	25-28	29-32	33-36	37-40	41-44
Number of plates	15	18	20	16	8	7

Solution:
Assumed mean A = 30.5, C = 4

Diameter class interval X	Mid value x_i	f_i	$d_i = x_i - A$	$d_i = \frac{x_i - A}{C}$	$f_i d_i$	d_i^2	$f_i d_i^2$
20.5–24.5	22.5	15	–8	–2	–30	4	60
24.5–28.5	26.5	18	–4	–1	–18	1	18
28.5–32.5	30.5	20	0	0	0	0	0
32.5–36.5	34.5	16	4	1	16	1	16
36.5–40.5	38.5	8	8	2	16	4	32
40.5–44.5	42.5	7	12	3	21	9	63
		N = 84			5		189

Standard deviation $\sigma = c \times \sqrt{\frac{\Sigma f_i d^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2}$

$$= 4 \times \sqrt{\frac{189}{84} - \left(\frac{5}{84}\right)^2}$$

$$= 4 \times \sqrt{2.25 - (0.059)^2}$$

$$= 4 \times \sqrt{2.25 - 0.0035}$$

$$= 4 \times \sqrt{2.2465}$$

$$= 4 \times 1.498$$

$$\cong 5.99$$

$$= 6$$

Question 13.
The time taken by 50 students to complete a 100 meter race are given below. Find its standard deviation.

Time taken (sec)	8.5-9.5	9.5-10.5	10.5-11.5	11.5-12.5	12.5-13.5
Number of students	6	8	17	10	9

Solution:
Assumed mean A = 11, C = 1

Time Taken X	Mid value x_i	No. of Students f_i	$d_i = x_i - A$	$d_i = \frac{x_i - A}{C}$	$f_i d_i$	d_i^2	$f_i d_i^2$
8.5–9.5	9	6	–2	–2	–12	4	24
9.5–10.5	10	8	–1	–1	–8	1	8
10.5–11.5	11	17	0	0	0	0	0
11.5 – 12.5	12	10	1	1	10	1	10
12.5 – 13.5	13	9	2	2	18	4	36
		N = 50			8		78

Standard deviation $\sigma = c \times \sqrt{\frac{\Sigma f_i d^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2}$

$$= 1 \times \sqrt{\frac{78}{50} - \left(\frac{8}{50}\right)^2}$$

$$= 1 \times \sqrt{1.56 - (0.16)^2}$$

$$= 1 \times \sqrt{1.56 - 0.0256}$$

$$= 1 \times \sqrt{1.534}$$

$= 1 \times 1.213$
$= 1.213$
$= 1.2$

Question 14.
For a group of 100 candidates the mean and standard deviation of their marks were found to be 60 and 15 respectively. Later on it was found that the scores 45 and 72 were wrongly entered as 40 and 27. Find the correct mean and standard deviation.
Solution:

$$n = 100$$

$$\bar{x} = \frac{\Sigma x}{n}$$

$$\bar{x} = 60, \sigma = 15$$

$$\therefore \quad \Sigma x = \bar{x} \times n = 60 \times 100 = 6000$$

$$\text{Correct } \Sigma x = 6000 + 45 + 72 - 40 - 27$$

$$= 6117 - 67$$

$$\Sigma x = 6050$$

$$n = 100$$

$$\text{Correct } \bar{x} = \frac{\Sigma x}{n} = \frac{6050}{100} = 60.5$$

$$\text{Standard deviation } \sigma = \sqrt{\left(\frac{\Sigma x^2}{n}\right) - \left(\frac{\Sigma x}{n}\right)^2}$$

$$\text{Incorrect value of } \sigma = 15 = \sqrt{\frac{\Sigma x^2}{100} - 60^2}$$

$$225 = \frac{\Sigma x^2}{100} - 3600 \text{ gives,}$$

$$\frac{\Sigma x^2}{100} = 3825$$

$$\Sigma x^2 = 382500$$

$$\text{Correct value of } \Sigma x^2 = 382500 + 45^2 + 72^2 - 40^2 - 27^2$$

$$= 382500 + 2025 + 5184 - 1600 - 729$$

$$= 389709 - 2329$$

$$= 387380$$

Correct standard deviation

$$\sigma = \sqrt{\frac{387380}{100} - (60.5)^2}$$

$$= \sqrt{3873.80 - 3660.25}$$

$$= \sqrt{213.55}$$

$$= 14.61$$

Question 15.
The mean and variance of seven observations are 8 and 16 respectively. If five of these are 2, 4, 10, 12 and 14, then find the remaining two observations.

Solution:
$\bar{x} = 8$, $\sigma^2 = 16$, $n = 7$. If five of these are 2, 4, 10, 12, 14.

$$\bar{x} = \frac{\Sigma x}{n} = \frac{2+4+10+12+14+a+b}{7}$$

$$8 = \frac{42+a+b}{7}$$

$$42 + a + b = 56$$

$$a + b = 56 - 42 = 14$$

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The given 5 number are 2, 4, a, b, 10, 12, 14.
a, b are 6 and 8.

Ex 8.2

Question 1.
The standard deviation and coefficient of variation of a data are 1.2 and 25.6 respectively. Find the value of mean.
Solution:
Co-efficient of variation C.V. = $\text{C.V} = \frac{\sigma}{\bar{x}} \times 100$

Co-efficient of variation C.V = $\frac{\sigma}{\bar{x}} \times 100$

$$\sigma = 6.5, \bar{x} = 12.5$$

$$\therefore \quad \text{C.V} = \frac{6.5}{12.5} \times 100\%$$
$$= 52\%.$$

Question 2.
The standard deviation and coefficient of variation of a data are 1.2 and 25.6 respectively. Find the value of mean.
Solution:

$\sigma = 1.2$, C.V. = 25.6, $\bar{x} = ?$

$$\text{C.V} = \frac{\sigma}{\bar{x}} \times 100\% \Rightarrow \bar{x} = \frac{\sigma}{\text{C.V.}} \times 100\%$$

$$\bar{x} = \frac{1.2}{25.6} \times 100$$
$$= 4.687 = 4.69$$

Question 3.
If the mean and coefficient of variation of a data are 15 and 48 respectively, then find the value of standard deviation.
Solution:
$\bar{x} = 15$, C.V. = 48, $\sigma = ?$

$$C.V = \frac{\sigma}{\bar{x}} \times 100$$

$$\Rightarrow \quad \sigma = \frac{C.V. \times \bar{x}}{100} = \frac{48 \times 15}{100}$$

$$= 7.2$$

Question 4.
If n = 5 , $\overline{x} = 6$, $\Sigma x^2 = 765$, then calculate the coefficient of variation.
Solution:
$n = 5, \bar{x} = 6, \Sigma x^2 = 765$, C.V = ?

$$\sigma = \sqrt{\left(\frac{\Sigma x^2}{n}\right) - \left(\frac{\Sigma x}{n}\right)^2}$$

$$= \sqrt{\frac{765}{5} - 6^2}$$

$$= \sqrt{153 - 36}$$

$$= 10.82$$

$$C.V = \frac{10.82}{6} \times 100\%$$

$$= 180.28\%$$

Question 5.
Find the coefficient of variation of 24, 26, 33, 37, 29,31.
Solution:

x	$d = x - \bar{x}$	d^2
24	–6	36
26	–4	16
33	3	9
37	7	49
29	–1	1
31	1	1
180	$\Sigma d = 0$	112

$$\bar{x} = \frac{\Sigma x}{n}$$

$$= \frac{180}{6}$$

$$= 30$$

$$\sigma = \sqrt{\frac{\Sigma d^2}{n}}$$

$$= \sqrt{\frac{112}{6}}$$

$$= \sqrt{18.66}$$

$$= 4.32$$

$\therefore$ Co-efficient of variation $\text{C.V} = \frac{\sigma}{x} \times 100\%$

$$\text{C.V} = \frac{4.32}{30} \times 100\%$$

$$= 14.4\%$$

Question 6.
The time taken (in minutes) to complete a homework by 8 students in a day are given by 38, 40, 47, 44, 46, 43, 49, 53. Find the coefficient of variation.
Solution:

$$\text{Mean} = \bar{x} = \frac{\Sigma x}{n}$$

$$= \frac{38+40+47+44+46+43+49+53}{8}$$

$$= \frac{360}{8}$$

$$= 45$$

$$\sigma = \sqrt{\frac{\Sigma d^2}{n}} = \sqrt{\frac{164}{8}} = \sqrt{20.5}$$

$$= 4.5$$

Co-efficient of variation

$$\text{C.V.} = \frac{\sigma}{\bar{x}} \times 100 = \frac{4.5}{45} \times 100 = 10.07\%$$

Question 7.
The total marks scored by two students Sathya and Vidhya in 5 subjects are 460 and 480 with standard deviation 4.6 and 2.4 respectively. Who is more consistent in performance?
Solution:

x	$d = x - \bar{x}$	d^2
38	–7	49
40	–5	25
47	2	4
44	–1	1
46	1	1
43	–2	4
49	4	16
53	8	64
360	0	164

Sathya	**Vidhya**	
$\Sigma x_1 = 460$	$\Sigma x_2 = 460$	
$\sigma_1 = 4.6$	$\sigma_2 = 2.4$	
$\bar{x}_1 = \frac{\Sigma x_1}{n}$	$\bar{x}_2 = \frac{\Sigma x_2}{n}$	C.V of Vidhya is less than C.V. of Sathya.
$= \frac{460}{5}$	$= \frac{480}{5}$	
$= 92$	$= 96$	$\therefore$ Vidhya is more consistent.
$\therefore \text{C.V}_1 = \frac{4.6}{92} \times 100$	$\text{C.V}_2 = \frac{2.4}{96} \times 100$	
$= 5\%$	$= 2.5\%$	

Question 8.
The mean and standard deviation of marks obtained by 40 students of a class in three subjects Mathematics, Science and Social Science are given below.

Subject	Mean	SD
Mathematics	56	12
Science	65	14
Social Science	60	10

Which of the three subjects shows highest variation and which shows lowest variation in marks?
Solution:

	Maths	Science	Social Science
Mean	56	65	60
S.D	12	14	10
$C.V = \frac{\sigma}{x} \times 100\%$	21.4%	21.54% Highest value	16.67% lower variation

Science subject shows highest variation. Social science shows lowest variation.

Question 9.
The temperature of two cities A and B in a winter season are given below.

Temperature of city A (in degree Celsius)	18	20	22	24	26
Temperature of city B (in degree Celsius)	11	14	15	17	18

Find which city is more consistent in temperature changes?
Solution:

City A			City B		
x_1	$d_1 = x - \bar{x}$	d_1^2	x_1	$d = x - \bar{x}$	d_2^2
18	–4	16	11	–4	16
20	–2	4	14	–1	1
22	0	0	15	0	0
24	2	4	17	2	4
26	4	16	18	3	9
110	0	40	75		20

$$\bar{x}_1 = \frac{\Sigma x_1}{n} = \frac{110}{5} = 22$$

$$\sigma_1 = \sqrt{\frac{\Sigma d_1^2}{n}} = \sqrt{\frac{40}{5}} = \sqrt{8} = 2\sqrt{2}$$

$$\bar{x}_2 = \frac{\Sigma x_1}{n} = \frac{75}{5} = 15$$

$$\sigma_2 = \sqrt{\frac{\Sigma d_2^2}{n}} = \sqrt{\frac{20}{5}} = \sqrt{4}$$

$$CV_1 = \frac{\sigma_1}{x_1} \times 100 = \frac{2.83}{22} \times 100 = 12.86$$

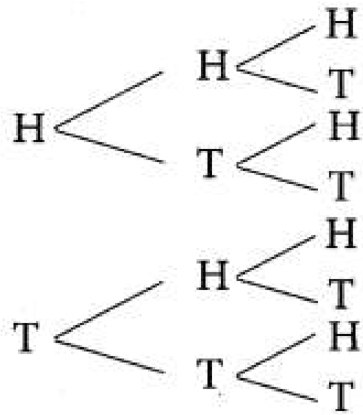
$$CV_2 = \frac{\sigma_2}{\bar{x}_2} \times 100 = \frac{2}{15} \times 100 = 13.33$$

∴ Co-efficient of variation of City A is less than C.V of City B.

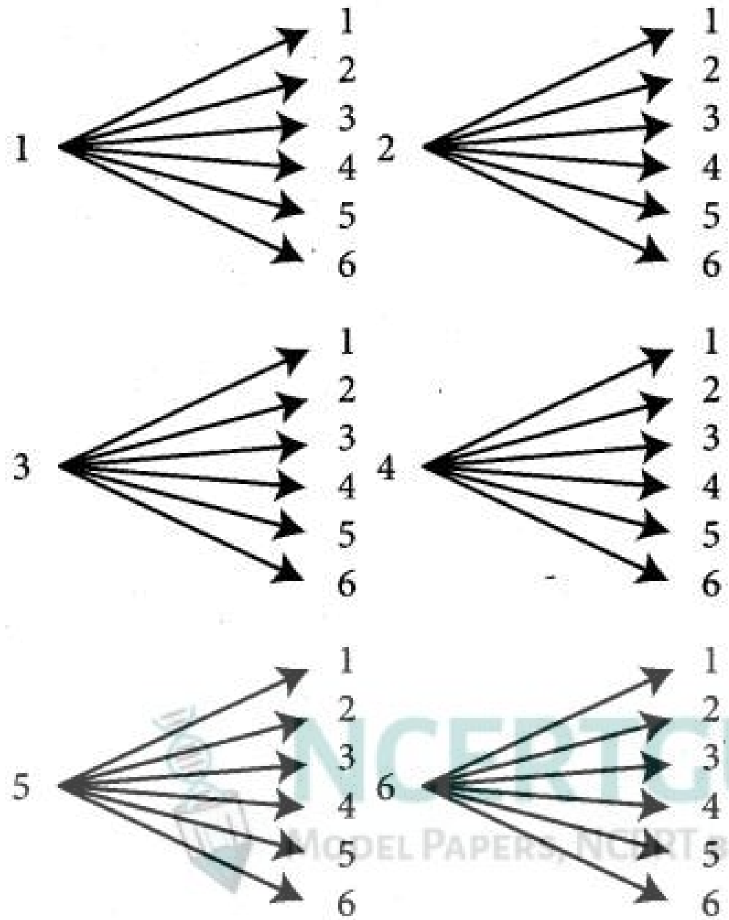
∴ City A is more consistent.

Ex 8.3

Question 1.
Write the sample space for tossing three coins using tree diagram.
Solution:



Question 2.
Write the sample space for selecting two balls from a bag containing 6 balls numbered 1 to 6 (using tree diagram).
Solution:



$S = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$

Question 3.
If A is an event of a random experiment such that $P(A) : P(\overline{A}) = 17 : 15$ and $n(S) = 640$ then find
(i) $P(\overline{A})$
(ii) $n(A)$.
Solution:

$P(A) : P(\overline{A}) = 17 : 15$

(i) $P(A) = \frac{17}{32}, P(\bar{A}) = \frac{15}{32}$

(ii) $P(A) = \frac{17}{32} = \frac{17 \times 20}{32 \times 20} = \frac{340}{640}$

$\Rightarrow \quad n(A) = 340$

Question 4.
A coin is tossed thrice. What is the probability of getting two consecutive tails?
Solution:
Outcomes {O}: {(HHH), (THH), (HTH), (HHT), (HTT), (THT), (TTH), (TTT)}
Two consecutive tails {F} : {(HTT), (TTH), (TTT)}
n{F} = 3
n{O} = 8

$\Rightarrow P = \frac{n\{F\}}{n\{O\}} = \frac{3}{8}$

Question 5.
At a fete, cards bearing numbers 1 to 1000, one number on one card are put in a box. Each player selects one card at random and that card is not replaced. If the selected card has a perfect square number greater than 500, the player wins a prize. What is the probability that
(i) the first player wins a prize
(ii) the second player wins a prize if the first has won?
Answer:
Sample space = {1, 2, 3,… ,1000}
n(S) = 1000
(i) Let A be the event of setting square number greater than 500
A = {529, 576, 625, 676, 729, 784, 841, 900, 961}
n(A) = 9
$P(A) = \frac{n(A)}{n(S)} = \frac{9}{1000}$
The probability that the first player wins prize = $\frac{9}{1000}$
(ii) If the first player wins, the number is excluded for the second player.
n(A) = 8 and n(S) = 999
$P(A) = \frac{n(A)}{n(S)} = \frac{8}{999}$
Probability the second player wins a prize = $\frac{8}{999}$

Question 6.
A bag contains 12 blue balls and x red balls. If one ball is drawn at random
(i) what is the probability that it will be a red ball?
(ii) If 8 more red balls are put in the bag, and if the probability of drawing a red ball will be twice that of the probability in (i), then find x.
Solution:
12 → blue balls
x → red balls
(i) P (red ball) = $\frac{x}{x+12}$
(ii) 8 red balls are added to the bag.
∴ 12 → blue balls
x + 8 → red balls

$$\therefore \text{P(red ball)}_{\text{new}} = \frac{x+8}{x+8+12} = \frac{x+8}{x+20}$$

Given that P(ii) = 2 × P(i)

$$\Rightarrow \frac{x+8}{x+20} = 2\times\frac{x}{x+12}$$

⇒ (x + 8)(x + 12) = 2x(x + 20)
⇒ (x^2 + 20x + 96) = $2x^2$ + 40x
⇒ x^2 + 20x – 96 = 0
⇒ x^2 + 24x – 4x – 96 = 0
⇒ x(x + 24) – 4(x + 24) = 0
⇒ (x – 4)(x + 24) = 0
∴ x = 4 (or) x = -24
x cannot be negative ⇒ x = 4
Substituting x = 4 in (i),

$$\text{we get P(red ball)} = \frac{4}{4+12} = \frac{1}{4}$$

Question 7.
Two unbiased dice are rolled once. Find the probability of getting
(i) a doublet (equal numbers on both dice)
(ii) the product as a prime number
(iii) the sum as a prime number
(iv) the sum as 1
Solution:
Doublet = {(1, 1) (2, 2) (3, 3) (4, 4) (5, 5) (6,6)}
Total number of outcomes = 6 × 6

n(S) = 36
Number of favourable outcomes = 6

$$\Rightarrow P(\text{doublet}) = \frac{6}{36} = \frac{1}{6}$$

(ii) Number of favourable outcomes = 6
as favourable outcomes = (1, 2), (2, 1), (1, 3), (3, 1),(1, 5),and (5, 1)

$$\Rightarrow P(\text{prime number as product}) = \frac{\cancel{6}}{\cancel{36}_6} = \frac{1}{6}$$

(iii) Sum as prime numbers = {(1, 1), (1, 2), (2, 3), (1, 4), (1, 6), (4, 3), (5, 6)}
Number of favourable outcomes = 7
⇒ Probability = $\frac{7}{36}$

(iv) With two dice, minimum sum possible = 2
∴ Prob (sum as 1) = 0 [Impossible event]

Question 8.
Three fair coins are tossed together. Find the probability of getting
(i) all heads
(ii) atleast one tail
(iii) atmost one head
(iv) atmost two tails
Answer:
Three fair coins are tossed together
Sample spade = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}
n(S) = 8
(i) Let A be the event of getting all heads
A = {HHH}
n(A) = 1
$P(A) = \frac{n(A)}{n(S)} = \frac{1}{8}$
(ii) Let B be the event of getting atleast one tail.
B = {HHT, HTH, HTT, THH, THT, TTH, TTT}
n(B) = 7
$P(B) = \frac{n(B)}{n(S)} = \frac{7}{8}$
(iii) Let C be the event of getting atmost one head
C = {HTT, THT, TTH, TTT}
n(C) = 4

$P(C) = \frac{n(C)}{n(S)} = \frac{4}{8} = \frac{1}{2}$
(iv) Let D be the event of getting atmost two tails.
D = {HTT, TTT, TTH, THT, THH, HHT, HTH}
n(D) = 7
$P(D) = \frac{n(D)}{n(S)} = \frac{7}{8}$

Question 9.
Two dice are numbered 1,2,3,4,5,6 and 1,1,2,2,3,3 respectively. They are rolled and the sum of the numbers on them is noted. Find the probability of getting each sum from 2 to 9 separately.
Solution:
Dice 1
S = {1,2, 3, 4, 5, 6}
Dice 2
S = {1,1,2, 2, 3, 3}
Total possible outcomes when they are rolled

$$S = \begin{Bmatrix} (1,1), & (1,1), & (1,2), & (1,2), & (1,3), & (1,3) \\ (2,1), & (2,1), & (2,2), & (2,2), & (2,3), & (2,3) \\ (3,1), & (3,1), & (3,2), & (3,2), & (3,3), & (3,3) \\ (4,1), & (4,1), & (4,2), & (4,2), & (4,3), & (4,3) \\ (5,1), & (5,1), & (5,2), & (5,2), & (5,3), & (5,3) \\ (6,1), & (6,1), & (6,2), & (6,2), & (6,3), & (6,3) \end{Bmatrix}$$

n(S) = 36
Event of sum (2) = A = {(1,1), (1,1)},
n(A) = 2,P(A) = $\frac{2}{36}$
Event of sum 3 is B = {(1, 2), (1, 2), (2, 1), (2, 1)}

$$n(B) = 4, P(B) = \frac{n(B)}{n(S)} = \frac{4}{36}$$

Event of sum 4 is C= {(1, 3), (1, 3), (2, 2), (2, 2), (3, 1) (3, 1)}
n(C) = 6

$$P(C) = \frac{\cancel{6}}{\cancel{36}_6}$$

Event of getting the sum 5 is
D = {(2, 3), (2, 3), (3, 2), (3, 2), (4, 1), (4, 1)}
n(D) = 6, P(D) = $\frac{6}{36}$.

Event of getting the sum 6 is
E = {(3, 3), (3, 3), (4, 2), (4, 2), (5, 1), (5, 1)}
n(E) = 6, P(E) = $\frac{6}{36}$
Event of getting the sum 7 is
F = {(4, 3), (4, 3), (5, 2), (5, 2), (6, 1), (6, 1)}
n(F) = 6
P(F) = $\frac{6}{36}$
Event of getting the sum 8 is
G = {(5, 3), (5, 3), (6, 2), (6, 2)}

$$n(G) = 4, P(G) = \frac{4}{36}$$

Event of getting the sum 9 is
H = {(6, 3), (6, 3), n(H) = 2

$$P(H) = \frac{2}{36}$$

$$\therefore \frac{2}{36}, \frac{4}{36}, \frac{6}{36}, \frac{6}{36}, \frac{6}{36}, \frac{6}{36}, \frac{4}{36}, \frac{2}{36}$$

Question 10.
A bag contains 5 red balls, 6 white balls, 7 green balls, 8 black balls. One ball is drawn at random from the bag. Find the probability that the ball is drawn
(i) white
(ii) black or red
(iii) not white
(iv) neither white nor black
Solution:

5 red 6 white 7 green 8 black total no. of balls = 5 + 6 + 7 +8= 26

(i) $\text{Prob(white)} = \frac{\cancel{6}^{3}}{\cancel{26}_{13}} = \frac{3}{13}$

(ii) $\text{Prob(black or red)} = \frac{5+8}{26} = \frac{13}{26} = \frac{1}{2}$

(iii) $\text{Prob(not white)} = 1 - \text{prob(white)}$

$$= 1 - \frac{3}{13} = \frac{10}{13}$$

(iv) prob (neither her white or black) = 1–prob(white or black)

$$= 1 - \frac{6+8}{26} = 1 - \frac{14}{26}$$

$$= \frac{26-14}{26} = \frac{\cancel{12}^{6}}{\cancel{26}_{13}} = \frac{6}{13}$$

Question 11.
In a box there are 20 non-defective and some defective bulbs. If the probability that a bulb selected at random from the box found to be defective is 3/8 then, find the number of defective bulbs.
Solution:
Let number of defective bulbs be 'x'
Total number of bulbs = x + 20

$$\text{Prob(defective)} = \frac{x}{x+20} = \frac{3}{8} \Rightarrow 8x = 3(x+20)$$

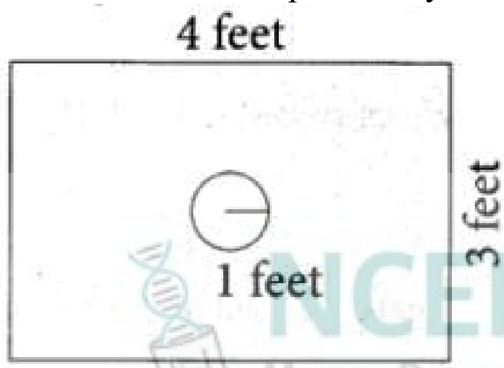
⇒ 8x = 3x + 60
⇒ 5x = 60
⇒ x = 12
∴ No.of defective bulbs are = 12.

Question 12.
The king and queen of diamonds, queen and jack of hearts, jack and king of spades are removed from a deck of 52 playing cards and then well shuffled. Now one card is drawn at random from the remaining cards. Determine the probability that the card is
(i) a clavor
(ii) a queen of red card
(iii) a king of black card

Solution:
Removed cards: K♦, Q♦, Q♥, J♥, J♠, K♠

(i.e) remaining number of cards = 52 – 6 = 46 13
(i) P(a clavor) = $\frac{13}{46}$
(ii) P(queen of red card) = 0 as both Queen of diamond and heart have been removed.
(iii) only K of clavor is in the deck
⇒ P(king of black card) = $\frac{1}{46}$

Question 13.
Some boys are playing a game, in which the stone was thrown by them landing in a circular region (given in the figure) is considered as a win and landing other than the circular region is considered as a loss. What'is the probability to win the game?



Solution:

$$\text{Area of circular region} = \pi R^2 = \pi(1)^2$$
$$= \pi \text{ sq.feet}$$
$$\text{Total area} = 4 \times 3 = 12 \text{ sq. feet}$$
$$\therefore \text{Prob(win the game)} = \frac{\pi}{12} = \frac{3.14}{12}$$
$$= \frac{314}{1200} = \frac{157}{600}$$

Question 14.
Two customers Priya and Amuthan are visiting a particular shop in the same week (Monday to Saturday). Each is equally likely to visit the shop on any one day as on another day. What is the probability that both will visit the shop on
(i) the same day
(ii) different days
(iii) consecutive days?

Solution:

Prob of Priya and Amurthan to visit shop on any day $= \frac{1}{6}$

(i) prob(visit in same day) $= \left(\frac{1}{6}\times\frac{1}{6}\right)\times 6 = \frac{1}{6}$

(ii) prob(different days) $= \left(\frac{1}{6}\times\frac{5}{6}\right)\times 6 = \frac{5}{6}$

(iii) prob(consequent days) $= \left(\frac{1}{6}\times\frac{1}{6}\right)\times 5 = \frac{5}{36}$

Question 15.
In a game, the entry fee is □ 150. The game consists of tossing a coin 3 times. Dhana bought a ticket for entry. If one or two heads show, she gets her entry fee back. If she throws 3 heads, she receives double the entry fees. Otherwise, she will lose. Find the probability that she
(i) gets double entry fee
(ii) just gets her entry fee
(iii) loses the entry fee.
Solution:

$$\text{prob(double entry fee)} = \text{prob(3H)} = \frac{1}{2\times2\times2} = \frac{1}{8}$$

$$\text{prob(first gets her entry fee)} = \text{prob(1H)} + \text{prob(2H)} = \frac{3}{8}+\frac{3}{8} = \frac{6}{8} = \frac{3}{4}$$

$$\text{prob(losing entry fee)} = \text{prob(0H)} = \frac{1}{8}$$

Ex 8.4

Question 1.
If $P(A) = \frac{2}{3}$ $P(B) =$,$\frac{2}{5}$$P(A \cup B) = 13$ then find $P(A \cap B)$.
Solution:
$P(A) = \frac{2}{3}$, $P(B) = \frac{2}{5}$, $P(A \cup B) = \frac{1}{3}$
$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$$= \frac{2}{3} + \frac{2}{5} - \frac{1}{3}$$

$$= \frac{10+6-5}{15} = \frac{11}{15}$$

Question 2.
A and B are two events such that, $P(A) = 0.42$, $P(B) = 0.48$, and $P(A \cap B) = 0.16$. Find
(i) P(not A)
(it) P(not B)
(iii) P(A or B)
Answer:
(i) P(not A) = 1 – P (A) = 1 – 0.42 = 0.58
(ii) P(not B) = 1 – P(B) = 1 – 0.48 = 0.52
(iii) P(A or B) = $P(A) + P(B) - P(A \cap B) = 0.42 + 0.48 - 0.16 = 0.90 - 0.16 = 0.74$

Question 3.
If A and B are two mutually exclusive events of a random experiment and P(not A) = 0.45, $P(A \cup B) = 0.65$, then find P(B).
Solution:
A and B are two mutually exclusive events of a random experiment.
P(not A) = 0.45,
P(A) = 1 – P(not A)

$P(A \cup B) = 0.65 = 1 - 0.45 = 0.55$
$P(A \cup B) = P(A) + P(B) = 0.65$
$0.55 + P(B) = 0.65$
$P(B) = 0.65 - 0.55$
$= 0.10$

Question 4.
The probability that atleast one of A and B occur is 0.6. If A and B occur simultaneously with probability 0.2, then find $P(\bar{A}) + P(\bar{B})$.
Answer:
Here $P(A \cup B) = 0.6$, $P(A \cap B) = 0.2$
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$0.6 = P(A) + P(B) - 0.2$
$P(A) + P(B) = 0.8$
$P(\bar{A}) + P(\bar{B}) = 1 - P(A) + 1 - P(B)$
$= 2 - [P(A) + P(B)]$
$= 2 - 0.8$
$= 1.2$

Question 5.
The probability of happening of an event A is 0.5 and that of B is 0.3. If A and B are mutually exclusive events, then find the probability that neither A nor B happen.
Solution:
$P(A) = 0.5$ Since A and B are mutually inclusive events
$P(B) = 0.3$ events.
$P(\overline{\mathbf{A}}) \cup P(\overline{\mathbf{B}}) = 1 - [P(A) + P(B)]$
$= 1 - [0.5 + 0.3] = 0.2$

Question 6.
Two dice are rolled once. Find the probability of getting an even number on the first die or a total of face sum 8.
Solution:
Two dice rolled once.

$$S = \begin{Bmatrix} (1,1), & (1,2), & (1,3), & (1,4), & (1,5), & (1,6) \\ (2,1), & (2,2), & (2,3), & (2,4), & (2,5), & (2,6) \\ (3,1), & (3,2), & (3,3), & (3,4), & (3,5), & (3,6) \\ (4,1), & (4,2), & (4,3), & (4,4), & (4,5), & (4,6) \\ (5,1), & (5,2), & (5,3), & (5,4), & (5,5), & (5,6) \\ (6,1), & (6,2), & (6,3), & (6,4), & (6,5), & (6,6) \end{Bmatrix}$$

$n(S) = 36$

Happening of an even number in the first die is A.

$$A = \begin{Bmatrix} (2,1), & (2,2), & (2,3), & (2,4), & (2,5), & (2,6) \\ (4,1), & (4,2), & (4,3), & (4,4), & (4,5), & (4,6) \\ (6,1), & (6,2), & (6,3), & (6,4), & (6,5), & (6,6) \end{Bmatrix}$$

$$n(A) = 18$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{18}{36}$$

Happening of a total of face sum is 8 is B.

$$B = \{(2, 6), (3,5), (4, 4), (5, 3), (6, 2)\}$$

$$n(B) = 5$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{5}{36}$$

$$(A \cap B) = \{(2, 6), (4, 4), (6, 2)\}$$

$$n(A \cap B) = 3$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{3}{36}$$

$$\therefore P(A \cup B) = P(A) + P(B) + P(A \cap B)$$

$$= \frac{18}{36} + \frac{5}{36} - \frac{3}{36} = \frac{18+5-3}{36}$$

$$= \frac{\cancel{20}^{5}}{\cancel{36}_{9}} = \frac{5}{9}$$

Question 7.
From a well-shuffled pack of 52 cards, a card is drawn at random. Find the probability of it being either a red king or a black queen.
Solution:
n(S) = 52
No. of Red cards = 26,
Red king cards = 2
No. of Black cards = 26,
Black queen cards = 2
No. of red king cards = n(K) = 2

$$\therefore P(K) = \frac{n(K)}{n(S)} = \frac{2}{52}$$

No. of black queen cards $n(Q) = 2$

$$P(Q) = \frac{n(Q)}{n(S)} = \frac{2}{52}$$

$$n(K \cap Q) = 0$$

$$\therefore \quad P(K \cup Q) = P(K) + P(Q) - P(K \cap Q)$$

$$= \frac{2}{52} + \frac{2}{52} - 0$$

$$= \frac{4}{\cancel{52}_{13}} = \frac{1}{13}$$

$\therefore$ The probability of being either a red king or a black queen = $\frac{1}{13}$.

Question 8.

A box contains cards numbered 3, 5, 7, 9,… 35, 37. A card is drawn at random from the box. Find the probability that the drawn card has either multiples of 7 or a prime number.

Solution:

S = {3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37}

n(S) = 18

Multiplies of seven cards (A) = {7, 21, 35}

= n(A) = 3

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{18}$$

Let the prime number cards B

B = {3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37} n(B) = 11

$$P(B) = \frac{11}{18}$$

$$\therefore \quad (A \cap B) = \{7\}$$

$$n(A \cap B) = 1$$

$$P(A \cap B) = \frac{1}{18}$$

$$\therefore P(A \cup B) = P(A) + P(B) + P(A \cap B)$$

$$= \frac{3}{18} + \frac{11}{18} - \frac{1}{18} = \frac{13}{18}$$

$\therefore$ Probability of the drawn card is either multiples of seven or a prime number $= \frac{13}{18}$

Question 9.
Three unbiased coins are tossed once. Find the probability of getting at most 2 tails or at least 2 heads.
Solution:
When we toss three coins, the sample space S = {HHH, TTT, HTT, THH, HHT, TTH, HTH, THT}
n(S) = 8
Event of getting at most 2 tails be A.

$\therefore$ A = { HHH, HTT, THH, HHT, TTH, HTH, THT}

$n(A) = 7$

$P(A) = \frac{n(A)}{n(S)} = \frac{7}{8}$

Event of getting atleast 2 heads be B.

$\therefore B = \{HHH, THH, HHT, HTH\}$

$n(B) = 4$

$P(B) = \frac{4}{8}$

$A \cap B = \{HHH, THH, HHT, HTH\}$

$n(A \cap B) = 4, P(A \cap B)$

$= \frac{n(A \cap B)}{n(S)} = \frac{4}{8}$

$\therefore \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{7}{8} + \frac{4}{8} - \frac{4}{8} = \frac{7}{8}$

Question 10.
The probability that a person will get an electrification contract is the probability that he will not get plumbing contract is $\frac{3}{5}$. The probability of getting at least one contract is $\frac{5}{8}$. What is the probability that he will get both?
Solution:

$$\text{Let } P(A) = \frac{3}{5}$$

$$P(B) = 1 - \frac{5}{8} = \frac{3}{8}$$

$$P(A \cup B) = \frac{5}{7}$$

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$= \frac{3}{5} + \frac{3}{8} - \frac{5}{7}$$

$$= \frac{168 + 105 - 200}{280} = \frac{73}{280}$$

$$\text{Probability of getting both offer} = \frac{73}{280}$$

Question 11.

In a town of 8000 people, 1300 are over 50 years and 3000 are females. It is known that 30% of the females are over 50 years. What is the probability that a chosen individual from the town is either a female or over 50 years ?

Solution:

$$n(S) = 8000$$

Over 50 years be A; $n(A) = 1300$

Females be B; $n(B) = 3000$

$30\% = \frac{30}{100}$ of 3000 are over 50 years.

$$\text{i.e. } \frac{30}{100} \times 3000 = 900.$$

$$n(A \cap B) = 900$$

$$\therefore \quad P(A) = \frac{n(A)}{n(S)} = \frac{1300}{8000}, P(B) = \frac{3000}{8000},$$

$$P(A \cap B) = \frac{900}{8000}$$

$$\therefore \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1300}{8000} + \frac{3000}{8000} - \frac{900}{8000}$$

$$= \frac{34\cancel{0}\cancel{0}}{80\cancel{0}\cancel{0}} = \frac{17}{40} = 0.425$$

Question 12.
A coin is tossed thrice. Find the probability of getting exactly two heads or at least one tail or two consecutive heads.
Solution:
Three coins tossed simultaneously.
S = { HHH, TTT, HHT, TTH, HTH, THT, HTT, THH}
n(S) = 8
Happening of getting exactly two heads be A.
A= {HHT, HTH, THH}
n(A) = 3

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{8}$$

Event of getting at least one tail be B.
∴ B = {TTT, HHT, TTH, HTH, THT, HTT, THH}

$$n(B) = 7$$

$$P(B) = \frac{7}{8}$$

Event of getting consecutively two heads be C.

C = {HHT, THH, HHH}

$$n(C) = 3$$

$$P(C) = \frac{3}{8}$$

$$A \cap C = \{HHT, THH\}$$

$$n(A \cap C) = 2, P(A \cap C) = \frac{2}{8}$$

$$A \cap B = \{HHT, HTH, THH\}$$

$$n(A \cap B) = 3, P(A \cap B) = \frac{3}{8}$$

$$B \cap C = \{HHT, THH\}$$

$$n(B \cap C) = 2, P(B \cap C) = \frac{2}{8}$$

$$\therefore (A \cap B \cap C) = \{HHT, THH\}$$

$$n(A \cap B \cap C) = 2$$

$$P(A \cap B \cap C) = \frac{2}{8}$$

$$P(A \cup B \cup C) = P(A)+P(B)+P(C)-P(A\cap B)-P(B \cap C)-P(A \cap C)+P(A \cap B \cap C)$$

$$= \frac{3}{8}+\frac{7}{8}+\cancel{\frac{3}{8}}-\cancel{\frac{3}{8}}-\frac{2}{8}-\cancel{\frac{2}{8}}+\cancel{\frac{2}{8}} = \frac{8}{8} = 1$$

Question 13.

If A, B, C are any three events such that probability of B is twice as that of probability of A and probability of C is thrice as that of probability of A and if $P(A\cap B) = \frac{1}{6}$, $P(B\cap C) = \frac{1}{4}$, $P(A\cap C) = \frac{1}{8}$, $\mathbf{P(A \cup B \cup C)} = \frac{9}{10}$, $\mathbf{P(A \cap B \cap C)} = \frac{1}{15}$, then find P(A), P(B) and P(C)?

Solution:

P(B) = 2P(A)
P(C) = 3P(A)

$$P(A \cap B) = \frac{1}{6}, P(B \cap C) = \frac{1}{4}, P(A \cap C) = \frac{1}{8},$$

$$P(A \cup B \cup C) = \frac{9}{10}, P(A \cap B \cap C)\ \frac{1}{15}$$

$$\therefore P(A \cup B \cup C) = P(A)+P(B)+P(C)-P(A\cap B)-P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

$$\frac{9}{10} = P(A) + 2P(A) + 3(P(A)) - \frac{1}{6} - \frac{1}{4} - \frac{1}{8} + \frac{1}{15}$$

$$\frac{9}{10} = 6P(A) + \left(\frac{-60-90-45+24}{360}\right)$$

$$\frac{9}{10} = 6P(A) + \frac{-171}{360}$$

$$6P(A) = \frac{9}{10} + \frac{171}{360} = \frac{324+171}{360} = \frac{495}{360}$$

$$P(A) = \frac{495}{360} \times \frac{1}{6} = \frac{11}{48}$$

$$\therefore P(B) = 2P(A) = 2 \times \frac{11}{48} = \frac{11}{24}$$

$$P(C) = 3P(A) = 3 \times \frac{11}{48} = \frac{11}{16}$$

$$P(A), P(B), P(C) = \frac{11}{48}, \frac{11}{24}, \frac{11}{16}$$

Question 14.
In a class of 35, students are numbered from 1 to 35. The ratio of boys to girls is 4:3. The roll numbers of students begin with boys and end with girls. Find the probability that a student selected is either a boy with a prime roll number or a girl with a composite roll number or an even roll number.
Solution:

$n(S) = 35$

$n(\mathrm{S}) = 35$

$$n(\mathrm{B}) = \frac{4}{7} \times 35 = 20$$

$$n(\mathrm{G}) = \frac{3}{7} \times 35 = 15$$

$$\mathrm{B} = \{1, 2, 3, 4, 5, 6, 7, \ldots 20\}$$
$$\mathrm{G} = \{21, 22, \ldots, 35\}$$

Boy with prime roll number $\rightarrow$ A

$$\mathrm{A} = \{2, 3, 5, 7, 11, 13, 17, 19\}$$
$$n(\mathrm{A}) = 8$$
$$\mathrm{P(A)} = \frac{8}{35}$$

Girl with composite roll number. $\rightarrow$ C

$$\mathrm{C} = \{21, 22, 24, 25, 26, 27, 28, 30, 32, 33, 34, 35\}$$

$$n(\mathrm{C}) = 12, \mathrm{P(GC)} = \frac{12}{35}$$

Student with even roll number - E

E = {2,4,6,8,10,12,14,16,18,20,22,24,26,28,30,
32,34}

$n(E) = 17$, $P(E) = \frac{17}{35}$

$A \cap C = \{\ \}$, $n(A \cap C) = 0$, $P(A \cap C) = \frac{0}{35}$

$C \cap E = \{22,24,26,28,30,32,34\}$, $n(C \cap E) = 7$

$P(C \cap E) = \frac{7}{35}$,

$E \cap A = \{2\}$, $n(E \cap A) = 1$, $P(E \cap A) = \frac{1}{35}$,

$P(A \cap C \cap E) = 0$

$\therefore P(A \cup C \cup E) = P(A) + P(C) + P(E) - P(A \cap C) - P(C \cap E) - P(E \cap A) + P(A \cap C \cap E)$

$= \frac{8}{35} + \frac{12}{35} + \frac{17}{35} - \frac{0}{35} - \frac{7}{35} - \frac{2}{35} + \frac{0}{35}$

$= \frac{37-9}{35} = \frac{28}{35} = \frac{4}{5}$

Ex 8.5

Multiple Choice Questions
Question 1.
Which of the following is not a measure of dispersion?
(1) Range
(2) Standard deviation
(3) Arithmetic mean
(4) Variance
Solution:
(3) Arithmetic mean

Question 2.
The range of the data 8, 8, 8, 8, 8. . . 8 is _____
(1) 0
(2) 1
(3) 8
(4) 3
Answer:
(1) 0
Hint:
Range = L – S = 8 – 8 = 0

Question 3.
The sum of all deviations of the data from its mean is
(1) Always positive
(2) always negative
(3) zero
(4) non-zero integer

Solution:
(3) zero

Question 4.
The mean of 100 observations is 40 and their standard deviation is 3. The sum of squares of all deviations is ______
(1) 40000
(2) 160900
(3) 160000
(4) 30000
Answer:
(2) 160900
Hint:

$$\bar{x} = \frac{\Sigma x}{n} = 40, n = 100, \Sigma x = 4000$$

$$\text{S.D}(\sigma) = \sqrt{\frac{\Sigma\left(x-\bar{x}\right)^2}{n}}$$

$$3 = \sqrt{\frac{\Sigma\left(x-\bar{x}\right)^2}{n}} \text{ on Squaring both sides}$$

$$\frac{\Sigma\left(x-\bar{x}\right)^2}{n} = 9$$

$$\Sigma(x-\bar{x})^2 = 9 \times n = 9 \times 100 = 900$$

$$\Sigma(x-\bar{x})^2 = \Sigma(x^2 - 2x\bar{x} + \bar{x}^2) = 900$$

$$\Rightarrow \Sigma x^2 - 2\bar{x}\Sigma x + \bar{x}^2 . n = 900$$

$$\Sigma x2 = 900 + 2\bar{x} . \Sigma x - \bar{x}^2 . n$$

$$= 900 + 2 \times 40 \times 4000 - 40 \times 40 \times 100$$

$$= 3,20,000 - 1,60,000 + 900 = 1,60,900$$

Question 5.
Variance of the first 20 natural numbers is
(1) 32.25
(2) 44.25

(3) 33.25
(4) 30
Solution:
(3) 33.25

Question 6.
The standard deviation of a data is 3. If each value is multiplied by 5 then the new variance is ______
(1) 3
(2) 15
(3) 5
(4) 225
Answer:
(4) 225
Hint:
Standard deviation = 3
Each value is multiplied by 5
New standard deviation = 3 × 5 = 15
New variance = 152 = 225

Question 7.
If the standard deviation of x, y, z is p then the standard deviation of 3x + 5, 3y + 5, 3z + 5 is
(1) 3p + 5
(2) 3p
(3) p + 5
(4) 9p + 15
Solution:
(2) 3p

Question 8.
If the mean and coefficient of variation of a data are 4 and 87.5% then the standard deviation is ______
(1) 3.5
(2) 3
(3) 4.5
(4) 2.5
Answer:
(1) 3.5

Hint:

$$CV = \frac{\sigma}{\bar{x}} \times 100$$

$$87.5 = \frac{\sigma}{4} \times 100$$

$$\sigma = \frac{87.5 \times 4}{100} = 3.5$$

Question 9.
Which of the following is incorrect?
(1) P (A) > 1
(2) $0 \leq P(A) \leq 1$
(3) $P(\phi) = 0$ (4)
(4) $P(A) + P(\overline{\mathbf{A}}) = 1$
Solution:
(1) P(A) > 1

Question 10.
The probability a red marble selected at random from a jar containing p red, q blue and r green marbles is

(1) $\frac{q}{p+q+r}$ (2) $\frac{p}{p+q+r}$

(3) $\frac{p+q}{p+q+r}$ (4) $\frac{p+r}{p+q+r}$

Solution:
(2) $\frac{p}{p+q+r}$

Question 11.
A page is selected at random from a book. The probability that the digit at units place of the page number chosen is less than 7 is

(1) $\frac{3}{10}$ (2) $\frac{7}{10}$

(3) $\frac{3}{9}$ (4) $\frac{7}{9}$

Solution:
(2) $\frac{7}{10}$

Question 12.
The probability of getting a job for a person is $\frac{x}{3}$. If the probability of not getting the job is $\frac{2}{3}$ then the value of x is ____
(1) 2
(2) 1
(3) 3
(4) 1.5
Answer:
(2) 1
Hint:

$$= P(500) + P(200) = \frac{15}{50} + \frac{25}{50} = \frac{40}{50} = \frac{4}{5}$$

$$P(\bar{J}) = \frac{2}{3} \quad = 1 - \frac{x}{3}$$

$$\Rightarrow 1 - \frac{x}{3} = \frac{2}{3}$$

$$\Rightarrow \frac{3-x}{3} = \frac{2}{3}$$

$$\Rightarrow 3 - x = 2 \Rightarrow x = 1$$

Question 13.
Kamalam went to play a lucky draw contest. 135 tickets of the lucky draw were sold. If the probability of Kamalam winning is $\frac{1}{9}$, then the number of tickets bought by Kamalam is
(1) 5
(2) 10
(3) 15
(4) 20
Solution:
(3) 15
Hint:
$= \frac{1}{9} \times 135 = 15$

Question 14.
If a letter is chosen at random from the English alphabets {a, b, ……, z} then the probability that the letter chosen precedes x _______

(1) $\frac{12}{13}$
(2) $\frac{1}{13}$
(3) $\frac{23}{26}$
(4) $\frac{3}{26}$

Answer:
(3) $\frac{23}{26}$

Hint:

$$= \quad 1 - \frac{3}{26} = \frac{23}{26}$$

Question 15.
A purse contains 10 notes of □ 2000, 15 notes of □ 500, and 25 notes of □ 200. One note is drawn at random. What is the probability that the note is either a □ 500 note or □ 200 note?

(1) $\frac{1}{5}$
(2) $\frac{3}{10}$
(3) $\frac{2}{3}$
(4) $\frac{4}{5}$

Solution:
(4) $\frac{4}{5}$

Unit Exercise 8

Question 1.
The mean of the following frequency distribution is 62.8 and the sum of all frequencies is 50. Compute the missing frequencies f_1 and f_2.

Class Interval	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	5	f_1	10	f_2	7	8

Solution:
Mean $\bar{x}$ = 62.8
Σx = 50

Class interval	Mid value of x_i	Frequency f_i	$\Sigma f_i x_i$
0–20	10	5	50
20–40	30	f_1	$30f_1$
40–60	50	10	500
60–80	70	f_2	$70f_2$
80–100	90	7	630
100–120	110	8	880
		$30 + f_1 + f_2$	$2060+30f_1+70f_2$

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{2060 + 30f_1 + 70f_2}{30 + f_1 + f_2}$$

$$\frac{2060 + 30f_1 + 70f_2}{30 + f_1 + f_2} = 62.8 \qquad ...(1)$$

$$
\begin{aligned}
30 + f_1 + f_2 &= 50 \text{ (given)} \\
f_1 + f_2 &= 20 \qquad \ldots(2) \\
2060 + 30f_1 + 70f_2 &= 3140 \\
30f_1 + 70f_2 &= 3140 - 2060 \\
30f_1 + 70f_2 &= 1080 \qquad \ldots(3)
\end{aligned}
$$

Solving (2) & (3) we get,

$$
\begin{aligned}
\cancel{30f_1} + 70f_2 &= 1080 \\
(2) \times 30 \Rightarrow \underset{(-)}{\cancel{30f_1}} \underset{(-)}{+} 30f_2 &= \underset{(-)}{600} \\
\hline
40f_2 &= 480 \\
f_2 &= 12
\end{aligned}
$$

Sub. $f_2 = 12$ in (2), we get

$f_1 + 12 = 20 \Rightarrow f_1 = 8$

$f_1 = 8, f_2 = 12$

Question 2.
The diameter of circles (in mm) drawn in a design are given below.

Diameters	33-36	37-40	41-44	45-48	49-52
Number of circles	15	17	21	22	25

Claculate the standard deviation.

Solution:

Class interval	Mid value x_i	No. of circles f_i	$d_1 = x_i - \bar{x}$	d^2	$f_i d^2$
32.5–36.5	34.5	15	–8	64	960
36.5–40.5	38.5	17	–4	16	272
40.5–44.5	42.5	21	0	0	0
44.5–48.5	46.5	22	4	16	352
48.5–52.5	50.5	25	8	64	1600
		100			$\Sigma =$ 3184

$$\sigma = \sqrt{\frac{\Sigma f_i (x_i - \bar{x})^2}{\Sigma f}}$$

$$= \sqrt{\frac{3184}{100}} = \sqrt{31.84} = 5.64$$

Question 3.
The frequency distribution is given below.

x	k	$2k$	$3k$	$4k$	$5k$	$6k$
f	2	1	1	1	1	1

In the table, k is a positive integer, has a varience of 160. Determine the value of k.
Solution:

x	f	fx	$d = x - \bar{x}$	d^2
k	2	$2k$	$\frac{-15}{7}k$	$\left(\frac{-15}{7}k\right)^2$
$2k$	1	$2k$	$\frac{-8}{7}k$	$\left(\frac{-8}{7}k\right)^2$
$3k$	1	$3k$	$\frac{-1}{7}k$	$\left(\frac{-1}{7}k\right)^2$
$4k$	1	$4k$	$\frac{6k}{7}$	$\left(\frac{6k}{7}\right)^2$
$5k$	1	$5k$	$\frac{13}{7}k$	$\left(\frac{13}{7}k\right)^2$
$6k$	1	$6k$	$\frac{20}{7}k$	$\left(\frac{20}{7}k\right)^2$

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{22k}{7}$$

$$\sigma^2 = \frac{\Sigma f_i d_1^2}{\Sigma f}$$

$$= \frac{\frac{k^2}{7^2}[1^2 + 6^2 + 8^2 + 13^2 + 15^2 + 20^2]}{7}$$

$$160 = \frac{k^2}{7^3} \times 1120$$

$$k^2 = \frac{160 \times 7^3}{1120}$$

$$= 49 \Rightarrow k = \pm 7$$

$$k = 7 \text{ since } k \text{ is a +ve number.}$$

Question 4.
The standard deviation of some temperature data in degree Celsius (°C) is 5. If the data were converted into degree Fahrenheit (°F) then what is the variance?
Answer:
Standard deviation (σ) = 5
Variance = $5^2 = 25$
We know the formula, F = $\frac{9}{5}$ C + 32
Variance (F) = Vanance $\frac{9}{5}$ C° + 32
[Variance of ax + b = a^2 (variance of x)]
= $\left(\frac{9}{5}\right)^2$. variance
= $\frac{81}{25}$ × 25
= 81° F
New variance = 81° F

Question 5.
If for a distribution, $\Sigma(x-5) = 3, \Sigma(x-5)^2 = 43$ and total number of observations is 18, find the mean and standard deviation.
Solution:

Mean $\bar{x} = \frac{\Sigma x}{n}, \Sigma (x-5) = 3, n = 18$

$$\Sigma x - 5 \times 18 = 3$$

$$\Sigma x = 3 + 90 = 93$$

$$\bar{x} = \frac{93}{18} = 5.17$$

$$\Sigma (x-5)^2 = 43$$

$$\Rightarrow \Sigma(x^2 - 10x + 25) = 43$$

$$\Rightarrow \Sigma x^2 - 10\Sigma x + \Sigma 25 = 43$$

$$\Rightarrow \Sigma x^2 - 10 \times 93 + 25 \times 18 = 43$$

$$\therefore \Sigma x^2 = 43 + 930 - 450 = 523$$

Standard Deviation $\sigma = \sqrt{\frac{\Sigma d^2}{n}} = \sqrt{\frac{\Sigma\left(x-\bar{x}\right)^2}{n}}$

ie $\sqrt{\frac{\Sigma\left(x-5.17\right)^2}{18}}$

$$= \sqrt{\frac{\Sigma\left(x^2 - 2\times 5.17x + 5.17^2\right)}{18}}$$

$$= \sqrt{\frac{\Sigma x^2 - 2\times 5.17\Sigma x + 5.17^2 \times 18}{18}}$$

$$= \sqrt{\frac{523 - 2\times 5.17\times 93 + 5.17^2 \times 18}{18}}$$

$$= \sqrt{\frac{523 - 961.62 + 481.12}{18}} = \sqrt{\frac{42.5}{18}} = \sqrt{2.36}$$

$\sigma = 1.53$

Question 6.
Prices of peanut packets in various places of two cities are given below. In which city, prices were more stable?

Prices in city A	20	22	19	23	16
Prices in city B	10	20	18	12	15

Solution:

City A			City B		
x_1	$d_1 = x - \bar{x}_1$	d_1^2	x_2	$d_2 = x - \bar{x}_2$	d_2^2
20	0	0	10	–5	25
22	2	4	20	5	25
19	–1	1	18	3	9
23	–3	9	12	–3	9
16	–4	16	15	0	0
100	6	30	75	0	68

$$\bar{x}_1 = \frac{100}{5} = 20 \qquad \bar{x}_2 = \frac{\sum x}{n} = \frac{75}{5} = 15$$

$$\sigma = \sqrt{\frac{\sum d^2}{n}} \qquad \sigma = \sqrt{\frac{\sum d^2}{n}}$$

$$= \sqrt{\frac{30}{5}} \qquad = \sqrt{\frac{68}{5}}$$

$$= \sqrt{6} \qquad = \sqrt{13.6}$$

$$= 2.44 \qquad = 3.68$$

$$CV = \frac{\sigma}{\bar{x}} \times 100 \qquad CV = \frac{\sigma}{\bar{x}} \times 10$$

$$= \frac{2.44}{20} \times 100 \qquad = \frac{3.68}{15} \times 100$$

$$= 12.2 \qquad = \frac{368}{15} = 24.53$$

C.V. of city A < C.V. of city B.

$\therefore$ City A is more consistents.

Question 7.
If the range and coefficient of range of the data are 20 and 0.2 respectively, then find the largest and smallest values of the data.

Solution:
Range = L – S = 20

$$\text{Co-efficient of range} = \frac{L-S}{L+S} = 0.2$$

$$\begin{aligned}
L - S &= 20 \quad \ldots(1)\\
L - S &= 0.2\,(L + S)\\
(L + S)\ 0.2 &= 20\\
0.2\,L + 0.2\,S &= 20 \quad \ldots(2)\\
(1) \times 0.2 \Rightarrow 0.2\,L - 0.2\,S &= 4\\
\hline
0.4\,L &= 24\\
L &= \frac{24}{0.4} = 60
\end{aligned}$$

Substitute $L = 60$ in (1)

$$\begin{aligned}
60 - S &= 20\\
-S &= 20 - 60 = -40\\
S &= 40
\end{aligned}$$

∴ The largest is 60, the smallest is 40°.

Question 8.
If two dice are rolled, then find the probability of getting the product of face value 6 or the difference of face values 5.
Solution:
Product of face values 6: {(1, 6), (2, 3), (6, 1), (3,2)}
Difference of face value 5: {(1, 6), (6, 1)}

$$P(\text{product } 6) = \frac{4}{6\times6} = \frac{4}{36} = \frac{1}{9}$$

$$p(\text{difference } 5) = \frac{2}{6\times6} = \frac{1}{18}$$

Question 9.
In a two children family, find the probability that there is at least one girl in a family.
Solution:
S = {BB, BG, GB, GG}
n(S) = 4
Event of atleast one girl in a family say A

A= {BG, GB, GG}
n(A) = 3

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{4}$$

Probability of at least one girl in a family is $\frac{3}{4}$

Question 10.
A bag contains 5 white and some black balls. If the probability of drawing a black ball from the bag is twice the probability of drawing a white ball then find the number of black balls.
Solution:
Let a number of black balls be 'x'.
Number of white balls = 5.

$$P(\text{black ball}) = \frac{x}{x+5}$$

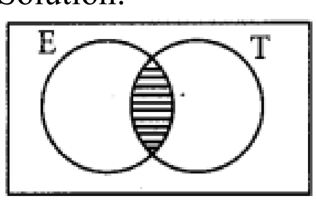
$$P(\text{white ball}) = \frac{5}{x+5}$$

$\because$ P(black ball) = 2 × P(white ball)

$$\Rightarrow \quad \frac{x}{x+5} = 2\times\frac{5}{x+5} \Rightarrow x = 10$$

Number of Black balls = 10.

Question 11.
The probability that a student will pass the final examination in both English and Tamil is 0.5 and the probability of passing neither is 0.1. If the probability of passing the English examination is 0.75, what is the probability of passing the Tamil examination?
Solution:



P(English) = 0.75
P(Tamil) = x(assume)
P(English ∪ Tamil) = P(English) + P(Tamil) – P(English ∩ Tamil)
⇒ 1 – 0.1 = 0.75 + x – 0.5

$\Rightarrow x = 0.9 - 0.25$

$$\Rightarrow \quad x = 0.65 = \frac{13}{20}$$

Question 12.
The King, Queen and Jack of the suit spade are removed from a deck of 52 cards. One card is selected from the remaining cards. Find the probability of getting
(i) a diamond
(ii) a queen
(iii) a spade
(iv) a heart card bearing the number 5.
Solution:
King spade, Queen spade, Jack spade are removed
∴ total number of cards = 52 – 3 = 49.

(i) Prob (diamond) $= \frac{13}{49}$

(ii) Prob(queen) $= \frac{4-1}{49} = \frac{3}{49}$

(iii) Prob(spade) $= \frac{13-3}{49} = \frac{10}{49}$

(iv) Prob(heart bearing number 5) $= \frac{1}{49}$

Additional Questions

Question 1.
Find the standard deviation of 30, 80, 60, 70, 20, 40, 50 using the direct method.
Solution:
Direct method:

x	x^2
30	900
80	6400
60	3600
70	4900
20	400
40	1600
50	2500
$\Sigma x = 350$	$\Sigma x^2 = 20300$

$$\sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$$

$$= \sqrt{\frac{20300}{7} - \left(\frac{350}{7}\right)^2}$$

$$= \sqrt{400} = 20$$

Question 2.
Find the standard deviation for the following data. 5, 10, 15, 20, 25. And also find the new

S.D. if three is added to each value.
Solution:

x	$d' = \frac{x-15}{5}$	d'^2
5	–2	4
10	–1	1
15	0	0
20	1	1
25	2	4
	$\Sigma d = 0$	$\Sigma d'^2 = 10$

$$\bar{x} = \frac{\Sigma x}{n} = \frac{75}{5} = 15$$

$$d' = \frac{x-\bar{x}}{c} = \frac{x-A}{c}$$

A is assumed mean c is common factor.

Here A = 15, C = 5

$$\sigma = \sqrt{\left(\frac{\Sigma d'^2}{n}\right)-\left(\frac{\Sigma d'}{n}\right)^2} \times c$$

$$= \sqrt{\frac{10}{5} - 0} \times c$$

$$= \sqrt{2} \times 5$$

$$= 5\sqrt{2}$$

If 3 is added to each value, we get 8, 13, 18, 23, 28 as new values.

x	$d' = \frac{x-18}{5}$	d'^2
8	–2	4
13	–1	1
18	0	0
23	1	1
28	2	4
	$\Sigma d' = 0$	$\Sigma d'^2 = 10$

$$\therefore \quad \sigma = \sqrt{\left(\frac{\Sigma d'^2}{n}\right) - \left(\frac{\Sigma d'}{n}\right)^2} \times c$$

$$= \sqrt{\frac{10}{5} - 0} \times 5$$

$$= \sqrt{2} \times 5 = 5\sqrt{2}$$

S.D. doesn't change when a number is added or subtracted to the values.

Question 3.
The marks scored by 5 students in a test for 50 marks are 20, 25, 30, 35, 40. Find the S.D for the marks. If the marks are converted for 100 marks, find the S.D. for newly obtained marks.
Solution:
Let assumed mean A = 30
C = 5

x	$d' = \frac{x-30}{5}$	d'^2
20	–2	4
25	–1	1
30	0	0
35	1	1
40	2	4
	$\Sigma d' = 0$	$\Sigma d'^2 = 10$

$$\sigma = \sqrt{\left(\frac{\Sigma d'^2}{n}\right) - \left(\frac{\Sigma d'}{n}\right)^2} \times c$$

$$= \sqrt{\frac{10}{5} - 0} \times 5$$

$$= \sqrt{2} \times 5 = 5\sqrt{2}$$

To convert the values for 100, all the values will be multiplied by 2. Therefore the new values are 40, 50, 60, 70, 80.
Let A = 60,
C = 10

x	$d' = \frac{x-60}{10}$	d'^2
40	–2	4
50	–1	1
60	0	0
70	1	1
80	2	4

$$\sigma = \sqrt{\left(\frac{\Sigma d'^2}{n}\right) - \left(\frac{\Sigma d'}{n}\right)^2} \times c$$

$$= \sqrt{\frac{10}{5} - 0} \times 10$$

$$= \sqrt{2} \times 10$$

$$= 10\sqrt{2}$$

S.D. also be multiplied by 2. It is also true for the division also.

Question 4.

$\Sigma x = 99$, $n = 9$, $\Sigma(x - 10)^2 = 79$, then find, (i) Σx^2 (ii) $\Sigma(x - \bar{x})^2$

Solution:

$$n = 9, \Sigma x = 99, \bar{x} = \frac{\Sigma x}{n} = \frac{99}{9} = 11$$

$$\Sigma(x - 10)^2 = 79 = \Sigma x^2 - 20x + 100 = 79$$

$$= \Sigma x^2 - 20\Sigma x + 100 \times 9 = 79$$

$$= \Sigma x^2 - 20 \times 99 + 900 = 79$$

$$\Sigma x^2 = 79 + 1980 - 900 = 1159$$

$$\Sigma(x - \bar{x})^2 = \Sigma(x - 11)^2 = \Sigma(x^2 - 22x + 121)$$

$$= \Sigma x^2 - 22\Sigma x + 121 \times 9$$

$$= 1159 - 22 \times 99 + 1089 = 70$$

$$\therefore \Sigma x^2 = 1159, \Sigma(x - \bar{x})^2 = 70$$

Question 5.
Find the co-efficient of variation for the following data: 16, 13, 17, 21, 18.

Solution:

$$\text{Mean } \bar{x} = \frac{16+13+17+21+18}{5} = \frac{85}{5} = 17$$

x	$d = x - 17$	d^2
16	–1	1
13	–4	16
17	0	0
21	4	16
18	1	1
	$\Sigma d = 0$	$\Sigma d^2 = 34$

$$\sigma = \sqrt{\frac{\Sigma d^2}{n}} = \sqrt{\frac{34}{5}} = \sqrt{6.8}$$

$$\sigma = 2.61$$

Co-efficient of variation

$$CV = \frac{\sigma}{\bar{x}} \times 100 = \frac{2.61}{17} \times 100$$

$$= 15.35\%$$

Question 6.
C.V. of a data is 69%, S.D. is 15.6, then find its mean.
Solution:

$$CV = \frac{\sigma}{\bar{x}} \times 100 \Rightarrow \bar{x} = \frac{\sigma}{CV} \times 100$$

$$\bar{x} = \frac{15.6}{6.9} \times 100 = 22.6$$

Question 7.
S.D. of a data is 2102, mean is 36.6, then find its C.V.

Solution:

$$\sigma = 21.2, \bar{x} = 36.6$$

$$CV = \frac{\sigma}{\bar{x}} \times 100 = \frac{21.2}{36.6} \times 100 = 57.92\%$$

Question 8.

Team A	**50**	**20**	**10**	**30**	**30**
Team B	**40**	**60**	**20**	**20**	**10**

Which team is more consistent?
Solution:

Team A		
x_1	$d_1 = x - 28$	d_1^2
50	22	484
20	–8	64
10	–18	324
30	2	4
30	2	4
140	$\Sigma d = 0$	880

$$\bar{x}_1 = \frac{140}{5} = 28$$

$$\sigma_1 = \sqrt{\frac{\Sigma d^2}{n}} = \sqrt{\frac{880}{5}}$$

$$= \sqrt{176}$$

$$= 13.27$$

$$CV_1 = \frac{\sigma_1}{\bar{x}_1} \times 100$$

$$CV_1 = \frac{13.27}{28} \times 100$$

$$= 47.39\%$$

$CV_1 < CV_2$

Team B		
x_2	$d_2 = x - 30$	d^2
40	10	100
60	30	900
20	–10	100
20	–10	100
10	–20	400
150	$\Sigma d = 0$	1600

$$\bar{x}_2 = \frac{150}{5} = 30$$

$$\sigma_2 = \sqrt{\frac{\Sigma d^2}{n}} = \sqrt{\frac{1600}{5}}$$

$$= \sqrt{320}$$

$$= 17.89$$

$$CV_1 = \frac{\sigma_2}{\bar{x}_2} \times 100$$

$$CV_2 = \frac{17.89}{30} \times 100$$

$$= 59.63\%$$

∴ Team A is more consistent.

Question 9.
Final the probability of choosing a spade or a heart card from a deck of cards.
Solution:
Total number of cards = 52
Event of selecting a spade card = A
Event of selecting a heart card = B
n(A) = 13,
n(B) = 13