Numbers and Sequences

Ex 2.1

Question 1.
Find all positive integers which when divided by 3 leaves remainder 2.
Answer:
The positive integers when divided by 3 leaves remainder 2.
By Euclid's division lemma $a = bq + r$, $0 \leq r < b$.
Here $a = 3q + 2$, where $0 \leq q < 3$, a leaves remainder 2 when divided by 3.
$\therefore$ 2, 5, 8, 11 ……………..

Question 2.
A man has 532 flower pots. He wants to arrange them in rows such that each row contains 21 flower pots. Find the number of completed rows and how many flower pots are left over?
Answer:
Here $a = 532$, $b = 21$
Using Euclid's division algorithm
$a = bq + r$
$532 = 21 \times 25 + 7$
Number of completed rows = 21
Number of flower pots left over = 7

Question 3.
Prove that the product of two consecutive positive integers is divisible by 2.
Solution:
Let $n - 1$ and n be two consecutive positive integers. Then their product is $(n - 1)n$.
$(n - 1)(n) = n^2 - n$.
We know that any positive integer is of the form $2q$ or $2q + 1$ for some integer q. So, following cases arise.
Case I. When $n = 2q$.

In this case, we have
$n^2 - n = (2q)^2 - 2q = 4q^2 - 2q = 2q(2q - 1)$
$\Rightarrow n^2 - n = 2r$, where $r = q(2q - 1)$
$\Rightarrow n^2 - n$ is divisible by 2.

Case II. When $n = 2q + 1$
In this case, we have
$n^2 - n = (2q + 1)^2 - (2q + 1)$
$= (2q + 1)(2q + 1 - 1) = 2q(2q + 1)$
$\Rightarrow n^2 - n = 2r$, where $r = q(2q + 1)$.
$\Rightarrow n^2 - n$ is divisible by 2.
Hence, $n^2 - n$ is divisible by 2 for every positive integer n.
Hence it is Proved

Question 4.
When the positive integers a, b and c are divided by 13, the respective remainders are 9,7 and 10.
Show that a + b + c is divisible by 13.
Answer:
Let the positive integer be a, b, and c
We know that by Euclid's division lemma
$a = bq + r$
$a = 13q + 9$ ….(1)
$b = 13q + 7$ ….(2)
$c = 13q + 10$ ….(3)
Add (1) (2) and (3)
$a + b + c = 13q + 9 + 13q + 7 + 13q + 10$
$= 39q + 26$
$a + b + c = 13 (3q + 2)$
This expansion will be divisible by 13
$\therefore$ a + b + c is divisible by 13

Question 5.
Prove that square of any integer leaves the remainder either 0 or 1 when divided by 4.
Solution:
Let x be any integer.
The square of x is x^2.
Let x be an even integer.
$x = 2q + 0$
then $x^2 = 4q^2 + 0$
When x be an odd integer
When $x = 2k + 1$ for some interger k.

$x^2 = (2k + 1)^2$
$= 4k^2 + 4k + 1$
$= 4k(k + 1) + 1$
$= 4q + 1$
where $q = k(k + 1)$ is some integer.
Hence it is proved.

Question 6.
Use Euclid's Division Algorithm to find the Highest Common Factor (H.C.F) of
(i) 340 and 412
Answer:
To find the HCF of 340 and 412 using Euclid's division algorithm. We get
$412 = 340 \times 1 + 72$
The remainder $72 \neq 0$
Again applying Euclid's division algorithm to the division of 340
$340 = 72 \times 4 + 52$
The remainder $52 \neq 0$
Again applying Euclid's division algorithm to the division 72 and remainder 52 we get
$72 = 52 \times 1 + 20$
The remainder $20 \neq 0$

Again applying Euclid's division algorithm
$52 = 20 \times 2 + 12$
The remainder $12 \neq 0$
Again applying Euclid's division algorithm
$20 = 12 \times 1 + 8$
The remainder $8 \neq 0$
Again applying Euclid's division algorithm
$12 = 8 \times 1 + 4$
The remainder $4 \neq 0$
Again applying Euclid's division algorithm
$8 = 4 \times 2 + 0$
The remainder is zero
∴ HCF of 340 and 412 is 4

(ii) 867 and 255
Answer:
To find the HCF of 867 and 255 using
Euclid's division algorithm. We get
$867 = 255 \times 3 + 102$
The remainder $102 \neq 0$
Using Euclid's division algorithm
$255 = 102 \times 2 + 51$
The remainder $51 \neq 0$

Again using Euclid's division algorithm
$102 = 51 \times 2 + 0$
The remainder is zero
$\therefore$ HCF = 51
$\therefore$ HCF of 867 and 255 is 51

(iii) 10224 and 9648
Answer:
Find the HCF of 10224 and 9648 using Euclid's division algorithm. We get
$10224 = 9648 \times 1 + 576$
The remainder $576 \neq 0$
Again using Euclid's division algorithm
$9648 = 576 \times 16 + 432$
The remainder $432 \neq 0$
Using Euclid's division algorithm
$576 = 432 \times 1 + 144$
The remainder $144 \neq 0$
Again using Euclid's division algorithm
$432 = 144 \times 3 + 0$
The remainder is 0
$\therefore$ HCF = 144
The HCF of 10224 and 9648 is 144

(iv) 84,90 and 120
Answer:
Find the HCF of 84, 90 and 120 using Euclid's division algorithm
$90 = 84 \times 1 + 6$
The remainder $6 \neq 0$
Using Euclid's division algorithm
$4 = 14 \times 6 + 0$
The remainder is 0
$\therefore$ HCF = 6
The HCF of 84 and 90 is 6
Find the HCF of 6 and 120
$120 = 6 \times 20 + 0$
The remainder is 0
$\therefore$ HCF of 120 and 6 is 6
$\therefore$ HCF of 84, 90 and 120 is 6

Question 7.
Find the largest number which divides 1230 and 1926 leaving remainder 12 in each case.
Solution:
The required number is the H.C.F. of the numbers.

1230 – 12 = 1218,
1926 – 12 = 1914
First we find the H.C.F. of 1218 & 1914 by Euclid's division algorithm.
1914 = 1218 × 1 + 696
The remainder 696 ≠ 0.
Again using Euclid's algorithm
1218 = 696 × 1 + 522
The remainder 522 ≠ 0.
Again using Euclid's algorithm.
696 = 522 × 1 + 174
The remainder 174 ≠ 0.
Again by Euclid's algorithm
522 = 174 × 3 + 0
The remainder is zero.
∴ The H.C.F. of 1218 and 1914 is 174.
∴ The required number is 174.

Question 8.
If d is the Highest Common Factor of 32 and 60, find x and y satisfying d = 32x + 60y.
Answer:

Find the HCF of 32 and 60
60 = 32 × 1 + 28 ….(1)
The remainder 28 ≠ 0
By applying Euclid's division lemma
32 = 28 × 1 + 4 ….(2)
The remainder 4 ≠ 0
Again by applying Euclid's division lemma
28 = 4 × 7 + 0….(3)
The remainder is 0
HCF of 32 and 60 is 4
From (2) we get
32 = 28 × 1 + 4
4 = 32 – 28
= 32 – (60 – 32)
4 = 32 – 60 + 32
4 = 32 × 2 -60
4 = 32 x 2 + (-1) 60
When compare with d = 32x + 60 y
x = 2 and y = -1
The value of x = 2 and y = -1

Question 9.
A positive integer when divided by 88 gives the remainder 61. What will be the remainder when

the same number is divided by 11?
Solution:
Let a (+ve) integer be x.
$x = 88 \times y + 61$
$61 = 11 \times 5 + 6$ ($\because$ 88 is multiple of 11)
$\therefore$ 6 is the remainder. (When the number is divided by 88 giving the remainder 61 and when divided by 11 giving the remainder 6).

Question 10.
Prove that two consecutive positive integers are always coprime.
Answer:

1. Let the consecutive positive integers be x and x + 1.
2. The two number are co – prime both the numbers are divided by 1.
3. If the two terms are x and x + 1 one is odd and the other one is even.
4. HCF of two consecutive number is always 1.
5. Two consecutive positive integer are always coprime.

Fundamental Theorem of Arithmetic
Every composite number can be written uniquely as the product of power of prime is called fundamental theorem of Arithmetic.

Ex 2.2

Question 1.
For what values of natural number n, 4n can end with the digit 6?
Solution:
$4^n = (2 \times 2)^n = 2^n \times 2^n$
2 is a factor of 4^n.
So, 4^n is always even and end with 4 and 6.
When n is an even number say 2, 4, 6, 8 then 4^n can end with the digit 6.
Example:
$4^2 = 16$
$4^3 = 64$
$4^4 = 256$
$4^5 = 1,024$
$4^6 = 4,096$
$4^7 = 16,384$
$4^8 = 65, 536$
$4^9 = 262,144$

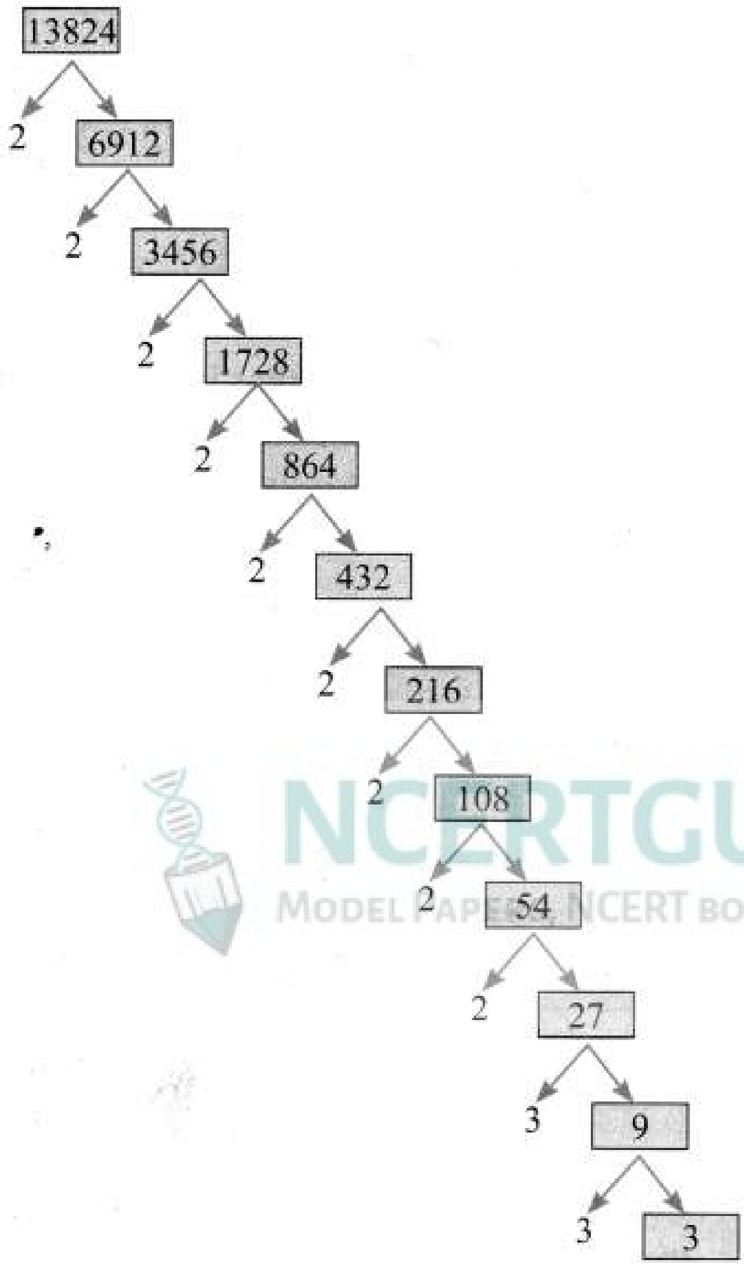
Question 2.
If m, n are natural numbers, for what values of m, does $2^n \times 5^m$ ends in 5?
Answer:
2^n is always even for any values of n.
[Example. $2^2 = 4$, $2^3 = 8$, $2^4 = 16$ etc]

5^m is always odd and it ends with 5.
[Example. $5^2 = 25$, $5^3 = 125$, $5^4 = 625$ etc]
But $2^n \times 5^m$ is always even and end in 0.

[Example. $2^3 \times 5^3 = 8 \times 125 = 1000$
$2^2 \times 5^2 = 4 \times 25 = 100$]
∴ $2^n \times 5^m$ cannot end with the digit 5 for any values of m.

Question 3.
Find the H.C.F. of 252525 and 363636.
Solution:
To find the H.C.F. of 252525 and 363636
Using Euclid's Division algorithm
$363636 = 252525 \times 1 + 111111$
The remainder $111111 \neq 0$.
∴ Again by division algorithm
$252525 = 111111 \times 2 + 30303$
The remainder $30303 \neq 0$.
∴ Again by division algorithm.
$111111 = 30303 \times 3 + 20202$
The remainder $20202 \neq 0$.
∴ Again by division algorithm
$30303 = 20202 \times 1 + 10101$
The remainder $10101 \neq 0$.
∴ Again using division algorithm
$20202 = 10101 \times 2 + 0$
The remainder is 0.
∴ 10101 is the H.C.F. of 363636 and 252525.

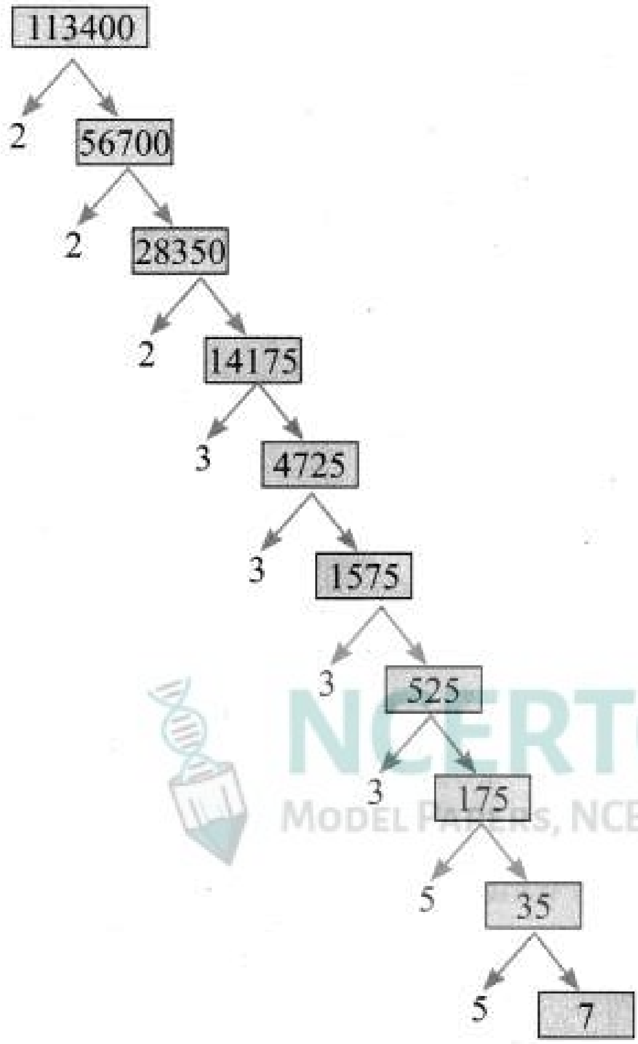
Question 4.
If $13824 = 2^a \times 3^b$ then find a and b.
Solution:
If $13824 = 2^a \times 3^b$
Using the prime factorisation tree



$13824 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$
$= 2^9 \times 3^3 = 2^a \times 3^b$
$\therefore a = 9, b = 3.$

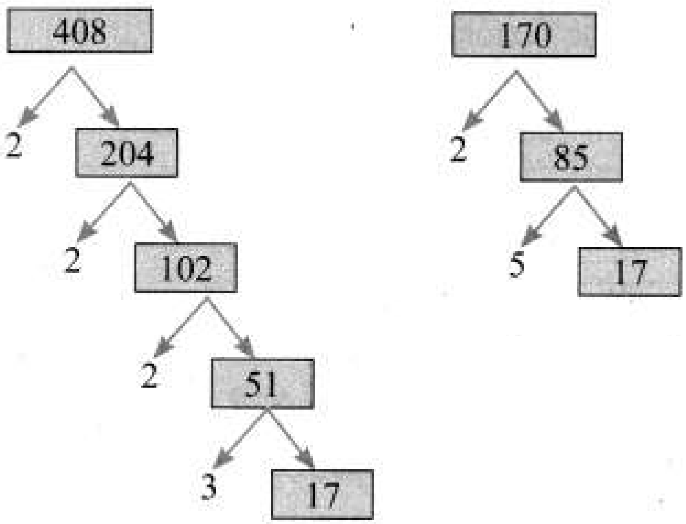
Question 5.
If $p_1^{x_1} \times p_2^{x_2} \times p_3^{x_3} \times p_4^{x_4} = 113400$ where p_1, p_2, p_3, p_4 are primes in ascending order and x_1, x_2, x_3, x_4 are integers, find the value of P_1, P_2, P_3, P_4 and x_1, x_2, x_3, x_4.
Solution:
If $p_1^{x_1} \times p_2^{x_2} \times p_3^{x_3} \times p_4^{x_4} = 113400$

p_1, p_2, p_3, P_4 are primes in ascending order, x_1, x_2, x_3, x_4 are integers.
using Prime factorisation tree.



$113400 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 7$
$= 23 \times 34 \times 52 \times 7$
$= p_1{}^{x}{}_1 \times p_2{}^{x}{}_2 \times p_3{}^{x}{}_3 \times p_4{}^{x}{}_4$
$\therefore p_1 = 2, p_2 = 3, p_3 = 5, p_4 = 7, x_1 = 3, x_2 = 4, x_3 = 2, x_4 = 1.$

Question 6.
Find the L.C.M. and H.C.F. of 408 and 170 by applying the fundamental theorem of arithmetic.
Solution:
408 and 170.



$408 = 2^3 \times 3^1 \times 17^1$
$170 = 2^1 \times 5^1 \times 17^1$

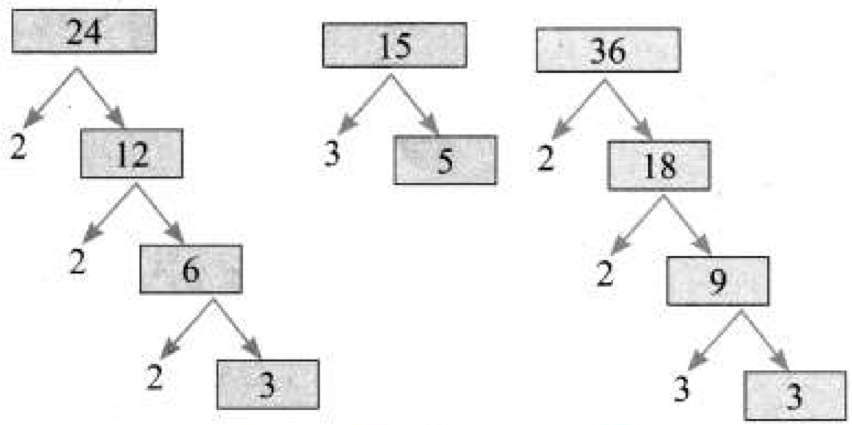
Common Prime Factors	Least Exponents
2	1
17	1

$\therefore$ H.C.F. $= 2^1 \times 17^1 = 34$.
To find L.C.M, we list all prime factors of 408 and 170, and their greatest exponents as follows.

Prime factors of 408 and 170	Greatest Exponents
2	3
3	1
5	1
17	1

$\therefore$ L.C.M. $= 2^3 \times 3^1 \times 5^1 \times 17^1$
$= 2040$.

Question 7.
Find the greatest number consisting of 6 digits which is exactly divisible by 24, 15, 36?
Solution:
To find L.C.M of 24, 15, 36



$24 = 2^3 \times 3$
$15 = 3 \times 5$
$36 = 2^2 \times 3^2$

Prime factors of 24, 15, 36	Greatest Exponents
2	3
3	2
5	1

$\therefore$ L.C.M $= 2^3 \times 3^2 \times 5^1$
$= 8 \times 9 \times 5$
$= 360$
If a number has to be exactly divisible by 24, 15, and 36, then it has to be divisible by 360. Greatest 6 digit number is 999999.
Common multiples of 24, 15, 36 with 6 digits are 103680, 116640, 115520, …933120, 999720 with six digits.
$\therefore$ The greatest number consisting 6 digits which is exactly divisible by 24, 15, 36 is 999720.

Question 8.
What is the smallest number that when divided by three numbers such as 35, 56 and 91 leaves remainder 7 in each case?
Answer:
Find the L.C.M of 35, 56, and 91
35 – 5 × 7 56
$56 = 2 \times 2 \times 2 \times 7$
$91 = 7 \times 13$
L.C.M = 23 × 5 × 7 × 13
= 3640

Since it leaves remainder 7
The required number = 3640 + 7
= 3647
The smallest number is = 3647

Question 9.
Find the least number that is divisible by the first ten natural numbers.
Solution:
The least number that is divisible by the first ten natural numbers is 2520.
Hint:
1,2, 3,4, 5, 6, 7, 8,9,10
The least multiple of 2 & 4 is 8
The least multiple of 3 is 9
The least multiple of 7 is 7
The least multiple of 5 is 5
$\therefore 5 \times 7 \times 9 \times 8 = 2520$.
L.C.M is $8 \times 9 \times 7 \times 5$
$= 40 \times 63$
$= 2520$

Ex 2.3

Question 1.
Find the least positive value of x such that
(i) $71 \equiv x \pmod 8$
(ii) $78 + x \equiv 3 \pmod 5$
(iii) $89 \equiv (x + 3) \pmod 4$
(iv) $96 = \frac{x}{7} \pmod 5$
(v) $5x \equiv 4 \pmod 6$
Solution:
To find the least value of x such that
(i) $71 \equiv x \pmod 8$
$71 \equiv 7 \pmod 8$
$\therefore x = 7$.[$\because 71 - 7 = 64$ which is divisible by 8]

(ii) $78 + x \equiv 3 \pmod 5$
$\Rightarrow 78 + x - 3 = 5n$ for some integer n.
$75 + x = 5n$
$75 + x$ is a multiple of 5.
$75 + 5 = 80$. 80 is a multiple of 5.
Therefore, the least value of x must be 5.

(iii) $89 \equiv (x + 3) \pmod 4$
$89 - (x + 3) = 4n$ for some integer n.
$86 - x = 4n$
$86 - x$ is a multiple of 4.
$\therefore$ The least value of x must be 2 then
$86 - 2 = 84$.
84 is a multiple of 4.
$\therefore$ x value must be 2.

(iv) $96 \equiv \frac{x}{7}$ (mod 5)
$96 - \frac{x}{7} = 5n$ for some integer n.
$\frac{672-x}{7} = 5n$
672 – x = 35n.
672 – x is a multiple of 35.
∴ The least value of x must be 7 i.e. 665 is a multiple of 35.

(v) 5x ≡ 4 (mod 6)
5x – 4 = 6M for some integer n.
5x = 6n + 4
$x = \frac{6n+4}{5}$
When we put 1, 6, 11, … as n values in $x = \frac{6n+4}{5}$ which is divisible by 5.
When n = 1, $x = \frac{10}{5} = 2$
When n = 6, $x = \frac{36+4}{5} = \frac{40}{5} = 8$ and so on.
∴ The solutions are 2, 8, 14…..
∴ Least value is 2.

Question 2.
If x is congruent to 13 modulo 17 then 7x – 3 is congruent to which number modulo 17?
Answer:
Given x ≡ 13 (mod 17) ……(1)
7x – 3 ≡ a (mod 17) ……..(2)
From (1) we get
x- 13 = 17 n (n may be any integer)
x – 13 is a multiple of 17
∴ The least value of x = 30
From (2) we get
7(30) – 3 ≡ a(mod 17)
210 – 3 ≡ a(mod 17)
207 ≡ a (mod 17)
207 ≡ 3(mod 17)
∴ The value of a = 3

Question 3.
Solve 5x ≡ 4 (mod 6)
Solution:
5x ≡ 4 (mod 6)
5x – 4 = 6M for some integer n.
5x = 6n + 4

$x = \frac{6n+4}{5}$ where n = 1, 6, 11,…..
∴ x = 2, 8, 14,…

Question 4.
Solve 3x – 2 ≡ 0 (mod 11)
Solution:
3x – 2 ≡ 0 (mod 11)
3x – 2 = 11 n for some integer n.
3x = 11n + 2

$$x = \frac{11n+2}{3} \text{ where } n = 2, 5, 8, \ldots$$

$$x = \frac{11 \times 2 + 2}{3} = 8$$

$$\therefore \quad x = \frac{11 \times 5 + 2}{3} = \frac{55+2}{3}$$

$$= \frac{57}{3} = 19$$

$$x = \frac{11 \times 8 + 2}{3} = \frac{88+2}{3}$$

$$= \frac{90}{3} = 30.$$

$$\therefore \quad x = 8, 19, 30, \ldots$$

Question 5.
What is the time 100 hours after 7 a.m.?
Solution:
100 ≡ x (mod 12) (∵7 comes in every 12 hrs)
100 ≡ 4 (mod 12) (∵ Least value of x is 4)
∴ The time 100 hrs after 7 O' clock is 7 + 4 = 11 O' clock i.e. 11 a.m

Question 6.
What is time 15 hours before 11 p.m.?
Answer:
15 ≡ x (mod 12)
15 ≡ 3 (mod 12)

The value of x must be 3.

$$\begin{array}{r|l} & \;\;12 \\ \hline 17 & 207 \\ & \underline{17} \\ & \;\;37 \\ & \;\;\underline{34} \\ & \;\;\;\;\underline{3} \end{array}$$

The time 15 hours before 11 o'clock is (11 – 3) 8 pm

Question 7.
Today is Tuesday. My uncle will come after 45 days. In which day my uncle will be coming?
Solution:
No. of days in a week = 7 days.
$45 \equiv x \pmod 7$
$45 - x = 7n$
$45 - x$ is a multiple of 7.
∴ Value of x must be 3.
∴ Three days after Tuesday is Friday. Uncle will come on Friday.

Question 8.
Prove that $2^n + 6 \times 9n$ is always divisible by 7 for any positive integer n.
Answer:
$9 = 2 \pmod 7$
$9^n = 2^n \pmod 7$ and $2^n = 2^n \pmod 7$
$2^n + 6 \times 9^n = 2^n \pmod 7 + 6\,[2^n \pmod 7]$
$= 2^n \pmod 7 + 6 \times 2^n \pmod 7$
$7 \times 2^n \pmod 7$
It is always divisible for any positive integer n

Question 9.
Find the remainder when 2^{81} is divided by 17.
Solution:
$2^{81} \equiv x \pmod{17}$
$2^{40} \times 2^{40} \times 2^{41} \equiv x \pmod{17}$
$(2^4)^{10} \times (2^4)^{10} \times 2^1 \equiv x \pmod{17}$
$(16)^{10} \times (16)^{10} \times 2 \equiv x \pmod{17}$
$(16^5)^2 \times (16^5)^2 \times 2$

$(16^5) \equiv 16 \pmod{17}$
$(16^5)^2 \equiv 16^2 \pmod{17}$
$(16^5)^2 \equiv 256 \pmod{17}$
$\equiv 1 \pmod{17}$ [$\because$ 255 is divisible by 17]
$(16^5)^2 \times (16^5)^2 \times 2 \equiv 1 \times 1 \times 2 \pmod{17}$
$\therefore 2^{81} \equiv 2 \pmod{17}$
$\therefore x = 2$

Question 10.
The duration of flight travel from Chennai to London through British Airlines is approximately 11 hours. The aeroplane begins its journey on Sunday at 23:30 hours. If the time at Chennai is four and a half hours ahead to that of London's time, then find the time in London, when will the flight lands at London Airport.
Solution:
The duration of the flight from Chennai to London is 11 hours.
Starting time at Chennai is 23.30 hrs. = 11.30 p.m.
Travelling time = 11.00 hrs. = 22.30 hrs = 10.30 a.m.
Chennai is $4\frac{1}{2}$ hrs ahead to London.
= 10.30 – 4.30 = 6.00
$\therefore$ At 6 a.m. on Monday the flight will reach at London Airport.

Ex 2.4

Question 1.
Find the next three terms of the following sequence.
(i) 8, 24, 72, …….
(ii) 5, 1, -3, …….
(iii) $\frac{1}{4}, \frac{2}{9}, \frac{3}{16}$ ………..
Solution:
(i) 8, 24, 72…
In an arithmetic sequence a = 8,
d = $t_1 - t_1 = t_3 - t_2$
= 24 – 8   72 – 24
= 16 ≠ 48
So, it is not an arithmetic sequence. In a geometric sequence,
r = $\frac{t_2}{t_1} = \frac{t_3}{t_2}$
⇒ $\frac{24}{8} = \frac{72}{24}$
⇒ 3 = 3
∴ It is a geometric sequence
∴ The n^{th} term of a G.P is $t_n = ar^{n-1}$
∴ $t_4 = 8 \times 3^{4-1}$
$= 8 \times 3^3$
$= 8 \times 27$
= 216

$t_5 = 8 \times 3^{5-1}$
$= 8 \times 3^4$
$= 8 \times 81$
= 648

$t_6 = 8 \times 3^{6-1}$
$= 8 \times 3^5$
$= 8 \times 243$
$= 1944$
The next 3 terms are 8, 24, 72, 216, 648, 1944.

(ii) 5, 1, -3, …
$d = t_2 - t_1 = t_3 - t_2$
$\Rightarrow 1 - 5 = -3-1$
$-4 = -4$ ∴ It is an A.P.
$t_n = a+(n-1)d$
$t_4 = 5 + 3 \times -4$
$= 5 - 12$
$= -7$
$15 = a + 4d$
$= 5 + 4 \times -4$
$= 5 - 16$
$= -11$
$t_6 = a + 5d$
$= 5 + 5 \times -4$
$= 5 - 20$
$= -15$
∴ The next three terms are 5, 1, -3, $\underline{-7}$, $\underline{-11}$, $\underline{-15.}$

(iii) $\frac{1}{4}, \frac{2}{9}, \frac{3}{16}, \ldots\ldots\ldots\ldots$
Here a_n = Numerators are natural numbers and denominators are squares of the next numbers
$\frac{1}{4}, \frac{2}{9}, \frac{3}{16}, \frac{4}{25}, \frac{5}{36}, \frac{6}{49} \ldots\ldots\ldots\ldots$

Question 2.
Find the first four terms of the sequences whose n^{th} terms are given by

(i) $a_n = n^3 - 2$
Answer:
$a_n = n^3 - 2$
$a_1 = 1^3 - 2 = 1 - 2 = -1$
$a_2 = 2^3 - 2 = 8 - 2 = 6$
$a_3 = 3^3 - 2 = 27 - 2 = 25$

$a_4 = 4^3 - 2 = 64 - 2 = 62$
The four terms are -1, 6, 25 and 62

(ii) $a_n = (-1)^{n+1}\ n(n + 1)$
Answer:
$a_n = (-1)^{n+1}\ n(n + 1)$
$a_1 = (-1)^2\ (1)\ (2) = 1 \times 1 \times 2 = 2$
$a_2 = (-1)^3\ (2)\ (3) = -1 \times 2 \times 3 = -6$
$a_3 = (-1)^4\ (3)\ (4) = 1 \times 3 \times 4 = 12$
$a_4 = (-1)^5\ (4)\ (5) = -1 \times 4 \times 5 = -20$
The four terms are 2, -6, 12 and -20

(iii) $a_n = 2n^2 - 6$
Answer:
$a_n = 2\ n^2 - 6$
$a_1 = 2(1)^2 - 6 = 2 - 6 = -4$
$a_2 = 2(2)^2 - 6 = 8 - 6 = 2$
$a_3 = 2(3)^2 - 6 = 18 - 6 = 12$
$a_4 = 2(4)^2 - 6 = 32 - 6 = 26$
The four terms are -4, 2, 12, 26

Question 3.
Find the n^{th} term of the following sequences
(i) 2, 5, 10, 17, ……….
(ii) 0, $\frac{1}{2}$, $\frac{2}{3}$,…..
(iii) 3, 8, 13, 18, ………
Solution:
(i) 2, 5, 10, 17
$= 1^2 + 1, 2^2 + 1, 3^2 + 1, 4^2 + 1$ ……….
$\therefore$ n^{th} term is n^2+1
(ii) 0, $\frac{1}{2}, \frac{2}{3}$,………….
$= \frac{1-1}{1}, \frac{2-1}{2}, \frac{3-1}{3}$ …..
$\Rightarrow \frac{n-1}{n}$
$\therefore$ nth term is $\frac{n-1}{n}$
(iii) 3, 8, 13, 18

a = 3
d = 5
$t_n = a + (n - 1)d$
$= 3 + (n - 1)5$
$= 3 + 5n - 5$
$= 5n - 2$
$\therefore n^{th}$ term is $5n - 2$

Question 4.
Find the indicated terms of the sequences whose n^{th} terms are given by
(i) $a_n = \frac{5n}{n+2}$; a_6 and a_{13}
(ii) $a_n = -(n^2 - 4)$; a_4 and a_{11}
Solution:

(i)
$$a_n = \frac{5n}{n+2}$$
$$a_{13} = \frac{5\times13}{13+2} = \frac{\cancel{65}^{13}}{\cancel{15}^{3}} = \frac{13}{3}$$
$$a_6 = \frac{5\times6}{6+2} = \frac{\cancel{30}^{15}}{\cancel{8}^{4}}$$
$$= \frac{15}{4}$$

(ii)
$$a_n = -(n^2 - 4);\ a_4 \text{ and } a_{11}$$
$$a_4 = -(4^2 - 4)$$
$$= -(16 - 4) = -12$$
$$a_{11} = -(11^2 - 4)$$
$$= -(121 - 4) = -117$$

Question 5.
Find a_8 and a_{15} whose n^{th} term is

$$a_n = \begin{cases} \dfrac{n^2-1}{n+3}; \; n \text{ is even}, n \in \mathbf{N} \\ \dfrac{n^2}{2n+1}; \; n \text{ is odd}, n \in \mathbf{N} \end{cases}$$

Solution:

$$a_n = \begin{cases} \dfrac{n^2-1}{n+3}, n \text{ is even} \\ \dfrac{n^2}{2n+1}, n \text{ is odd} \end{cases}$$

$$a_8 = \frac{n^2-1}{n+3} = \frac{8^2-1}{8+3} = \frac{64-1}{11} = \frac{63}{11}$$

$$a_{15} = \frac{n^2}{2n+1} = \frac{15^2}{2\times15+1} = \frac{225}{30+1} = \frac{225}{31}$$

Question 6.

If $a_1 = 1$, $a_2 = 1$ and $a_n = 2a_{n-1} + a_{n-2}$ $n \geq 3$, $n \in N$. Then find the first six terms of the sequence.

Answer:

$a_1 = a_2 = 1$

$a_n = 2a_{n-1} + a_{n-2}$

$a_3 = 2a_{3-1} + a_{3-2} = 2a_2 + a_1$

$= 2(1) + 1 = 3$

$a_4 = 2a_{4-1} + a_{4-2}$

$= 2a_3 + a_2$

$= 2(3) + 1 = 6 + 1 = 7$

$a_5 = 2\, a_{5-1} + a_{5-2}$

$= 2a_4 + a_3$

$= 2(7) + 3 = 17$

$a_6 = 2a_{6-1} + a_{6-2}$

$= 2a_5 + a_4$

$= 2(17) + 7$

$= 34 + 7 = 41$

The sequence is 1, 1, 3, 7, 17,41, …

Ex 2.5

Question 1.
Check whether the following sequences are in A.P.
(i) $a - 3, a - 5, a - 7, \ldots\ldots\ldots$
(ii) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{4}, \ldots\ldots\ldots$
(iii) 9, 13, 17, 21, 25, ………
(iv) $\frac{-1}{3}, 0, \frac{1}{3}, \frac{2}{3}, \ldots\ldots\ldots$
(v) 1,-1, 1,-1, 1,-1,…
Solution:
To prove it is an A.P, we have to show $d = t_2 - t_1 = t_3 - t_2$.
(i) $a - 3, a - 5, a - 7\ldots\ldots\ldots$
t_1, t_2, t_3
$d = t_2 - t_1 = a - 5 - (a - 3) = a - 5 - a + 3 = -2$
$\therefore d = -2 \quad \therefore$ It is an A.P.
$d = t_3 - t_2 = a - 7 - (a - 5) = a - 7 - a + 5 = -2$

(ii) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{4}, \ldots\ldots\ldots\ldots$

$d = t_2 - t_1$	$d = t_3 - t_2$
$\Rightarrow \frac{1}{3} - \frac{1}{2}$	$\frac{1}{4} - \frac{1}{3}$
$= \frac{2-3}{6}$	$= \frac{3-4}{12} = \frac{-1}{12}$
$= \frac{-1}{6}$	

$$\frac{-1}{6} \neq \frac{-1}{12}$$

$\Rightarrow t_2 - t_1 \neq t_3 - t_2$ $\quad\therefore$ It is not an A.P.

(iii) 9, 13, 17, 21, 25, . . .

$$d = t_2 - t_1 = 13 - 9 = 4$$
$$d = t_3 - t_2 = 17 - 13 = 4$$
$$4 = 4$$

$\therefore$ It is an A.P.

(iv) $\frac{-1}{3}, 0, \frac{1}{3}, \frac{2}{3}, \ldots$

$$d = t_2 - t_1 = 0 - \left(-\frac{1}{3}\right) = \frac{1}{3}$$
$$d = t_3 - t_2 = \frac{1}{3} - 0 = \frac{1}{3}$$
$$\frac{1}{3} = \frac{1}{3}$$

$\therefore$ It is an A.P.

(v) 1,-1, 1,-1, 1,-1,…

$d = t_2 - t_1 = -1 - 1 = -2$

$d = t_3 - t_2 = 1 - (-1) = 2$

$-2 \neq 2$ $\therefore$ It is not an A.P.

Question 2.

First term a and common difference d are given below. Find the corresponding A.P.

(i) a = 5, d = 6
(ii) a = 7, d = 5
(iii) a = $\frac{3}{4}$, d = $\frac{1}{2}$
Solution:
(i) a = 5, d = 6
A.P a, a + d, a + 2d, ...
= 5, 5 + 6, 5 + 2 × 6, ...
= 5, 11, 17,...
(ii) a = 7,d = -5
A.P. = a,a + d,a + 2d,...
= 7,7 + (-5), 7 + 2(-5), ...
= 7, 2, -3, ...,...
(iii) a = $\frac{3}{4}$, d = $\frac{1}{2}$

$$\begin{aligned} A.P &= a, a+d, a+2d, \ldots \\ &= \frac{3}{4}, \frac{3}{4}+\frac{1}{2}, \frac{3}{4}+2\left(\frac{1}{2}\right), \ldots \\ &= \frac{3}{4}, \frac{3+2}{4}, \frac{3+4}{4}, \ldots \\ A.P &= \frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \ldots \end{aligned}$$

Question 3.
Find the first term and common difference of the Arithmetic Progressions whose nthterms are given below

(i) t_n = -3 + 2n
Answer:
t_n = -3 + 2 n
t_1 = -3 + 2(1) = -3 + 2
= -1
t_2 = -3 + 2(2) = -3 + 4
= 1
First term (a) = -1 and
Common difference
(d) = 1 – (-1) = 1 + 1 = 2

(ii) t_n = 4 – 7n
Answer:

$t_n = 4 – 7n$
$t_1 = 4 – 7(1)$
$= 4 – 7 = -3$
$t_2 = 4 – 7(2)$
$= 4 – 14 = -10$
First term (a) = – 3 and
Common difference (d) = 10 – (-3)
= – 10 + 3
= – 7

Question 4.
Find the 19th term of an A.P. -11, -15, -19, ………..
Solution:
A.P = -11, -15, -19, ……..
a = -11
$d = t_2 – t_1$ =-15-(-11)
= -15 + 11
= -4
n = 19
$\therefore t_n = a + (n – 1)d$
$t_{19} = -11 + (19 – 1)(-4)$
= -11 + 18 × -4
= -11 – 72
= -83

Question 5.
Which term of an A.P. 16, 11, 6, 1,… is -54?
Answer:
First term (a) = 16
Common difference (d) = 11 – 16 = -5
$t_n = – 54$
a + (n – 1) d = -54
16 + (n – 1) (-5) = -54
54 + 21 = -54
54 + 21 = 5n
75 = 5n
$n = \frac{75}{5} = 15$
The 15^{th} term is – 54

Question 6.
Find the middle term(s) of an A.P. 9, 15, 21, 27, ……. ,183.

Solution:
A.P = 9, 15, 21, 27,…, 183
No. of terms in an A.P. is
$n = \frac{l-a}{d} + 1$
a = 9, l = 183, d = 15 – 9 = 6
$\therefore n = \frac{183-9}{6} + 1$
$= \frac{174}{6} + 1$
= 29 + 1 = 30
∴ No. of terms = 30. The middle must be 15th term and 16th term.
$\therefore t_{15} = a + (n - 1)d$
= 9 + 14 × 6
=9 + 84
= 93
$t_{16} = a + 15\ d$
= 9 + 15 × 6
= 9 + 90 = 99
∴ The middle terms are 93, 99.

Question 7.
If nine times the ninth term is equal to the fifteen times fifteenth term, Show that six times twenty fourth term is zero.
Answer:
$t_n = a + (n - 1)d$
9 times 9^{th} term = 15 times 15^{th} term
$9t_9 = 15\ t_{15}$
9[a + 8d] = 15[a + 14d]
9a + 72d = 15a + 210d
9a – 15a + 72 d – 210 d = 0
-6a – 138 d = 0
6a + 138 d = 0
6 [a + 23 d] = 0
6 [a + (24 – 1)d] = 0
$6\ t_{24} = 0$
∴ Six times 24th terms is 0.

Question 8.
If 3 + k, 18 – k, 5k + 1 are in A.P. then find k.
Solution:
3 + k, 18 – k, 5k + 1 are in A.P

$\Rightarrow$ 2b = a + c if a, b, c are in A.P

$$\therefore \underbrace{3+k}_{a}, \underbrace{18-k}_{b}, \underbrace{5k+1}_{c}$$

$$\begin{aligned} 2b &= a+c \\ \Rightarrow \quad 2(18-k) &= 3+k+5k+1 \\ 36-2k &= 4+6k. \\ 6k+2k &= 36-4 \\ 8k &= 32 \\ k &= \frac{32}{8} = 4 \end{aligned}$$

Question 9.
Find x,y and z gave that the numbers x, 10, y, 24, z are in A.P.

Answer:
x, 10, y, 24, z are in A.P
$t_2 - t_1 = 10 - x$
d = 10 – x …..(1)
$t_3 - t_2 = y - 10$
d = y – 10 ……(2)
$t_4 - t_3 = 24 - y$
d = 24 – y …..(3)
$t_5 - t_4 = z - 24$
d = z – 24 …..(4)
From (2) and (3) we get
y – 10 = 24 – y
2y = 24 + 10
2y = 34
y = 17
From (1) and (2) we get
10 – x = y – 10
– x – y = -10 -10
-x -y = -20
x + y = 20
x + 17 = 20(y = 17)
x = 20 – 17 = 3
From (1) and (4) we get
z – 24 = 10 – x
z – 24 = 10 – 3 (x = 3)

$z - 24 = 7$
$z = 7 + 24$
$z = 31$
The value of $x = 3$, $y = 17$ and $z = 31$

Question 10.
In a theatre, there are 20 seats in the front row and 30 rows were allotted. Each successive row contains two additional seats than its front row. How many seats are there in the last row?
Solution:
$t_1 = a = 20$
$t_2 = a + 2 = 22$
$t_3 = a + 2 + 2 = 24 \Rightarrow d = 2$
$\therefore$ There are 30 rows.
$t_{30} = a + 29d$
$= 20 + 29 \times 2$
$= 20 + 58$
$= 78$
$\therefore$ There will be 78 seats in the last row.

Question 11.
The Sum of three consecutive terms that are in A.P. is 27 and their product is 288. Find the three terms.
Answer:
Let the three consecutive terms be $a - d$, a and $a + d$
By the given first condition
$a - d + a + a + d = 27$
$3a = 27$
$a = \frac{27}{3} = 9$
Again by the second condition
$(a - d)(a)(a + d) = 288$
$a(a^2 - d^2) = 288$
$9(81 - d^2) = 288$ $(a = 9)$
$81 - d^2 = \frac{288}{9}$
$81 - d^2 = 32$
$\therefore d^2 = 81 - 32$
$= 49$
$d = \sqrt{49} = \pm 7$
When $a = 9$, $d = 7$
$a + d = 9 + 7 = 16$
$a = 9$

a – d = 9 – 7 = 2
When a = 9, d = -7
a + d = 9 – 7 = 2
a = 9
a – d = 9 – (-7) = 9 + 7 = 16
The three terms are 2, 9, 16 (or) 16, 9, 2

Question 12.
The ratio of 6th and 8th term of an A.P is 7:9. Find the ratio of 9th term to 13th term.
Solution:
$\frac{t_6}{t_8} = \frac{7}{9}$
$\frac{a+5d}{a+7d} = \frac{7}{9}$
9a + 45d = 7a + 49d
9 a + 45 – 7d = 7a + 49 d
9a + 45d – 7a – 49d = 0
2a – 4d = 0 ⇒ 2a = 4d
a = 2d
Substitue a = 2d in

$$\frac{t_9}{t_{13}} = \frac{a+8d}{a+12d} = \frac{2d+8d}{2d+12d} = \frac{10\cancel{d}}{14\cancel{d}} = \frac{5}{7}$$

$$\therefore \quad t_9 : t_{13} = 5:7.$$

Question 13.
In a winter season let us take the temperature of Ooty from Monday to Friday to be in A.P. The sum of temperatures from Monday to Wednesday is 0° C and the sum of the temperatures from Wednesday to Friday is 18° C. Find the temperature on each of the five days.
Solution:
Let the five days temperature be (a – d), a, a + d, a + 2d, a + 3d.
The three days sum = a – d + a + a + d = 0
⇒ 3a = 0 ⇒ a = 0. (given)
a + d + a + 2d + a + 3d = 18

$3a + 6d = 18$
$3(0) + 6d = 18$
$6d = 18$
$d = \frac{18}{6} = 3$
∴ The temperature of each five days is $a - d, a, a + d, a + 2d, a + 3d$
$0 - 3, 0, 0 + 3, 0 + 2(3), 0 + 3(3) = -3°C, 0°C, 3°C, 6°C, 9°C$

Question 14.
Priya earned ₹ 15,000 in the first month. Thereafter her salary increased by ₹1500 per year. Her expenses are ₹13,000 during the first year and the expenses increases by ₹900 per year. How long will it take for her to save ₹20,000 per month.
Solution:

	Yearly Salary	Yearly expenses	Yearly savings
1st year	15000	13000	2000
2nd year	16500	13900	2600
3rd year	18000	14800	3200

We find that the yearly savings is in A.P with $a_1 = 2000$ and $d = 600$.
We are required to find how many years are required to save 20,000 a year …………
$a_n = 20,000$
$a_n = a + (n - 1)d$
$20000 = 2000 + (n - 1)600$
$(n - 1)600 = 18000$
$n - 1 = \frac{18000}{600} = 30$
$n = 31$ years

Ex 2.6

Question 1.
Find the sum of the following
(i) 3, 7, 11, …….. up to 40 terms.
(ii) 102, 97, 92, ……… up to 27 terms.
(iii) 6 + 13 + 20 + ……… + 97
Solution:
(i) 3, 7, 11,. . . upto 40 terms.
a = 3, d = $t_2 - t_1$ = 7 – 3 = 4
n = 40
$S_n = \frac{n}{2}$ (2a + (n – 1)d)
$S_{40} = \frac{20}{2}$ (2× 3 + 39d)
= 20(6 + 39 × 4)
= 20(6 + 156)
= 20 × 162
= 3240

(ii) 102, 97, 952,… up to 27 terms
a = 102,
d = $t_2 - t_1$
= 97 – 102 = -5
n = 27

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_{27} = \frac{27}{2}(2 \times 102 + 26 \times -5)$$

$$= \frac{27}{\cancel{2}}(\cancel{74})^{37}$$

$$= 27 \times 37 = 999.$$

(iii) 6 + 13 + 20 + … + 97
a = 6,d = 7, l = 97

$$n = \frac{l-a}{d}+1$$

$$= \frac{97-6}{7}+1 = \frac{91}{7}+1$$

$$= \frac{91+7}{7} = \frac{98}{7} = 14$$

$$\therefore \quad S_n = \frac{n}{2}(a+l)$$

$$S_{14} = \frac{14}{2}(6+97)$$

$$= 7 \times 103 = 721$$

Question 2.
How many consecutive odd integers beginning with 5 will sum to 480?
Answer:
5,7,9, 11, 13,…
S_n = 480
a = 5, d = 2, S_n = 480

$$S_n = \frac{n}{2}(2a + (n-1)d)$$
$$480 = \frac{n}{2}[2 \times 5 + (n-1)2]$$
$$= \frac{n}{2}[10 + 2n - 2]$$
$$480 = \frac{n}{2}[8 + 2n]$$
$$8n + 2n^2 = 960$$

$2n^2 + 8n - 960 = 0$
$\Rightarrow n^2 + 4n - 480 = 0$
$\Rightarrow n^2 + 24n - 20n - 480 = 0$
$\Rightarrow n(n + 24) - 20(n + 24) = 0$
$\Rightarrow (n - 20)(n + 24) = 0$
$\Rightarrow n = 20,-24$
No. of terms cannot be -ve.
$\therefore$ No. of consecutive odd integers beginning with 5 will sum to 480 is 20.

Question 3.
Find the sum of first 28 terms of an A.P. whose n^{th} term is $4n - 3$.
Answer:
Number of terns (n) = 28
$t_n = 4n - 3$
$t_1 = 4(1) - 3 = 4 - 3 = 1$
$t_2 = 4(2) - 3 = 8 - 3 = 5$
$t_3 = 4(3) - 3 = 12 - 3 = 9$
Here a = 1, d = 5 – 1 = 4
$S_{28} = \frac{n}{2}\ [2a + (n - 1)d]$
$= \frac{28}{2}\ [2 + (27)(4)]$
= 14 [2 + 108]
= 14 × 110
= 1540
Sum of 28 terms = 1540

Question 4.
The sum of first n terms of a certain series is given as 2n2 – 3n. Show that the series is an A.P.
Solution:

Given $S_n = 2n^2 - 3n$

$S_1 = 2(1)^2 - 3(1) = 2 - 3 = -1$

$\Rightarrow t_1 = a = -1$

$S_2 = 2(2^2) - 3(2) = 8 - 6 = 2$

$t_2 = S_2 - S_1 = 2 - (-1) = 3$

$\therefore d = t_2 - t_1 = 3 - (-1) = 4$

Consider a, a + d, a + 2d, ……..

-1, -1 + 4, -1 + 2(4), ……..

-1, 3, 7,….

Clearly this is an A.P with a = – 1, and d = 4.

Question 5.

The 104^{th} term and 4th term of an A.P are 125 and 0. Find the sum of first 35 terms.

Solution:

$t_{104} = 125$

$t_4 = 0$

$a + (n - 1)d = t_n$

$$\begin{aligned} a + 103d &= 125 \quad &...(1) \\ a + 3d &= 0 \quad &...(2) \\ (-)\ \ (-) \quad & \quad (-) & \end{aligned}$$

$$(1) - (2) \Rightarrow 100d = 125$$

$$d = \frac{125}{100} = \frac{5}{4}$$

$$\text{Substitute } d = \frac{5}{4} \text{ in (2)}$$

$$a + 3 \times \frac{5}{4} = 0$$

$$a + \frac{15}{4} = 0 \Rightarrow a = -\frac{15}{4}$$

$$\therefore S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_{35} = \frac{35}{2}\left(\cancel{2} \times \frac{-15}{\cancel{4}_2} + \cancel{34}^{17} \times \frac{5}{\cancel{4}_2}\right)$$

$$= \frac{35}{2}\left(\frac{-15}{2} + \frac{85}{2}\right)$$

$$= \frac{35}{2}\left(\frac{70}{2}\right) = \frac{35}{2} \times 35$$

$$= \frac{1225}{2}$$

$$= 612.5$$

Question 6.
Find the sum of all odd positive integers less than 450.
Solution:
Sum of all odd positive integers less than 450 is given by
$1 + 3 + 5 + \ldots + 449$
$a = 1$
$d = 2$
$l = 449$

$$\therefore \quad n = \frac{l-a}{d}+1 = \frac{449-1}{2}+1$$
$$= \frac{448}{2}+1$$
$$= 224 + 1 = 225$$
$$\therefore \quad S_n = \frac{n}{2}(a+l)$$
$$S_{225} = \frac{225}{2}(1+449)$$
$$= \frac{225}{\cancel{2}} \times \cancel{450}^{225}$$
$$= 225^2$$
= 50625
Another method:
Sum of all +ve odd integers = n^2.
We can use the formula $n^2 = 225^2$
= 50625

Question 7.
Find the sum of all natural numbers between 602 and 902 which are not divisible by 4.
Answer:
Natural numbers between 602 and 902
603,604, …, 901
a = 603, l = 901, d = 1,

$$\therefore \quad n = \frac{l-a}{d}+1 = \frac{901-603}{1}+1$$

$$= 298 + 1 = 299$$

$$S_n = \frac{n}{2}(a+l)$$

$$S_{299} = \frac{299}{2}(603+901)$$

$$= \frac{299}{2} \times 1504$$

$$= 224848$$

Sum of all natural numbers between 602 and 902 which are not divisible by 4.
= Sum of all natural numbers between 602 and 902
= Sum of all natural numbers between 602 and 902 which are divisible by 4.
l = 902 – 2 = 900
To make 602 divisible by 4 we have to add 2 to 602.
∴ 602 + 2 = 604 which is divisible by 4.
To make 902 divisible by 4, subtract 2 from 902.
∴ 900 is the last number divisible by 4.

$a = 604, l = 900, d = 4, n = \frac{l-a}{d} + 1$

$$\begin{array}{r|l} & 150 \\ \hline 4 & 602 \\ & 600 \\ \hline & 2 \end{array} \qquad \begin{array}{r|l} & 225 \\ \hline 4 & 902 \\ & 900 \\ \hline & 2 \end{array}$$

$$n = \frac{900-604}{4} + 1 = \frac{296}{4} + 1$$
$$= 74 + 1 = 75$$
$$S_n = \frac{n}{2}(a + l)$$
$$S_{75} = \frac{75}{2}(604 + 900)$$
$$= \frac{75}{2}(1504)$$
$$= 56400$$

Sum of all natural numbers between 602 and 902 which are not divisible 4.
= 224848 – 56400
= 168448

Question 8.
Raghu wish to buy a Laptop. He can buy it by paying ₹40,000 cash or by making 10 installments as ₹4800 in the first month, ₹4750 in the second month, ₹4700 in the third month and so on. If he pays the money in this fashion, Find
(i) Total amount paid in 10 installments.
(ii) How much extra amount that he pay in installments.
Answer:
(i) Amount paid in 10 installments
4800 + 4750 + 4700 + ……………. 10
Here a = 4800; d = – 50 ; n = 10
$S_n = \frac{n}{2}$ [2a +(n – 1)d]
$S_{10} = \frac{10}{2}$ [2 × 4800 +9(-50)]
= $\frac{10}{2}$ [9600 – 450]
= 5 [9150]
= 45750
Amount paid in 10 installments

= □45750
(ii) Extra amount paid = amount paid in 10
installment – cost of the laptop
= □45750 – 40,000
= □ 5750
(i) Amount paid in 10 installments = □ 45750
(ii) Difference in payment = □ 5750

Question 9.
A man repays a loan of □65,000 by paying □400 in the first month and then increasing the payment by □300 every month. How long will it take for him to clear the loan?
Solution:
Loan Amount = □ 65,000
Repayment through installments
400 + 700 + 1000 + 1300 + …
a = 400
d = 300
S_n = 65000
$S_n = \frac{n}{2}(2a + (n - 1)d)$
= 65000
$\frac{n}{2}(2 \times 400 + (n - 1)300) = 65000$
n(800 + 300n – 300) = 130000
n(500 + 300n) = 130000
$500n + 300n^2 = 130000$

$$3\cancel{00}n^2 + 5\cancel{00}n = 1300\cancel{00}$$

$$3n^2 + 5n - 1300 = 0$$

$$(n - 20)(3n + 65) = 0$$

$$n = 20,\ n = \frac{-65}{3}$$

$$-3900 \quad (5)$$
$$-60 \quad 65$$
$$\frac{-60}{3} \quad \frac{65}{3}$$
$$-20 \quad \frac{65}{3}$$

$$\therefore n = 20$$

Number of terms should be (+ve) and cannot be (-ve) or fractional number.
∴ He will take 20 months to clear the loans.

Question 10.
A brick staircase has a total of 30 steps. The bottom step requires 100 bricks. Each successive step requires two bricks less than the previous step.

(i) How many bricks are required for the top most step?
(ii) How many bricks are required to build the stair case?
Answer:
Total number of steps = 30
∴ $n = 30$
Number of bricks for the bottom = 100
$a = 100$
2 bricks is less for each step

(i) Number of bricks required for the top most step
$t_n = a + (n - 1)d$
$t_{30} = 100 + 29 (-2)$
$= 100 - 58$
$= 42$

(ii) Number of bricks required
$S_n = \frac{n}{2} [2a + (n-1) d]$
$S_{30} = \frac{30}{2} [200 + 29 (-2)]$
$= 15[200 - 58]$
$= 2130$
(i) Number of bricks required for the top most step = 42 bricks
(ii) Number of bricks required = 2130

Question 11.
If $S_1, S_2, S_3, \ldots, S_m$ are the sums of n terms of m A.P.'s whose first terms are 1,2,3,…,m and whose common differences are 1, 3, 5,…, (2m – 1) respectively, then show that
$S_1 + S_2 + S_3 + \ldots + Sm = \frac{1}{2} mn(mn + 1)$.
Solution:

First term	d	No. of terms	Sum of n terms
1	1	n	$S_1 = \frac{n}{2}(2\times1+(n-1)1)$
2	3	n	$S_2 = \frac{n}{2}(2\times2+(n-1)3)$
3	5	n	
.	.	.	.
.	.	.	.
.	.	.	.
m	$(2m-1)$	n	$S_m = \frac{n}{2}[2m+(n-1)(2m-1)]$

$$S_1 = \frac{n}{2}(2+(n-1)) = \frac{n}{2}(n+1)$$

$$S_2 = \frac{n}{2}(4+3n-3) \quad = \frac{n}{2}(3n+1)$$

$$S_3 = \frac{n}{2}(6+5n-5) \quad = \frac{n}{2}(5n+1)$$

$$S_m = \frac{n}{2}\ [2m + 2mn - 2m - n + 1]$$

$$= \frac{n}{2}(n(2m-1)+1)$$

$$\therefore S_1 + S_2 + S_3 + \ldots + S_m$$

$$= \frac{n}{2}[n + 3n + 5n + \ldots (2m-1)n + m \times 1]$$

$$= \frac{n}{2}[n\ (1 + 3 + 5 + \ldots + (2m-1) + m]$$

$$= \frac{n}{2}\left[n \times \frac{m}{\cancel{2}}(\cancel{2}m - \cancel{1} + \cancel{1}) + m\right]$$

$$= \frac{n}{2}[m^2n + m]$$

$$= \frac{1}{2}mn\ (mn+1)$$

Hence proved.

Question 12.
Find the sum

$$\left[\frac{a-b}{a+b} + \frac{3a-2b}{a+b} + \frac{5a-3b}{a+b} + \ldots \text{ to 12 terms}\right]$$

Solution:

$$= \frac{1}{a+b}\ [(a-b)+(3a-2b)+(5a-3b)+\ldots \text{ 12 terms}]$$

$$\text{Here } a = \frac{a-b}{a+b},\ d = t_2 - t_1$$

$$= \frac{3a-2b}{a+b} - \frac{a-b}{a+b}$$

$$d = \frac{2a-b}{a+b}$$

$$\therefore \quad S_n = \frac{n}{2}(2a+(n-1)d)$$

$$S_{12} = \frac{12}{2}\left[2\left(\frac{a-b}{a+b}\right)+11\times\left(\frac{2a-b}{a+b}\right)\right]$$

$$= 6\left[\frac{2a-2b+22a-11b}{a+b}\right]$$

$$= 6\left[\frac{24a-13b}{a+b}\right]$$

Ex 2.7

Question 1.
Which of the following sequences are in G.P?
(i) 3, 9, 27, 81, ……..
(ii) 4, 44, 444, 4444, ………
(iii) 0.5, 0.05, 0.005, ……..
(iv) $\frac{1}{3}, \frac{1}{6}, \frac{1}{12}$ ………
(v) 1, -5, 25, -125, …….
(vi) 120, 60, 30, 18, …….
(vii) 16, 4, 1, $\frac{1}{4}$, ……
Solution:
(i) 3, 9, 27, 81
r = Common ratio

$$r = \frac{t_2}{t_1} = \frac{t_3}{t_2} \text{ in G.P}$$

$$\text{Here } \frac{t_2}{t_1} = \frac{9}{3} = 3$$

$$\frac{t_3}{t_2} = \frac{27}{9} = 3$$

$\therefore$ It is a G.P.

(ii) 4, 44, 444, 4444,......

$$r = \frac{t_2}{t_1} = \frac{44}{4} = 11$$

$$r = \frac{t_3}{t_2} = \frac{444}{44} = \frac{111}{11}$$

$$11 \neq \frac{111}{11}$$

$\therefore$ It is not a G.P.

(iii) 0.5, 0.05, 0.005, . . .

$$r = \frac{t_2}{t_1} = \frac{0.05}{0.5} = \frac{0.05\times100}{0.5\times100}$$

$$= \frac{5}{50} = \frac{1}{10}$$

$$r = \frac{t_3}{t_2} = \frac{0.005}{0.05} = \frac{0.005\times1000}{0.05\times1000}$$

$$= \frac{5}{50} = \frac{1}{10}$$

$$r = \frac{1}{10} = \frac{1}{10}.$$

$\therefore$ It is a G.P.

(iv) $\frac{1}{3}, \frac{1}{6}, \frac{1}{12} \ldots$

$$r = \frac{t_2}{t_1} = \frac{\frac{1}{6}}{\frac{1}{3}} = \frac{1}{\cancel{6}_2} \times \frac{\cancel{3}}{1} = \frac{1}{2}$$

$$r = \frac{t_3}{t_2} = \frac{\frac{1}{12}}{\frac{1}{6}} = \frac{1}{\cancel{12}_2} \times \frac{\cancel{6}}{1} = \frac{1}{2}$$

$$r = \frac{1}{2} = \frac{1}{2}$$

$\therefore$ It is a G.P.

(v) $1, -5, 25, -125$

$$r = \frac{t_2}{t_1} = \frac{-5}{1} = -5$$

$$r = \frac{t_3}{t_2} = \frac{25}{-5} = -5$$

$$-5 = -5$$

$\therefore$ It is a G.P

(vi) 120, 60, 30, 18, . . .

$$r = \frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_4}{t_3}$$

Here r is not equal i.e $\frac{60}{120} = \frac{30}{60} \neq \frac{18}{30}$

$\therefore$ It is not a G.P

(vii) 16, 4, 1, $\frac{1}{4}$, . . .

$$r = \frac{t_2}{t_1} = \frac{4}{16} = \frac{1}{4}$$

$$r = \frac{t_3}{t_2} = \frac{1}{4}$$

$$r = \frac{1}{4} = \frac{1}{4}$$

$\therefore$ It is a G.P

Question 2.
Write the first three terms of the G.P. whose first term and the common ratio are given below.
(i) a = 6, r = 3
(ii) a = $\sqrt{2}$, r = $\sqrt{2}$
(iii) a = 1000, r = $\frac{2}{5}$
Solution:
(i) a = 6, r = 3
$t_n = ar^{n-1}$
$t_1 = ar^{1-1} = ar^0 = a = 6$
$t_2 = ar^{2-1} = ar^1 = 6 \times 3 = 18$
$t_3 = ar^{3-1} = ar^2 = 6 \times 3^2 = 54$
$\therefore$ The 3 terms are 6, 18, 54, ….

(ii) $a = \sqrt{2}$, $r = \sqrt{2}$

$$t_n = ar^{n-1}$$

$$t_1 = ar^{1-1} = ar^0 = \sqrt{2} \times 1 = \sqrt{2}$$

$$t_2 = ar^{2-1} = ar^1 = \sqrt{2} \times \sqrt{2} = 2$$

$$t_3 = ar^{3-1} = ar^2 = \sqrt{2} \times (\sqrt{2})^2$$

$$= \sqrt{2} \times 2 = 2\sqrt{2}$$

$\therefore$ The 3 terms are $\sqrt{2}$, 2, $2\sqrt{2}$, . . .

(iii) $a = 1000$, $r = \frac{2}{5}$

$$t_n = ar^{n-1}$$

$$t_1 = ar^{1-1} = ar^0 = 1000 \times 1 = 1000$$

$$t_2 = ar^{2-1} = ar = \cancel{1000}^{200} \times \frac{2}{\cancel{5}} = 400$$

$$t_3 = ar^{3-1} = ar^2 = 1000\left(\frac{2}{5}\right)^2$$

$$= \cancel{1000}^{40} \times \frac{4}{\cancel{25}}$$

$$= 160$$

The 3 terms are 1000, 400, 160, ……………

Question 3.
In a G.P. 729, 243, 81,… find t_7.

Solution:
G.P = 729, 243, 81 ……

$t_7 = ?$

$$t_n = ar^{n-1}, \text{ here } a = 729, r = \frac{t_2}{t_1}$$

$$r = \frac{243}{729} = \frac{1}{3}$$

$$\therefore\ t_7 = 729\left(\frac{1}{3}\right)^{7-1} = 729 \times \left(\frac{1}{3}\right)^6$$

$$= 729 \times \frac{1}{729}$$

$$= 1$$

Question 4.
Find x so that x + 6, x + 12 and x + 15 are consecutive terms of a Geometric Progression.
Answer:
$\frac{t_2}{t_1} = \frac{x+12}{x+6}, \frac{t_3}{t_2} = \frac{x+15}{x+12}$
Since it is a G.P.
$\frac{x+12}{x+6} = \frac{x+15}{x+12}$
$(x + 12)^2 = (x + 6)(x + 15)$
$x^2 + 24x + 144 = x^2 + 21x + 90$
$3x = -54 \Rightarrow x = \frac{-54}{3} = -18$

Question 5.
Find the number of terms in the following G.P.
(i),4, 8, 16, …, 8192
(ii) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots\ldots \frac{1}{2187}$
Solution:
(i) 4, 8, 16, …… 8192

$$a = 4$$

$$r = \frac{8}{4} = 2$$

$$t_n = 8192$$

$$t_n = ar^{n-1}$$

$$8192 = 4 \times (2)^{n-1}$$

$$\cancel{4} \times 2^{n-1} = \cancel{8192}^{2048}$$

$$2^{n-1} = 2048$$

$$2^{n-1} = 2^{11}$$

$$n - 1 = 11$$

$$n = 11 + 1 = 12$$

$\therefore$ No. of terms $= 12$

Hint:

$$\begin{array}{r|l} 2 & 2048 \\ \hline 2 & 1024 \\ \hline 2 & 512 \\ \hline 2 & 256 \\ \hline 2 & 128 \\ \hline 2 & 64 \\ \hline 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

(ii) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots, \frac{1}{2187}$

Hint:

$$\begin{array}{r|l} 3 & 729 \\ \hline 3 & 243 \\ \hline 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

Here $a = \frac{1}{3}$, $r = \frac{\frac{1}{9}}{\frac{1}{3}} = \frac{1}{\cancel{9}_3} \times \frac{\cancel{3}}{1} = \frac{1}{3}$

$$t_n = ar^{n-1}$$

$$\frac{1}{3} \times \left(\frac{1}{3}\right)^{n-1} = \frac{1}{2187}$$

$$\left(\frac{1}{3}\right)^{n-1} = \frac{1}{\cancel{2187}_{729}} \times \cancel{3}$$

$$= \frac{1}{729}$$

$$\left(\frac{1}{3}\right)^{n-1} = \frac{1}{3^6} = \left(\frac{1}{3}\right)^6$$

$$n - 1 = 6$$

$$n = 6 + 1 = 7$$

$$\therefore \text{No. of terms} = 7$$

Question 6.

In a G.P. the 9^{th} term is 32805 and 6^{th} term is 1215. Find the 12th term.

Solution:

In a G.P

$t_n = ar^{n-1}$

$t_9 = 32805$

$t_6 = 1215$

$t_{12} = ?$

Let

$t_9 = ar^8 = 32805$ ………(1)

$t_6 = ar^5 = 1215$ ………. (2)

$$\frac{(1)}{(2)} = \frac{\cancel{a}r^8}{\cancel{a}r^5} = \frac{32805}{1215}$$

$$r^{8-5} = 27$$

$$r^3 = 3^3$$

$$\Rightarrow \quad r = 3$$

Substitute $r = 3$ in (2)

$$a \times 3^5 = 1215$$

$$\therefore \quad a = 5$$

$$t_{12} = ar^{11}$$

$$= 5 \times 3^{11}$$

$$= 5 \times 177147$$

$$= 885735$$

Hint:

$$\begin{array}{r|l} 3 & 1215 \\ \hline 3 & 405 \\ \hline 3 & 135 \\ \hline 3 & 45 \\ \hline 3 & 15 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

Question 7.

Find the 10th term of a G.P. whose 8^{th} term is 768 and the common ratio is 2.

Answer:

Here $r = 2$, $t_8 = 768$

$t_8 = 768$ ($t_n = ar^{n-1}$)

$a.\ r^{8-1} = 768$

$ar^7 = 768$ …..(1)

10^{th} term of a G.P. $= a.r\ 10^{-1}$

$= ar^9$

$= (ar^7) \times (r^2)$

$= 768 \times 2^2$ (from 1)

$= 768 \times 4 = 3072$

$\therefore$ 10^{th} term of a G.P. $= 3072$

Question 8.

If a, b, c are in A.P. then show that $3^a, 3^b, 3^c$ are in G.P.

Solution:

If a, b, c are in A.P

$t_2 - t_1 = t_3 - t_2$

$b - a = c - b$

$2b = c + a$

To prove that 3^a, 3^b, 3^c are in G.P
⇒ $3^{2b} = 3^{c+a}$ + a [Raising the power both sides]
⇒ $3^b.3^b = 3^c.3^a$
⇒ $\frac{3^b}{3^a} = \frac{3^c}{3^b}$
⇒ $\frac{t_2}{t_1} = \frac{t_3}{t_1}$
⇒ Common ratio is same for 3^a,3^b,3^c
⇒ 3^a, 3^b, 3^c forms a G.P
∴ Hence it is proved .

Question 9.
In a G.P. the product of three consecutive terms is 27 and the sum of the product of two terms taken at a time is $\frac{57}{2}$. Find the three terms.
Solution:
Let the three consecutive terms in a G.P are $\frac{a}{r}$, a, ar.
Their Product = $\frac{a}{r}$ × a × ar = 27
$a^3 = 27 = 3^3$
a = 3
Sum of the product of terms taken two at a time is $\frac{57}{2}$

$$\frac{a}{r}\times a + a\times ar + a\cancel{r}\times\frac{a}{\cancel{r}} = \frac{57}{2}$$

$$\frac{a^2}{r} + a^2r + a^2 = \frac{57}{2}$$

$$3^2\left(\frac{1}{r} + r + 1\right) = \frac{57}{2}$$

$$\frac{1 + r^2 + r}{r} = \frac{57}{2}\times\frac{1}{9} = \frac{57}{18}$$

$$18 + 18r^2 + 18r = 57r$$

$$18r^2 + 18r - 57r + 18 = 0$$

$$18r^2 - 39r + 18 = 0 \div 3$$

Hint:

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$$\frac{-\cancel{4}^{2}}{\cancel{6}_{3}} \qquad \frac{-\cancel{9}^{3}}{\cancel{6}_{2}}$$

$$\Rightarrow \quad 6r^2 - 13r + 6 = 0$$

$$\left(r - \frac{2}{3}\right)\left(r - \frac{3}{2}\right) = 0$$

$$r = \frac{2}{3}, \frac{3}{2}$$

$$\text{If } a = 3, r = \frac{2}{3}$$

$\therefore$ The three numbers are $\frac{3}{\frac{2}{3}}, 3, 3 \times \frac{2}{3}$

(or) $3 \times \frac{2}{3}, 3, \cancel{3} \times \frac{2}{\cancel{3}}$

$\frac{9}{2}, 3, 2$

If $a = 3, r = \frac{3}{2}$, the three numbers are

$$\frac{a}{r}, a, ar = \frac{3}{\frac{3}{2}}, 3, 3 \times \frac{3}{2}$$

$$= \frac{6}{3}, 3, \frac{9}{2}$$

$$= 2, 3, \frac{9}{2}$$

Question 10.
A man joined a company as Assistant Manager. The company gave him a starting salary of ₹60,000 and agreed to increase his salary 5% annually. What will be his salary after 5 years?
Solution:
Starting salary = ₹ 60,000
Increase per year = 5%
∴ At the end of 1 year the increase
= 60,0,00 × $\frac{5}{100}$
₹ 3000
∴ At the end of first year his salary
= ₹ 60,000 + 3000

I year salary = □ 63,000
II Year increase = 63000 × $\frac{5}{100}$
At the end of II year, salary
= 63000 + 3150
= □ 66150
III Year increase = 66150 × $\frac{5}{100}$
= 3307.50
At the end of III year, salary = 66150 + 3307.50
= □ 69457.50
IV year increase = 69457.50 × $\frac{5}{100}$
= □ 3472.87

At the end of IV year, salary = 69457.50 + 3472.87
= 72930.37

V year increase = ₹ 72930.37 × $\frac{5}{100}$
= ₹ 3646.51

At the end of V year, salary = 72930.37 + 3646.51
= 76576.88
= ₹ 76577

Question 11.
Sivamani is attending an interview for a job and the company gave two offers to him. Offer A: □ 20,000 to start with followed by a guaranteed annual increase of 3% for the first 5 years.
Offer B: □ 22,000 to start with followed by a guaranteed annual increase of 3% for the first 5 years.
What is his salary in the 4th year with respect to the offers A and B?
Solution:
Offer A
Starting salary □ 20,000
Annual increase 6%

i.e. $₹\ 20{,}0\cancel{00} \times \frac{6}{1\cancel{00}}$

$= ₹\ 1200$

At the end of I year salary = 20000 + 1200

$= ₹\ 21200$

II year increase $= 212\cancel{00} \times \frac{6}{1\cancel{00}}$

$= ₹\ 1272$

At the end of II year salary

$= 21200 + 1272 = 22472$

III year increase $= 22472 \times \frac{6}{100} = 1348.32$

At the end of
III year ,salary = 22472 + 1348 = 23820
∴ IV year salary = ₹ 23820
Offer B
Starting salary = ₹ 22,000

Annual increase $= 3\% = \frac{3}{100}$

I year, increase $= 220\cancel{00} \times \frac{3}{1\cancel{00}} = ₹\ 660$

At the end of

I year, salary $= 22000 + 660$

$= ₹\ 22660$

II year increase $= 2266\cancel{0} \times \frac{3}{10\cancel{0}}$

$= ₹\ 679.8$

At the end of

II year, salary $= ₹\ 23339.80$

III year increase $= 23339.8 \times \frac{3}{100}$

$= ₹\ 700$

At the end of

III year, salary $= ₹\ 24039.80$

$\therefore$ IV year salary $= \boxed{₹\ 24040}$

Salary as per Option A = ₹ 23820

Salary as per Option B = ₹ 24040

∴ Option B is better.

Question 12.

If a, b, c are three consecutive terms of an A.P. and x, y, z are three consecutive terms of a G.P. then prove that $x^{b-c} \times y^{c-a} \times z^{a-b} = 1$.

Solution:

a, b, c are three consecutive terms of an AP.

∴ Let a, b, c be a, a + d, a + 2d respectively ………… (1)

x, y, z are three consecutive terms of a GP.

∴ Assume x, y, z as x, x.r, $x.r^2$ respectively ……… (2)

PT : x^{b-c} , y^{c-a} , $z^{a-b} = 1$

Substituting (1) and (2) in LHS, we get

LHS = $x^{a+d-a-2d} \times (xr)^{a+2d-a} \times (xr^2)^{a-a-d}$

$= (x)^{-d} . (xr)^{2d} (xr^2)^{-d}$

$= \frac{1}{x^d} \times x^{2d} . r^{2d} \times \frac{1}{x^d r^{2d}} = 1 =$ RHS

Ex 2.8

Question 1.
Find the sum of first n terms of the G.P.
(i) 5, -3, $\frac{9}{5}$, $-\frac{27}{25}$, ……….
(ii) 256, 64, 16, ……..
Solution:
(i) 5, -3, $\frac{9}{5}$, $\frac{-27}{25}$, ………

$$\text{Here } a = 5, r = \frac{t_2}{t_1} = \frac{-3}{5} < 1$$

$$S_n = a\left(\frac{1-r^n}{1-r}\right)$$

$$= 5\left[\frac{1-\left(\frac{-3}{5}\right)^n}{1-\left(\frac{-3}{5}\right)}\right] = 5\left[\frac{1-\left(\frac{-3}{5}\right)^n}{1-\frac{-3}{5}}\right]$$

$$= 5\,\frac{\left[1-\left(\frac{-3}{5}\right)^n\right]}{\frac{8}{5}} = 5\times\frac{5}{8}\left(1-\left(\frac{-3}{5}\right)^n\right)$$

$$\therefore S_n = \frac{25}{8}\left(1-\left(\frac{-3}{5}\right)^n\right)$$

(ii) 256, 64, 16,…….

a = 256

$$r = \frac{64}{256} = \frac{4}{16} = \frac{1}{4} < 1$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$= 256\left(\frac{1-\left(\frac{1}{4}\right)^n}{1-\frac{1}{4}}\right)$$

$$= \frac{256\left(1-\left(\frac{1}{4}\right)^n\right)}{\frac{3}{4}}$$

$$= \frac{1024}{3}\left(1-\left(\frac{1}{4}\right)^n\right)$$

Question 2.
Find the sum of first six terms of the G.P. 5, 15, 45, …
Solution:

G.P.= 5, 15, 45, 15

$$n = 6, a = 5, r = \frac{15}{5} = 3 > 1$$

$$\therefore \quad S_n = a\frac{(r^n - 1)}{r-1}$$

$$S_6 = 5\left(\frac{3^6 - 1}{3-1}\right)$$

$$= 5\frac{(3^6 - 1)}{2} = \frac{5}{2}(729-1)$$

$$= \frac{5}{2} \times 728 = 5 \times 364$$

$$= 1820$$

Question 3.
Find the first term of the GP. whose common ratio 5 and whose sum to first 6 terms is 46872.
Solution:
Common ratio, r = 5
S_6 = 46872

$$\therefore \quad \frac{a(r^6 - 1)}{r-1} = 46872$$

$$\Rightarrow \quad S_6 = \frac{a(5^6 - 1)}{5-1} = 46872$$

$$\Rightarrow \quad a = \frac{46872 \times 4}{[25 \times 25 \times 25 - 1]}$$

$$\Rightarrow \quad a = \frac{187488}{15624} = 12$$

$$\Rightarrow \quad \text{first term} = 12$$

Question 4.
Find the sum to infinity of (i) 9 + 3 + 1 + ….
(ii) 21 + 14 + $\frac{28}{3}$ + ….
Solution:
(i) 9 + 3 + 1 + ….

$a = 9, r = \frac{3}{9} = \frac{1}{3} < 1$

$$S_\infty = \frac{a}{1-r} = \frac{9}{1-\frac{1}{3}}$$

$$= \frac{9}{\frac{3-1}{3}} = \frac{\frac{9}{2}}{3} = 9 \times \frac{3}{2} = \frac{27}{2}$$

(ii) $21 + 14 + \frac{28}{3} + ...$

Here $a = 21, r = \frac{14}{21} = \frac{2}{3}$

$$S_\infty = \frac{a}{1-r} = \frac{21}{1-\frac{2}{3}} = \frac{21}{\frac{3-2}{3}}$$

$$= \frac{21}{\frac{1}{3}} = 21 \times 3 = 63$$

$$S_\infty = 63$$

Question 5.
If the first term of an infinite G.P. is 8 and its sum to infinity is $\frac{32}{3}$ then find the common ratio.
Solution:

a = 8

$$S_\infty = \frac{32}{3} \Rightarrow \frac{a}{1-r} = \frac{32}{3}$$

$$\frac{8}{1-r} = \frac{32}{3}$$

$$32(1-r) = 24$$

$$1 - r = \frac{24}{32} = \frac{6}{8} = \frac{3}{4}$$

$$-r = \frac{3}{4} - 1 = \frac{3-4}{4}$$

$$-r = \frac{-1}{4} \Rightarrow r = \frac{1}{4}$$

Question 6.
Find the sum to n terms of the series
(i) 0.4 + 0.44 + 0.444 +…. to n terms
(ii) 3 + 33 + 333 + …. to n terms
Solution:
(i) 0.4 + 0.44 + 0.444 + … to n terms
= 4 (0.1 + 0.11 + 0.111 + … to n terms)
= $\frac{4}{9}$ (0.9 + 0.99 + 0.999 + … to n terms)

$= \frac{4}{9}$ (1 -0.1) + (1 – 0.01) +(1 – 0.001) + … n terms)

$$= \frac{4}{9}(1 + 1 + 1 + \ldots n \text{ terms}) - (0.1 + 0.1^2 + 0.1^3 + \ldots n \text{ terms})$$

$$= \frac{4}{9}\left[n - 0.1\left[\frac{1-(0.1)^n}{1-0.1}\right]\right]$$

G.P

$a = 0.1$

$r = 0.1$

$$= \frac{4}{9}\left[n - \frac{1}{10}\left[\frac{1-\left(\frac{1}{10}\right)^n}{\frac{9}{10}}\right]\right]$$

$$S_n = a\frac{(1-r^n)}{1-r}$$

$$= \frac{4}{9}\left[n - \frac{1}{9}\left[1-\left(\frac{1}{10}\right)^n\right]\right]$$

$$= \frac{4}{9}n - \frac{4}{81}\left(1-\left(\frac{1}{10}\right)^n\right]$$

(ii) 3 + 33 + 333 + … to n terms.
= 3 (1 + 11 + 111 + …….. to n terms)
$= \frac{3}{9}$ (9 + 99 + 999 +… to n terms)
$\frac{1}{3}$ = [(10 – 1) + (100 – 1) + (1000 – 1) + … to n terms]
$= \frac{1}{3}$ [(10 + 100 + 1000 + ..) + (-1)n]

$$= \frac{1}{3}\left(\frac{10(10^n - 1)}{9} - n\right)$$

$$= \frac{10}{27}(10^n - 1) - \frac{n}{3}$$

Hint:

Here $r = 10 > 1$

$a = 10$

$$S_n = a\frac{(r^n - 1)}{r-1}$$

$$= \frac{10(10^n - 1)}{9}$$

Question 7.
Find the sum of the Geometric series
3 + 6 + 12 + …… + 1536.

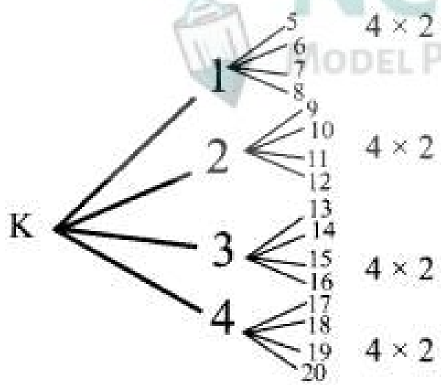
Solution:
3 + 6 + 12 + …… + 1536
Here a = 3

Hint:

$$t_n = 1536$$
$$ar^{n-1} = 1536$$
$$\cancel{3}(2)^{n-1} = \cancel{1536}^{512}$$
$$2^{n-1} = 512$$
$$2^{n-1} = 2^9$$
$$n - 1 = 9$$
$$n = 10$$

$$r = \frac{6}{3} = 2 > 1$$
$$S_n = \frac{a(r^n - 1)}{(r - 1)}$$
$$S_{10} = \frac{3(2^{10} - 1)}{2 - 1}$$
$$= 3(1024 - 1)$$
$$= 3 \times 1023 = 3069$$

Question 8.
Kumar writes a letter to four of his friends. He asks each one of them to copy the letter and mail to four different persons with the instruction that they continue the process similarly. Assuming that the process is unaltered and it costs □2 to mail one letter, find the amount spent on postage when 8th set of letters is mailed.



Solution:
Kumar (1) 2 × 4
Cost of posting 1^{st} set of Letters.
Cost of posting 2^{nd} set of letters

(1) (2) (3) (4)
$= 2 \times 4 + 2 \times 4 + 2 \times 4 + 2 \times 4$
$= 2(4 \times 4) = 2 \times 4^2$
Cost of posting 3rd set of letters
(5) (6) (7) (8) (9) (20)
$= 2\times4 + 2\times4 + 2\times4 + 2\times4 + 2\times4 + \ldots + 2\times4$
$= 2 \times (4 \times 4^2) = 2 \times 4^3$
$\therefore 2 \times 4 + 2 \times 4^2 + 2 \times 4^3 + \ldots 2 \times 4^8 =$
1st 2nd 3rd 8th

Amount spent
$[a + ar + ar^2 + \ldots ar^{n-1}]$
$= 4\ [2 + 2 \times 4 + 2 \times 42 \div + 2 \times 4^7]$
$= 4\ [S_n]$ Here n = 8, r = 4
It is a G.P

Sum of the G.P $= \dfrac{a(r^n - 1)}{r-1}$ if $r > 1$

$$S_n = S_8 = 2\left(\frac{4^8 - 1}{4-1}\right)$$
$$= 2\left(\frac{65536 - 1}{3}\right)$$
$$= 2\left(\frac{65535}{3}\right) = \frac{131070}{3}$$
$$= 43690$$

$\therefore$ Cost of postage after posting 8th set of letters
$= 4 \times 43690 =$ □ 174760

Question 9.
Find the rational form of the number $0.\overline{123}$
Solution:
Let x = 0.123123123 ………. $\Rightarrow x = 0.\overline{123}$ …..(1)
Multiplying 1000 on both rides
1000 x = 123.123123… $\Rightarrow 1000x = 123.\overline{123}$ …..(2)
(2) – (1) = 1000x – x $123.\overline{123} - 0.\overline{123}$.
$\Rightarrow 999x = 123$

$\Rightarrow x = \frac{123}{999}$
$\Rightarrow x = \frac{41}{333}$ Rational number

Question 10.
If Sn = $(x + y)+(x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3) + \ldots$n terms then prove that $(x - y)$

$$S_n = \left[\frac{x^2(x^n - 1)}{x - 1} - \frac{y^2(y^n - 1)}{y - 1}\right]$$

Solution:
Sn = $(x + y) + (x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3) + \ldots$n terms
$\Rightarrow x. S_n = (x + y)x + (x^2 + xy + y^2)x + (x^3 + x^2y + xy^2 + y^3) x + \ldots.$
$\Rightarrow x. S_n = x^2 +xy + x^3 + x^2y + y^2x + x^4 + x^3y$
$+ x^2 y^2 + y^3 x +\ldots\ldots \ldots.(1)$
Multiplying 'y' on both sides,
$= y.S_n = (x + y)y + (x^2 + xy + y^2) y+$
$(x^3 + x^2y + xy^2 + y^3) y + \ldots.$
$= y.S_n = xy + y^2 + x^2y + xy^2 + y^3 + x^3y$
$+ x^3y + x^2y^2 + xy^3 + y^4 + \ldots\ldots\ldots.$
(1) – (2) $\Rightarrow$

$$x S_n - y S_n = (x^2 + \cancel{xy} + x^3 + \cancel{x^2y} + \cancel{y^2x} + x^4 + \cancel{x^3y} + \cancel{x^2y^2} + \cancel{y^3x} + \ldots..) - (\cancel{xy} + y^2 + \cancel{x^2y} + \cancel{xy^2} + y^3 + \cancel{x^3y} + \cancel{x^2y^2} + \cancel{xy^3} + y^4 + \ldots..$$

$$\Rightarrow (x - y) S_n = (x^2 + x^3 + x^4 + \ldots\ldots) - (y^2 + y^3 + y^4 + \ldots.)$$

$$= \frac{x^2(x^n - 1)}{x - 1} - \frac{y^2(y^n - 1)}{y - 1}$$

$$\Rightarrow (x - y) S_n = \left[\frac{x^2(x^n - 1)}{x - 1} - \frac{y^2(y^n - 1)}{y - 1}\right]$$

Hence proved.

Ex 2.8

Question 1.
Find the sum of first n terms of the G.P.
(i) 5, -3, $\frac{9}{5}$, $-\frac{27}{25}$, ……….
(ii) 256, 64, 16, ……..
Solution:
(i) 5, -3, $\frac{9}{5}$, $\frac{-27}{25}$, ………

$$\text{Here } a = 5, r = \frac{t_2}{t_1} = \frac{-3}{5} < 1$$

$$S_n = a\left(\frac{1-r^n}{1-r}\right)$$

$$= 5\left[\frac{1-\left(\frac{-3}{5}\right)^n}{1-\left(\frac{-3}{5}\right)}\right] = 5\left[\frac{1-\left(\frac{-3}{5}\right)^n}{1-\frac{-3}{5}}\right]$$

$$= 5\,\frac{\left[1-\left(\frac{-3}{5}\right)^n\right]}{\frac{8}{5}} = 5\times\frac{5}{8}\left(1-\left(\frac{-3}{5}\right)^n\right)$$

$$\therefore S_n = \frac{25}{8}\left(1-\left(\frac{-3}{5}\right)^n\right)$$

(ii) 256, 64, 16,…….

a = 256

$$r = \frac{64}{256} = \frac{4}{16} = \frac{1}{4} < 1$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$= 256\left(\frac{1-\left(\frac{1}{4}\right)^n}{1-\frac{1}{4}}\right)$$

$$= \frac{256\left(1-\left(\frac{1}{4}\right)^n\right)}{\frac{3}{4}}$$

$$= \frac{1024}{3}\left(1-\left(\frac{1}{4}\right)^n\right)$$

Question 2.
Find the sum of first six terms of the G.P. 5, 15, 45, …
Solution:

G.P.= 5, 15, 45, 15

$$n = 6, a = 5, r = \frac{15}{5} = 3 > 1$$

$$\therefore \quad S_n = a\frac{(r^n - 1)}{r - 1}$$

$$S_6 = 5\left(\frac{3^6 - 1}{3 - 1}\right)$$

$$= 5\frac{(3^6 - 1)}{2} = \frac{5}{2}(729 - 1)$$

$$= \frac{5}{2} \times 728 = 5 \times 364$$

$$= 1820$$

Question 3.
Find the first term of the GP. whose common ratio 5 and whose sum to first 6 terms is 46872.
Solution:
Common ratio, r = 5
$S_6 = 46872$

$$\therefore \quad \frac{a(r^6 - 1)}{r - 1} = 46872$$

$$\Rightarrow \quad S_6 = \frac{a(5^6 - 1)}{5 - 1} = 46872$$

$$\Rightarrow \quad a = \frac{46872 \times 4}{[25 \times 25 \times 25 - 1]}$$

$$\Rightarrow \quad a = \frac{187488}{15624} = 12$$

$$\Rightarrow \quad \text{first term} = 12$$

Question 4.
Find the sum to infinity of (i) 9 + 3 + 1 + ….
(ii) 21 + 14 + $\frac{28}{3}$ + ….
Solution:
(i) 9 + 3 + 1 + ….

$a = 9, r = \frac{3}{9} = \frac{1}{3} < 1$

$$S_\infty = \frac{a}{1-r} = \frac{9}{1-\frac{1}{3}}$$

$$= \frac{9}{\frac{3-1}{3}} = \frac{\frac{9}{2}}{3} = 9 \times \frac{3}{2} = \frac{27}{2}$$

(ii) $21 + 14 + \frac{28}{3} + \ldots.$

Here $a = 21, r = \frac{14}{21} = \frac{2}{3}$

$$S_\infty = \frac{a}{1-r} = \frac{21}{1-\frac{2}{3}} = \frac{21}{\frac{3-2}{3}}$$

$$= \frac{21}{\frac{1}{3}} = 21 \times 3 = 63$$

$$S_\infty = 63$$

Question 5.
If the first term of an infinite G.P. is 8 and its sum to infinity is $\frac{32}{3}$ then find the common ratio.
Solution:

a = 8

$$S_\infty = \frac{32}{3} \Rightarrow \frac{a}{1-r} = \frac{32}{3}$$

$$\frac{8}{1-r} = \frac{32}{3}$$

$$32(1-r) = 24$$

$$1 - r = \frac{24}{32} = \frac{6}{8} = \frac{3}{4}$$

$$-r = \frac{3}{4} - 1 = \frac{3-4}{4}$$

$$-r = \frac{-1}{4} \Rightarrow r = \frac{1}{4}$$

Question 6.
Find the sum to n terms of the series
(i) 0.4 + 0.44 + 0.444 +…. to n terms
(ii) 3 + 33 + 333 + …. to n terms
Solution:
(i) 0.4 + 0.44 + 0.444 + … to n terms
= 4 (0.1 + 0.11 + 0.111 + … to n terms)
= $\frac{4}{9}$ (0.9 + 0.99 + 0.999 + … to n terms)

$= \frac{4}{9}$ (1 -0.1) + (1 – 0.01) +(1 – 0.001) + … n terms)

$$= \frac{4}{9}(1 + 1 + 1 + \ldots n \text{ terms}) - (0.1 + 0.1^2 + 0.1^3 + \ldots n \text{ terms})$$

G.P

$a = 0.1$

$r = 0.1$

$$S_n = a\frac{(1-r^n)}{1-r}$$

$$= \frac{4}{9}\left[n - 0.1\left[\frac{1-(0.1)^n}{1-0.1}\right]\right]$$

$$= \frac{4}{9}\left[n - \frac{1}{10}\left[\frac{1-\left(\frac{1}{10}\right)^n}{\frac{9}{10}}\right]\right]$$

$$= \frac{4}{9}\left[n - \frac{1}{9}\left[1-\left(\frac{1}{10}\right)^n\right]\right]$$

$$= \frac{4}{9}n - \frac{4}{81}\left(1-\left(\frac{1}{10}\right)^n\right]$$

(ii) 3 + 33 + 333 + … to n terms.
= 3 (1 + 11 + 111 + …….. to n terms)
$= \frac{3}{9}$ (9 + 99 + 999 +… to n terms)
$\frac{1}{3}$ = [(10 – 1) + (100 – 1) + (1000 – 1) + … to n terms]
$= \frac{1}{3}$ [(10 + 100 + 1000 + ..) + (-1)n]

$$= \frac{1}{3}\left(\frac{10(10^n-1)}{9} - n\right)$$

$$= \frac{10}{27}(10^n - 1) - \frac{n}{3}$$

Hint:
Here $r = 10 > 1$
$a = 10$

$$S_n = a\frac{(r^n - 1)}{r-1} = \frac{10(10^n-1)}{9}$$

Question 7.
Find the sum of the Geometric series
3 + 6 + 12 + …… + 1536.

Solution:
3 + 6 + 12 + …… + 1536
Here a = 3

Hint:

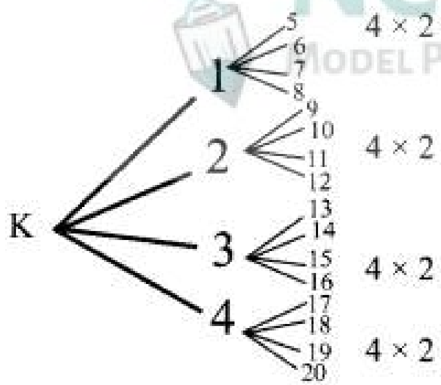
$$t_n = 1536$$
$$ar^{n-1} = 1536$$
$$\cancel{3}(2)^{n-1} = \cancel{1536}^{512}$$
$$2^{n-1} = 512$$
$$2^{n-1} = 2^9$$
$$n - 1 = 9$$
$$n = 10$$

$$r = \frac{6}{3} = 2 > 1$$

$$S_n = \frac{a(r^n - 1)}{(r-1)}$$

$$S_{10} = \frac{3(2^{10} - 1)}{2 - 1}$$
$$= 3(1024 - 1)$$
$$= 3 \times 1023 = 3069$$

Question 8.
Kumar writes a letter to four of his friends. He asks each one of them to copy the letter and mail to four different persons with the instruction that they continue the process similarly. Assuming that the process is unaltered and it costs □2 to mail one letter, find the amount spent on postage when 8th set of letters is mailed.



Solution:
Kumar (1) 2 × 4
Cost of posting 1^{st} set of Letters.
Cost of posting 2^{nd} set of letters

$\overset{(1)}{}\quad\overset{(2)}{}\quad\overset{(3)}{}\quad\overset{(4)}{}$

$= 2 \times 4 + 2 \times 4 + 2 \times 4 + 2 \times 4$

$= 2(4 \times 4) = 2 \times 4^2$

Cost of posting 3rd set of letters

$\overset{(5)}{}\quad\overset{(6)}{}\quad\overset{(7)}{}\quad\overset{(8)}{}\quad\overset{(9)}{}\quad\overset{(20)}{}$

$= 2\times4 + 2\times4 + 2\times4 + 2\times4 + 2\times4 + \ldots + 2\times4$

$= 2 \times (4 \times 4^2) = 2 \times 4^3$

$\therefore \underset{1st}{2 \times 4} + \underset{2nd}{2 \times 4^2} + \underset{3rd}{2 \times 4^3} + \ldots \underset{8th}{2 \times 4^8} =$

Amount spent

$[a + ar + ar^2 + \ldots ar^{n-1}]$

$= 4\ [2 + 2 \times 4 + 2 \times 42 \div + 2 \times 4^7]$

$= 4\ [S_n]$ Here $n = 8$, $r = 4$

It is a G.P

Sum of the G.P $= \dfrac{a(r^n - 1)}{r - 1}$ if $r > 1$

$$S_n = S_8 = 2\left(\frac{4^8 - 1}{4 - 1}\right)$$

$$= 2\left(\frac{65536 - 1}{3}\right)$$

$$= 2\left(\frac{65535}{3}\right) = \frac{131070}{3}$$

$$= 43690$$

$\therefore$ Cost of postage after posting 8th set of letters

$= 4 \times 43690 = $ □ 174760

Question 9.

Find the rational form of the number $0.\overline{123}$

Solution:

Let $x = 0.123123123 \ldots\ldots\ldots \Rightarrow x = 0.\overline{123}$ …..(1)

Multiplying 1000 on both rides

$1000\,x = 123.123123\ldots \Rightarrow 1000x = 123.\overline{123}$ …..(2)

$(2) - (1) = 1000x - x\ 123.\overline{123} - 0.\overline{123}.$

$\Rightarrow 999\,x = 123$

$\Rightarrow x = \frac{123}{999}$
$\Rightarrow x = \frac{41}{333}$ Rational number

Question 10.
If Sn = $(x + y)+(x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3) + \ldots$n terms then prove that $(x - y)$

$$S_n = \left[\frac{x^2(x^n - 1)}{x-1} - \frac{y^2(y^n - 1)}{y-1}\right]$$

Solution:
Sn = $(x + y) + (x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3) + \ldots$n terms
$\Rightarrow x.\ S_n = (x + y)x + (x^2 + xy + y^2)x + (x^3 + x^2y + xy^2 + y^3)\ x + \ldots.$
$\Rightarrow x.\ S_n = x^2 + xy + x^3 + x^2y + y^2x + x^4 + x^3y$
$+ x^2\ y^2 + y^3\ x + \ldots\ldots\ \ldots.(1)$
Multiplying 'y' on both sides,
$= y.S_n = (x + y)y + (x^2 + xy + y^2)\ y+$
$(x^3 + x^2y + xy^2 + y^3)\ y + \ldots.$
$= y.S_n = xy + y^2 + x^2y + xy^2 + y^3 + x^3y$
$+ x^3y + x^2y^2 + xy^3 + y^4 + \ldots\ldots\ldots.$
(1) – (2) $\Rightarrow$

$$x\,S_n - y\,S_n = (x^2 + \cancel{xy} + x^3 + \cancel{x^2y} + \cancel{y^2y} + x^4 + \cancel{x^3y}$$
$$+ \cancel{x^2y^2} + \cancel{y^2x} + \ldots..) - (\cancel{xy} + y^2 + \cancel{x^2y} + \cancel{xy^2} + y^3 +$$
$$\cancel{x^3y} + \cancel{x^2y^2} + \cancel{xy^3} + y^4 + \ldots..$$

$$\Rightarrow (x - y)\,S_n = (x^2 + x^3 + x^4 + \ldots\ldots) - (y^2 + y^3 + y^4 + \ldots.)$$

$$= \frac{x^2(x^n - 1)}{x-1} - \frac{y^2(y^n - 1)}{y-1}$$

$$\Rightarrow (x - y)\,S_n = \left[\frac{x^2(x^n - 1)}{x-1} - \frac{y^2(y^n - 1)}{y-1}\right]$$

Hence proved.

Ex 2.10

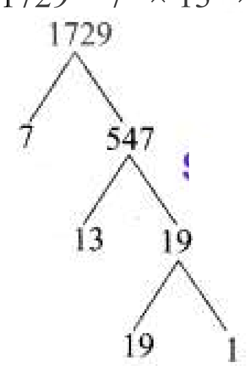
Multiple choice questions
Question 1.
Euclid's division lemma states that for positive integers a and b, there exist unique integers q and r such that a = bq + r, where r must satisfy.
(1) $1 < r < b$
(2) $0 < r < b$
(3) $0 \leq r < b$
(4) $0 < r \leq b$
Answer:
(3) $0 \leq r < b$

Question 2.
Using Euclid's division lemma, if the cube of any positive integer is divided by 9 then the possible remainders are
(1) 0, 1, 8
(2) 1, 4, 8
(3) 0, 1, 3
(4) 1, 3, 5
Answer:
(1) 0,1,8
Hint:
Cube of any +ve integers 1^3, 2^3, 3^3, 4^3,. . .
1, 8, 27, 64, 125, 216 …
Remainders when 27, 64, 125 are divided by 9.

Question 3.
If the H.C.F of 65 and 117 is expressible in the form of 65m -117 , then the value of m is ………………….
(1) 4
(2) 2

(3) 1
(4) 3
Answer:
(2) 2
Hint:
$117 = 3 \times 3 \times 13$
$65 = 5 \times 13$
H.C.F = 13
$65m - 117 = 13 \Rightarrow 65m = 117 + 13 = 130$
$m = \frac{130}{65} = 2$
The value of m = 2

Question 4.
The sum of the exponents of the prime factors in the prime factorization of 1729 is
(1) 1
(2) 2
(3) 3
(4) 4
Answer:
(3) 3
Hint:
$1729 = 7^1 \times 13^1 \times 19^1$



Question 5.
The least number that is divisible by all the numbers from 1 to 10 (both inclusive) is
(1) 2025
(2) 5220
(3) 5025
(4) 2520
Answer:
(4) 2520

Hint:

2	1, 2, 3, 4, 5, 6, 7, 8, 9, 10
2	1, 1, 3, 2, 5, 3, 7, 4, 9, 5
3	1, 1, 3, 1, 5, 1, 7, 2, 9, 5
5	1, 1, 1, 1, 5, 1, 7, 2, 3, 5
7	1, 1, 1, 1, 1, 1, 7, 2, 3, 1
2	1, 1, 1, 1, 1, 1, 1, 2, 3, 1
3	1, 1, 1, 1, 1, 1, 1, 1, 3, 1
	1, 1, 1, 1, 1, 1, 1, 1, 1, 1

∴ L.C.M. of 1,2,3,4,…,10 is $2 \times 2 \times 3 \times 5 \times 7 \times 2 \times 3 = 2520$

Question 6.
$7^{4k} \equiv$ …………………. (mod 100)
(1) 1
(2) 2
(3) 3
(4) 4
Answer:
(1) 1
Hint:
$7^{4k} \equiv$ . . . . . (mod 100)
$7^{4k} = (7^4)^k \equiv$ ……….. (mod 100) (7^4 – 2401)
The value is 1.

Question 7.
Given $F_1 = 1$, $F_2 = 3$ and $Fn = F_{n-1} + F_{n-2}$ then
(1) 3
(2) 5
(3) 8
(4) 11
Answer:
(4) 11
Answer:
$F_1 = 1, F_2 = 3$
$F_n = F_{n-1} + F_{n-2}$
$F_5 = F_{5-1} + F_{5-2} = F_4 + F_3$
$= F_3 + F_2 + F_2 + F_1$

$= F_2 + F_1 + F_2 + F_2 + F_1$
$= 3 + 1 + 3 + 3 + 1 = 11$

Question 8.
The first term of an arithmetic progression is unity and the common difference is 4. Which of the following will be a term of this A.P …………..
(1) 4551
(2) 10091
(3) 7881
(4) 13531
Answer:
(3) 7881
Hint:
Here a = 1, d = 4
$t_n = a + (n – 1) d = 1 + (n – 1) 4$
$= 1 + 4n – 4$
$= 4n – 3$
4554
(i) $4n – 3 = 4551 \Rightarrow 4n = 4551 + 3 \Rightarrow n = \frac{4554}{4} = 1138.5$.
It is not a term of A.P.

(ii) $4n – 3 = 10091 \Rightarrow 4n = 10091 + 3 = 10094$
$n = \frac{10094}{4} = 2523.5$ it is a term of A.P.

(iii) $4n – 3 = 7881 \Rightarrow 4n = 7881 + 3$
$n = \frac{7884}{4} = 1971$.
$\therefore$ 7881 is a term of the A.P.

Question 9.
If 6 times of 6^{th} term of an A.P is equal to 7 times the 7th term, then the 13th term of the A.P. is
(1) 0
(2) 6
(3) 7
(4) 13
Answer:
(1) 0
Hint:
$6t_6 = 7t_7$
$6(a + 5d) = 7(a + 6d)$

6a + 30d = 7a + 42d
7a + 42d – 6a – 30d = 0
a + 12d = 0 = t_{13}

Question 10.
An A.P consists of 31 terms. If its 16^{th} term is m, then the sum of all the terms of this A.P. is …………..
(1) 16m
(2) 62m
(3) 31m
(4) $\frac{31}{2}$ m
Answer:
(3) 31m
Hint:
M = 31
t_{16} = m ⇒ a + 15d = m
$S_n = \frac{n}{2}[2a + (n – 1)d]$
$S_n = \frac{31}{2}[2a + 30d] = \frac{31}{2} \times 2[a + 15d]$
= 31 (m) = 31m

Question 11.
In an A.P., the first term is 1 and the common difference is 4. How many terms of the A.P must be taken for their sum to be equal to 120?
(1) 6
(2) 7
(3) 8
(4) 9
Answer:
(3) 8
Hint:

$a = 1, d = 4$

$$S_n = 120 = \frac{n}{2}(2a + (n-1)d)$$

$$120 = \frac{n}{2}(2 \times 1 + (n-1)4)$$

$$120 = \frac{n}{2}(2 + 4n - 4) = \frac{n}{2}(4n-2)$$

$$= \frac{n}{\cancel{2}}.\cancel{2}(2n-1)$$

$$= n(2n-1)$$

$$120 = 2n^2 - n$$

$$2n^2 - n - 120 = 0$$

$$(n-8)(2n+15) = 0$$

$$\therefore \quad n = 8, n = \frac{-15}{2}$$

-240

$\frac{-16}{2}$ $\quad \frac{15}{2}$

$(n-8)\left(n + \frac{15}{2}\right)$

Question 12.
If A = 2^{65} and B = $2^{64} + 2^{63} + 2^{62} + +2^0$ which of the following is true?
(1) B is 264 more than A
(2) A and B are equal
(3) B is larger than A by 1
(4) A is larger than B by 1
Answer:
(4) A is larger than B by 1
Hint:
A = 2^{65}
B = $2^{64} + 2^{63} + 2^{62} + \ldots + 20$
B = $2^0 + 2^1 + 2^2 + \ldots + 264$
G.P = $1 + 2^1 + 2^2 + \ldots + 2^{64}$ it is a G.P
Here a = 1, r = 2, n = 65

$\therefore$ Sum of the G.P = $S_{65} = \frac{a(r^n - 1)}{r-1}$

$$= \frac{1(2^{65}-1)}{2-1} = 2^{65} - 1$$

A = 2^{65}, B = $2^{65} - 1$
∴ B is smaller.
A is larger than B by 1.

Question 13.
The next term of the sequence $\frac{3}{16}, \frac{1}{8}, \frac{1}{12}, \frac{1}{18}$, ….
(1) $\frac{1}{24}$
(2) $\frac{1}{27}$
(3) $\frac{2}{3}$
(4) $\frac{1}{81}$
Answer:
(2) $\frac{1}{27}$
Hint:

$$\frac{3}{16}, \frac{1}{8}, \frac{1}{12}, \frac{1}{18}, \ldots$$

$$r = \frac{\frac{1}{8}}{\frac{3}{16}} = \frac{1}{\cancel{8}} \times \frac{\cancel{16}^{2}}{3} = \frac{2}{3}$$

$$r = \frac{\frac{1}{12}}{\frac{1}{8}} = \frac{1}{\cancel{12}_3} \times \frac{\cancel{8}^{2}}{1} = \frac{2}{3}$$

$$\therefore \text{The next term is } \frac{1}{18} \times \frac{2}{3} = \frac{2}{54} = \frac{1}{27}$$

Question 14.
If the sequence t_1, t_2, t_3, ………..are in A.P. then the sequence t_6, t_{12}, t_{18}, …… is …………
(1) a Geometric progression

(2) an Arithmetic progression
(3) neither an Arithmetic progression nor a Geometric progression
(4) a constant sequence
Answer:
(2) an Arithmetic progression
Hint: t_1, t_2, t_3 …. are in A.P
t_6, t_{12}, t_{18} …… is also an A.P. (6, 12, 18 …….. is an A.P.)

Question 15.
The value of $(1^3 + 2^3 + 3^3 + \ldots + 15^3) - (1 + 2 + 3 + \ldots + 15)$ is
(1) 14400
(2) 14200
(3) 14280
(4) 14520
Answer:
(3) 14280
Hint:
$\left(\frac{15\times16}{2}\right)^2 - \frac{15\times16}{2} = (120)^2 - 120 = 14280$

Unit Exercise 2

Question 1.
Prove that $n^2 - n$ divisible by 2 for every positive integer n.
Answer:
To prove $n^2 - n$ divisible by 2 for every positive integer n.
We know that any positive integer is of the form 2q or 2q + 1, for some integer q.
So, following cases arise:
Case I. When n = 2q.
In this case, we have
$n^2 - n = (2q)^2 - 2q = 4q^2 - 2q = 2q(2q - 1)$
$\Rightarrow n^2 - n = 2r$ where $r = q(2q - 1)$
$\Rightarrow n^2 - n$ is divisible by 2.

Case II. When n = 2q + 1.
In this case, we have
$n^2 - n = (2q + 1)^2 - (2q + 1)$
$= (2q + 1)(2q + 1 - 1) = (2q + 1)2q$
$\Rightarrow n^2 - n = 2r$ where $r = q(2q + 1)$
$\Rightarrow n^2 - n$ is divisible by 2.
Hence $n^2 - n$ is divisible by 2 for every positive integer n.

Question 2.
A milk man has 175 litres of cow's milk and 105 litres of buffalow's milk. He wishes to sell the milk by filling the two types of milk in cans of equal capacity. Calculate the following (i) Capacity of a can
(ii) Number of cans of cow's milk
(iii) Number of cans of buffalow's milk.
Answer:
Cow's milk = 175 litres

Buffalow’s milk = 105 litres
Find the H.C.F. of 175 and 105 using Euclid’s division method of factorisation method.

$$\begin{array}{r|l} 5 & 175 \\ \hline 5 & 35 \\ \hline & 7 \\ \hline \end{array}$$

175 = 5 × 5 × 7
105 = 3 × 5 × 7
H.C.F. of 175 and 105 = 5 × 7 = 35
(i) The capacity of the milk can’s is 35 litres

(ii) Cows milk = 175 litres
Number of cans = $\frac{175}{35}$ = 5

$$\begin{array}{r|l} 5 & 105 \\ \hline 3 & 21 \\ \hline & 7 \\ \hline \end{array}$$

(iii) Buffalow’s milk = 105 litres
Number of cans = $\frac{105}{35}$ = 3
(i) Capacity of one can = 35 litres
(ii) Number of can’s for cow’s milk= 5 litres
(iii) Number of can’s for Buffalow’s milk = 3 litres

Question 3.
When the positive integers a, b and c are divided by 13 the respective remainders are 9, 7 and 10. Find the remainder when a + 2b + 3c is divided by 13.
Answer:
Let the positive integers be a, b, and c.
a = 13q + 9
b = 13q + 7
c = 13q + 10
a + 2b + 3c = 13 q + 9 + 2(13q + 7) + 3(13q + 10)
= 13q + 9 + 269 + 14 + 39q + 30
= 78q + 53 = (13 × 6)q + 53
The remainder is 53.
But 53 = 13 × 4 + 1
∴ The remainder is 1

Question 4.
Show that 107 is of the form 4q + 3 for any integer q.
Answer:

$$\begin{array}{l} 4)\ 107\ (26 \\ \quad \underline{8} \\ \quad 27 \\ \quad \underline{24} \\ \quad\ \ 3 \end{array}$$

107 = 4 x 26 + 3
This is in the form of a = bq + r
Hence it is proved.

Question 5.
If $(m + 1)^{th}$ term of an A.P. is twice the $(n + 1)^{th}$ term, then prove that $(3m + 1)^{th}$ term is twice the $(m + n + 1)^{th}$ term.
Solution:
$t_n = a + (n – 1)d$
$t_{m+1} = a + (m + 1 – 1)d$
$= a + md$
$t_{n+1} = a + (n + 1 – 1)d$
$= a + nd$
$2(t_{n+1}) = 2(a + nd)$
$t_{m+1} = 2t_{n+1}$ …………… (1)
$\Rightarrow a + md = 2(a + nd)$
$2a + 2nd – a – md = 0$
$a + (2n – m)d = 0$
$t_{(3m+1)} = a + (3m + 1 – 1)d$
$= a + 3md$
$t_{(m+n+1)} = a + (m + n + 1 – 1)d$
$= a + (m + n)d$
$2(t_{(m+n+1)}) = 2(a + (m + n)d)$
$= 2a + 2md + 2nd$
$t_{(3m+1)} = 2t_{(m+n+1)}$ ………….. (2)
$a + 3md = 2a + 2md + 2nd$
$2a + 2md + 2nd – a – 3md = 0$
$a – md + 2nd = 0$

$a + (2n - m)d = 0$
∴ It is proved that $t_{(3m+1)} = 2t_{(m+n+1)}$

Question 6.
Find the 12^{th} term from the last term of the A.P -2, -4, -6, … -100.
Solution:

$$n = \frac{l-a}{d} + 1 = \frac{-100-(-2)}{-2} + 1$$

$$= \frac{-100+2}{-2} + 1 = \frac{-98}{-2} + 1$$

$$n = 49 + 1 = 50$$

12^{th} term from the last = 39^{th} term from the beginning
∴ $t_{39} = a + 38d$
$= -2 + 38(-2)$
$= -2 - 76$
$= -78$

Question 7.
Two A.P.'s have the same common difference. The first term of one A.P. is 2 and that of the other is 7. Show that the difference between their 10^{th} terms is the same as the difference between their 21st terms, which is the same as the difference between any two corresponding terms.
Answer:
Let the common difference for the 2 A.P be "d"
For the first A.P
$a = 2, d = d, n = 10$
$t_n = a + (n - 1) d$
$t_{10} = 2 + 9 d$ ….(1)
For the 2^{nd} A.P
$a = 7, d = d\ n = 10$
$t_{10} = 7 + (9)d$
$= 7 + 9d$ ….(2)
Difference between their 10th term ⇒ (1) – (2)
$= 2 + 9d - (7 + 9d)$
$= 2 + 9d - 7 - 9d$
$= -5$
For first A.P when $n = 21, a = 2, d = d$
$t_{21} = 2 + 20d$ …….(3)
For second A.P when $n = 21, a = 7, d = d$

$t_{21} = 7 + 20d$ …….(4)

Difference between the 21^{st} term ⇒ (3) – (4)
$= 2 + 20d – (7 + 20d)$
$= 2 + 20d – 7 – 20d$
$= -5$

Difference between their 10th term and 21^{st} term = -5
Hence it is proved.

Question 8.
A man saved □16500 in ten years. In each year after the first he saved □100 more than he did in the preceding year. How much did he save in the first year?
Solution:
S_{10} = □16500
a, a + d, a + 2d…
d = 100
n = 10
$S_n = \frac{n}{2}(2a+(n-1)d)$
$S_{10} = 16500$
$S_{10} = \frac{10}{2}(2\times a+9\times 100)$
$16500 = 5(2a+900)$
$16500 = 10a + 4500$
$10a = 16500 – 4500$
$10a = 12000$
$a = \frac{12000}{10}$ = □1200
∴ He saved □1200 in the first year

Question 9.
Find the G.P. in which the 2nd term is $\sqrt{6}$ and the 6th term is $9\sqrt{6}$.
Solution:

$$t_2 = \sqrt{6}$$
$$t_6 = 9\sqrt{6}$$
$$t_n = ar^{n-1} \text{ in G.P}$$
$$\therefore t_2 = ar^{2-1} = \sqrt{6}$$
$$ar = \sqrt{6} \qquad \text{...(1)}$$
$$t_6 = ar^{6-1} = 9\sqrt{6}$$
$$ar^5 = 9\sqrt{6} \qquad \text{...(2)}$$
$$\frac{(2)}{(1)} = \frac{\cancel{a}\cancel{r}^{\cancel{5}4}}{\cancel{ar}} = \frac{9\cancel{\sqrt{6}}}{\cancel{\sqrt{6}}}$$
$$r^4 = 9 \Rightarrow r^2 = 3 \Rightarrow r = \sqrt{3}$$

Substitute $r = \sqrt{3}$ in (1)

$$ar = \sqrt{6}$$
$$a\sqrt{3} = \sqrt{6}$$
$$a = \frac{\sqrt{6}}{\sqrt{3}} = \sqrt{\frac{6}{3}} = \sqrt{2}$$
$$\therefore \text{G.P} = a, ar, ar^2, \ldots$$
$$= \sqrt{2}, \sqrt{6}, \sqrt{2}\sqrt{3}^2, \cdots$$
$$= \sqrt{2}, \sqrt{6}, 3\sqrt{2}, \ldots$$

Question 10.
The value of a motorcycle depreciates at the rate of 15% per year. What will be the value of the motor cycle 3 year hence, which is now purchased for □45,000?
Answer:
Value of the motor cycyle = □ 45000
a = 45000
Depreciation = 15% of the cost value
$= \frac{15}{100} \times 45000$
$= 15 \times 450$

= 6750

d = – 6750 (decrease it is depreciation

Value of the motor cycle lightning of the 2nd year = 45000 – 6750

= □ 38250

Depreciation for the 2nd year = $\frac{15}{100}$ × 38250

= □ 57370.50

Additional Questions

Question 1.
Use Euclid's algorithm to find the HCF of 4052 and 12756.
Solution:
Since 12576 > 4052 we apply the division lemma to 12576 and 4052, to get HCF
12576 = 4052 × 3 + 420.
Since the remainder 420 ≠ 0, we apply the division lemma to 4052
4052 = 420 × 9 + 272.
We consider the new divisor 420 and the new remainder 272 and apply the division lemma to get
420 = 272 × 1 + 148, 148 ≠ 0.
∴ Again by division lemma
272 = 148 × 1 + 124, here 124 ≠ 0.
∴ Again by division lemma
148 = 124 × 1 + 24, Here 24 ≠ 0.
∴ Again by division lemma
124 = 24 × 5 + 4, Here 4 ≠ 0.
∴ Again by division lemma
24 = 4 × 6 + 0.
The remainder has now become zero. So our procedure stops. Since the divisor at this stage is 4.
∴ The HCF of 12576 and 4052 is 4.

Question 2.
If the HCF of 65 and 117 is in the form (65m – 117) then find the value of m.
Answer:
By Euclid's algorithm 117 > 65
117 = 65 × 1 + 52
52 = 13 × 4 × 0
65 = 52 × 1 + 13
H.C.F. of 65 and 117 is 13
65m – 117 = 13
65 m = 130

$m = \frac{130}{65} = 2$
The value of m = 2

Question 3.
Find the LCM and HCF of 6 and 20 by the prime factorisation method.
Solution:
We have $6 = 2^1 \times 3^1$ and
$20 = 2 \times 2 \times 5 = 2^2 \times 5^1$
You can find HCF (6, 20) = 2 and LCM (6, 20) = 2 × 2 × 3 × 5 = 60. As done in your earlier classes. Note that HCF (6, 20) = 2^1 = product of the smallest power of each common prime factor in the numbers.
LCM (6, 20) = $2^2 \times 3^1 \times 5^1 = 60$.
= Product of the greatest power of each prime factor, involved in the numbers.

Question 4.
Prove that $\sqrt{3}$ is irrational.
Answer:

Let us assume the opposite, (1) $\sqrt{3}$ is irrational.
Hence $\sqrt{3} = \frac{p}{q}$
Where p and q($q \neq 0$) are co-prime (no common factor other than 1)

Hence, $\sqrt{3} = \frac{p}{q}$

$$\sqrt{3}\,q = p$$

Squaring both side

$$(\sqrt{3}\,q)^2 = p^2$$
$$3q^2 = p^2$$
$$q^2 = \frac{p^2}{3}$$

Hence, 3 divides p^2
So 3 divides p also …………….. (1)
Hence we can say
$\frac{p}{3}$ = c where c is some integer
p = 3c
Now we know that
$3q^2 = p^2$

Putting = 3c
$3q^2 = (3c)^2$
$3q^2 = 9c^2$
$q^2 = \frac{1}{3} \times 9c^2$
$q^2 = 3c^2$
$\frac{q^2}{3} = C^2$
Hence 3 divides q^2
So, 3 divides q also ……………. (2)
By (1) and (2) 3 divides both p and q
By contradiction $\sqrt{3}$ is irrational.

Question 5.
Which of the following list of numbers form an AP? If they form an AP, write the next two terms:
(i) 4, 10, 16, 22, …
(ii) 1, -1,-3, -5,…
(iii) -2, 2, -2, 2, -2, …
(iv) 1, 1, 1, 2, 2, 2, 3, 3, 3,…
Solution:
(i) 4, 10, 16, 22, …….
We have $a_2 - a_1 = 10 - 4 = 6$
$a_3 - a_2 = 16 - 10 = 6$
$a_4 - a_3 = 22 - 16 = 6$
∴ It is an A.P. with common difference 6.
∴ The next two terms are, 28, 34

(ii) 1, -1, -3, -5
$t_2 - t_1 = -1 - 1 = -2$
$t_3 - t_2 = -3 - (-1) = -2$
$t_4 - t_3 = -5 - (-3) = -2$
The given list of numbers form an A.P with the common difference -2.
The next two terms are (-5 + (-2)) = -7, -7 + (-2) = -9.

(iii) -2, 2,-2, 2,-2
$t_2 - t_1 = 2-(-2) = 4$
$t_3 - t_2 = -2 -2 = -4$
$t_4 - t_3 = 2 - (-2) = 4$
It is not an A.P.

(iv) 1, 1, 1, 2, 2, 2, 3, 3, 3
$t_2 - t_1 = 1 - 1 = 0$
$t_3 - t_2 = 1 - 1 = 0$
$t_4 - t_3 = 2 - 1 = 1$
Here $t_2 - t_1 \neq t_3 - t_2$
∴ It is not an A.P.

Question 6.
Find n so that the n^{th} terms of the following two A.P.'s are the same.
1, 7,13,19,… and 100, 95,90,…
Answer:
The given A.P. is 1, 7, 13, 19,….
$a = 1, d = 7 - 1 = 6$
$t_{n1} = a + (n - 1)d$
$t_{n1} = 1 + (n - 1) 6$
$= 1 + 6n - 6 = 6n - 5$ … (1)
The given A.P. is 100, 95, 90,….
$a = 100, d = 95 - 100 = - 5$
$tn_2 = 100 + (n - 1) (-5)$
$= 100 - 5n + 5$
$= 105 - 5n$ …..(2)
Given that, $t_{n1} = t_{n2}$
$6n - 5 = 105 - 5n$
$6n + 5n = 105 + 5$
$11 n = 110$
$n = 10$
∴ 10^{th} term are same for both the A.P's.

Question 7.
In a flower bed, there are 23 rose plants in the first row, 21 in the second, 19 is the third, and so on. There are 5 rose plants in the last row. How many rows are there in the flower bed?
Answer:
The number of rose plants in the 1st, 2nd, 3rd,… rows are
23, 21, 19,………….. 5
It forms an A.P.
Let the number of rows in the flower bed be n.
Then $a = 23, d = 21 - 23 = -2, l = 5$.
As, $a_n = a + (n - 1)d$ i.e. $t_n = a + (n - 1)d$
We have $5 = 23 + (n - 1)(-2)$
i.e. $-18 = (n - 1)(-2)$

$n = 10$
∴ There are 10 rows in the flower bed.

Question 8.
Find the sum of the first 30 terms of an A.P. whose n^{th} term is $3 + 2n$.
Answer:
Given,
$t_n = 3 + 2n$
$t_1 = 3 + 2(1) = 3 + 2 = 5$
$t_2 = 3 + 2(2) = 3 + 4 = 7$
$t_3 = 3 + 2(3) = 3 + 6 = 9$
Here $a = 5, d = 7 – 5 = 2, n = 30$
$S_n = \frac{n}{2}[2a + (n – 1)d]$
$S_{30} = \frac{30}{2}[10 + 29(2)]$
$= 15[10 + 58] = 15 \times 68 = 1020$
∴ Sum of first 30 terms = 1020

Question 9.
How many terms of the AP: 24, 21, 18, . must be taken so that their sum is 78?
Solution:
Here $a = 24$, $d = 21 – 24 = -3$, $S_n = 78$. We need to find n.
We know that,
$S_n = \frac{n}{2}(2a + (n – 1)d)$
$78 = \frac{n}{2}(48 + 13(-3))$
$78 = \frac{n}{2}(51 – 3n)$
or $3n^2 – 51n + 156 = 0$
$n^2 – 17n + 52 = 0$
$(n – 4)(n – 13) = 0$
$n = 4$ or 13
The number of terms are 4 or 13.

Question 10.
The sum of first n terms of a certain series is given as $3n^2 – 2n$. Show that the series is an arithmetic series.
Solution:
Given, $S_n = 3n^2 – 2n$
$S_1 = 3(1)^2 – 2(1)$

$= 3 - 2 = 1$
ie; $t_1 = 1$ ($\therefore S_1 = t_1$)
$S_2 = 3(2)^2 - 2(2) = 12 - 4 = 8$
ie; $t_1 + t_2 = 8$ ($\therefore S_2 = t_1 + t_2$)
$\therefore t_2 = 8 - 1 = 7$
$S_3 = 3(3)^2 - 2(3) = 27 - 6 = 21$
$t_1 + t_2 + t_3 = 21$ ($\therefore S_3 = t_1 + t_2 + t_3$)
$8 + t_3 = 21$ (Substitute $t_1 + t_2 = 8$)
$t_3 = 21 - 8 \Rightarrow t_3 = 13$
$\therefore$ The series is 1,7,13, ………… and this series is an A.P. with common difference 6.