Mensuration

Ex 7.1

Question 1.
The radius and height of a cylinder are in the ratio 5:7 and its curved surface area is 5500 sq.cm. Find its radius and height.
Solution:
r = 5x
h = 7x
CSA of a cylinder = 2πrh

$$= 2x\frac{22}{7} \times 5x \times 7x = 5500$$

$$22\cancel{0}x^2 = 550\cancel{0}$$

$$x^2 = \frac{550}{22} = 25$$

$$\therefore \quad x = 5$$

∴ Radius = 5 × 5 = 25 cm
height = 7 × 5 = 35 cm

Question 2.
A solid iron cylinder has total surface area of 1848 sq.m. Its curved surface area is five – sixth of its total surface area. Find the radius and height of the iron cylinder.
The external radius and the length of a hollow

Solution:

$$\begin{aligned}
\text{C.S.A.} &= \frac{5}{6}\text{ T.S.A} \\
2\pi r(h+r) &= 1848 \text{ m}^2 \\
2\pi rh + 2\pi r^2 &= 1848 \text{ m}^2 \\
\frac{5}{6} \times 1848 + 2\pi r^2 &= 1848 \\
1540 + 2\pi r^2 &= 1848 \\
2\pi r^2 &= 1848 - 1540 \\
&= 308 \\
r^2 &= 308 \times \frac{1}{2} \times \frac{7}{22} \\
r^2 &= 49 \\
r &= 7\, m. \\
2\pi rh &= \frac{5}{6} \times 1848 \\
2 \times \frac{22}{\cancel{7}} \times \cancel{7} \times h &= \frac{5}{6} \times 1848 \\
h &= 35 \text{ m}
\end{aligned}$$

$\therefore$ Radius $r = 7$ m, Height = 35 m.

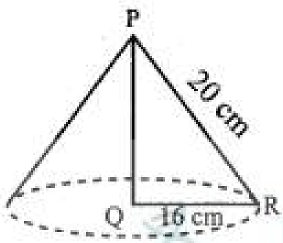
Question 3.
The external radius and the length of a hollow wooden log are 16 cm and 13 cm respectively. If its thickness is 4 cm then find its T.S.A.
Solution:
R = 16 cm
r = R – thickness
r = 12 cm
= 16 – 4 = 12 cm
h = 13 cm
Total surface area of hollow cylinder = $2\pi(R + r)(R - r + h)$ sq. units.

$= 2x\frac{22}{7}\ (16 + 12)\ (16 - 12 + 13)$

$= \frac{44}{\cancel{7}}\left(\overset{4}{\cancel{28}}\right)(17)$

∴ T.S.A = 2992 cm^2

Question 4.
A right angled triangle PQR where ∠Q = 90° is rotated about QR and PQ. If QR = 16 cm and PR = 20 cm, compare the curved surface areas of the right circular cones so formed by the triangle.
Solution:
When it is rotated about PQ the C.S.A of the cone formed = πrl.



$= \frac{22}{7} \times 16 \times 20$

$= \frac{7040}{7}$

$= 1005.71\ cm^2$

When it is rotated about QR CSA of the cone formed.

$= \pi rl.$

here $r = \sqrt{l^2 - h^2}$

$= \sqrt{20^2 - 16^2}$

$= \sqrt{400 - 256}$

$= \sqrt{144} = 12\ cm$

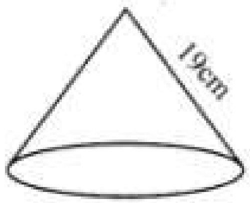
CSA $= \pi rl = \frac{22}{7} \times 12 \times 20$

$= \frac{5280}{7}$

$= 754.28\ cm^2.$

1005.71 > 754.28
∴ CSA of the cone rotated about its PQ is larger than the CSA of the cone rotated about QR.

Question 5.
4 persons live in a conical tent whose slant height is 19 cm. If each person require 22 cm^2 of the floor area, then find the height of the tent.
Solution:
Base area of the cone = πr^2 = sq units.



$$\pi r^2 = 4 \times 22 \text{ cm}^2$$
$$r^2 = 88 \times \frac{7}{22}$$
$$= \frac{616}{22} = 28 \text{ cm}^2$$
$$l = 19 \text{ cm}$$
$$h = \sqrt{l^2 - r^2}$$
$$= \sqrt{19^2 - 28}$$
$$= \sqrt{361 - 28}$$
$$= \sqrt{333}$$
$$= 18.25 \text{ cm.}$$

Hint:

	18.2
1	33300
	1
28 × 8	233
	224
362 × 2	900
	724
	17600

Question 6.
A girl wishes to prepare birthday caps in the form of right circular cones for her birthday party, using a sheet of paper whose area is 5720 cm^2, how many caps can be made with radius 5 cm and height 12 cm.

Solution:

Hint:

$$= \frac{4290}{\pi r l}$$

$$= \frac{4290}{\frac{22}{7} \times 5 \times 13}$$

$$= \frac{4290}{1430/7} = 429\cancel{0} \times \frac{7}{143\cancel{0}}$$

$$r = 5 \text{ cm}$$

$$h = 12 \text{ cm}$$

$$= l = \sqrt{r^2 + h^2}$$

$$= \sqrt{25+144}$$

$$= \sqrt{169}$$

$$= 13 \text{ cm}$$

$$\text{Required no. of caps} = \frac{\text{Area of the paper}}{\text{Area of 1 cap}}$$

$$= \frac{5720}{\pi r l} = \frac{5720}{\frac{22}{7} \times 5 \times 13}$$

$$= \frac{5720}{\frac{1430}{7}} = 572\cancel{0} \times \frac{7}{143\cancel{0}}$$

$$= \frac{4004}{143} = 28 \text{ caps}$$

Question 7.
The ratio of the radii of two right circular cones of same height is 1 : 3. Find the ratio of their curved surface area when the height of each cone is 3 times the radius of the smaller cone.
Solution:

$$r_1 : r_2 = 1 : 3 \Rightarrow 1x : 3x$$

$$h_1 = 3x, h_2 = 3x, l_1 = \sqrt{r_1^2 + h_1^2}$$

$$= \sqrt{x^2 + (3x)^2}$$

$$= \sqrt{x^2 + 9x^2} = \sqrt{10x^2}$$

$$= \sqrt{10}\,x$$

$$l^2 = \sqrt{r_2^2 + h_2^2}$$

$$= \sqrt{(3x)^2 + (3x)^2}$$

$$= \sqrt{9x^2 + 9x^2}$$

$$= \sqrt{18x^2} = 3\sqrt{2}x$$

$\therefore$ $CSA_1 : CSA_2$

$$= \frac{\pi r_1 l_1}{\pi r_2 l_2} = \frac{\pi \times 1x \times \sqrt{10}x}{\pi \times 3x \times 3\sqrt{2}x}$$

$$= \frac{\cancel{\pi} \times 1\cancel{x} \times \sqrt{10}\cancel{x}}{\cancel{\pi} \times 3\cancel{x} \times 3\sqrt{2}\cancel{x}}$$

$$= \frac{\sqrt{10}}{9\sqrt{2}} = \frac{\sqrt{2 \times 5}}{9\sqrt{2}}$$

$$= \frac{\cancel{\sqrt{2}} \times \sqrt{5}}{9\cancel{\sqrt{2}}} = \frac{\sqrt{5}}{9}$$

$$= \sqrt{5} : 9.$$

$\therefore$ The ratio of their curved surface areas

$\Rightarrow \sqrt{5} : 9$

Question 8.
The radius of a sphere increases by 25%. Find the percentage increase in its surface area.

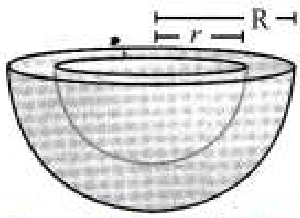
Solution:
Surface area of sphere A = $4\pi r^2$
New radius = r' = 1.25r,
[∵ r + 0.25r] (25% = 0.25)
New surface area = A' = $4\pi(r')^2$
= $4\pi(1.25\ r)^2$
$4\pi r^2\ [1.5625]$
$$\%\text{ increase in S.A} = \frac{\text{New SA} - \text{old SA}}{\text{Old surface Area}} \times 100$$
$$= \frac{A' - A}{A} \times 100$$
$$= \left(\frac{1.5625A - A}{A}\right) \times 100$$
$$= \frac{0.5625\ \cancel{A}}{\cancel{A}} \times 100$$
$$= 56.25\%$$

Question 9.
The internal and external diameters of a hollow hemispherical vessel are 20 cm and 28 cm respectively. Find the cost to paint the vessel all over at □ 0.14 per cm^2.
Solution:
External diameter D = 28 cm

Internal diameter d = 20 cm

$$\therefore R = \frac{28}{2} = 14 \text{ cm}, r = \frac{20}{2} = 10 \text{ cm}.$$

T.S.A of the hemispherical vessel = $\pi(3R^2 + r^2)$



$$= \frac{22}{7}(3 \times 196 + 100)$$

$$= \frac{22}{7} \times 688$$

$$= \frac{15136}{7} = 2162.28 \text{ cm}^2$$

Cost of painting @ ₹ 0.14 per cm^2

$$= 2162.28 \times 0.14$$

$$= ₹\ 302.72$$

Question 10.
The frustum shaped outer portion of the table lamp has to be painted including the top part. Find the total cost of painting the lamp if the cost of painting 1 sq.cm is □ 2.

Solution:
Here given that R = 12 m
r = 6 m
h = 8 m

$$l = \sqrt{h^2 + (R-r)^2}$$

$$= \sqrt{8^2 + 6^2} = \sqrt{64+36}$$

$$= \sqrt{100} = 10 \text{ m}$$

$\therefore$ CSA of the frustum

$$= \pi(R + r)l$$

$$= \frac{22}{7}(12 + 6)10$$

$$= \frac{220}{7} \times 18$$

$$= \frac{3960}{7} = 565.71 \text{ m}^2$$

Area of the top part $= \pi r^2$

$$= \frac{22}{7} \times 6 \times 6$$

$$= \frac{792}{7} = 113.14 \text{ m}^2$$

$\therefore$ The total area to be painted

$$= 565.71 + 113.14$$

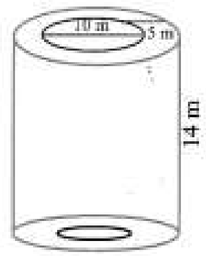
$$= 678.85 \text{ m}^2$$

$\therefore$ The cost of painting ₹ 2 per m^2

$$= ₹\ 1357.72$$

Ex 7.2

Question 1.
A 14 m deep well with inner diameter 10 m is dug and the earth taken out is evenly spread all around the well to form an embankment of width 5 m. Find the height of the embankment.
Solution:
Inner diameter = 10 m
Inner radius = 5 m
Inner height = 14 m



Volume of the cylinder = $\pi r^2 h$ cubic units
$= \frac{22}{7} \times 5 \times 5 \times 14$
= 1100 m^3
Volume of the hollow = n(R^2 – r^2)h cubic units
R = 10 m
r = 5 m
$\Rightarrow \frac{22}{7} \times \left(10^2 - 5^2\right) h = 1100 \mathrm{m}^3$

($\because$ the earth taken out = the earth spread all around)

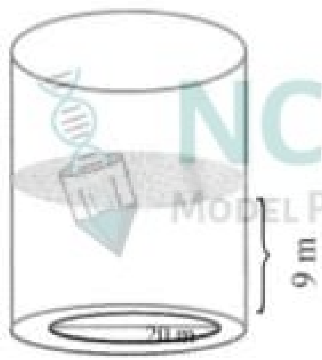
$$\Rightarrow \frac{22}{7} \times (100 - 25)h = 1100 \text{ m}^3$$

$$h = 1100 \times \frac{7}{22} \times \frac{1}{75}$$

$$= 4.66 = 4.67 \text{ m}$$

The height of the $\cong$ 4.7 m embankment.

Question 2.
A cylindrical glass with diameter 20 cm has water to a height of 9 cm. A small cylindrical metal of radius 5 cm and height 4 cm is immersed it completely. Calculate the raise of the water in the glass?
Solution:
The volume of the water raised = Volume of the cylindrical metal.



$$\pi r_2^2 h_2 = \pi r_1^1 h_1$$

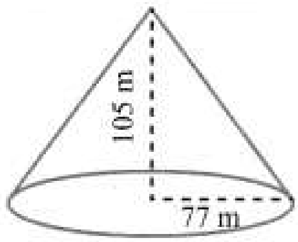
$$\pi \times 10^2 \times h_2 = \pi \times 5^2 \times 4$$

$$h_2 = \frac{100}{100} = 1 \text{ cm.}$$

$\therefore$ The height of the raised water in the glass = 1 cm.

Question 3.
If the circumference of a conical wooden piece is 484 cm then find its volume when its height is 105 cm.
Solution:
Circumference of the base of the cone = 484 cm
height = 105 cm

$\therefore 2\pi r = 484$



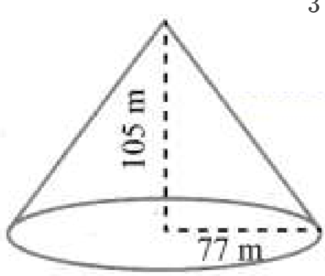
$r = 484 \times \frac{1}{2} \times \frac{7}{22}$
= 77 cm

$$\therefore \quad \text{Its volume} = \frac{1}{3}\pi r^2 h \text{ cubic units}$$

$$= \frac{1}{\cancel{3}} \times \frac{22}{\cancel{7}} \times \overset{11}{\cancel{77}} \times 77 \times \overset{35}{\cancel{105}}$$

$$= 652190 \text{ cm}^3.$$

Question 4.
A conical container is fully filled with petrol. The radius is 10m and the height is 15 m. If the container can release the petrol through its bottom at the rate of 25 cu. meter per minute, in how many minutes the container will be emptied. Round off your answer to the nearest minute.
Solution:
Volume of the cone = $\frac{1}{3}\pi r^2 h$ cu. units.



Volume of the given conical container = $\frac{1}{3} \times \pi \times 10 \times 10 \times 15$
= 500π m^3

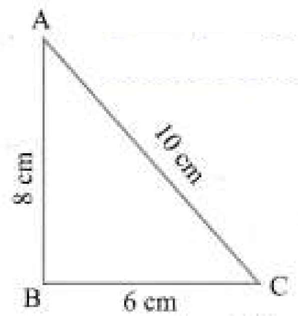
To empty 25 m^3, the time taken = 1 mt.

To empty 500π m^3 the time taken = $\dfrac{500 \times \frac{22}{7} \times 1}{25}$

= 62.857 mts.

$\cong$ 63 minutes.(approx.)

Question 5.
A right angled triangle whose sides are 6 cm, 8 cm and 10 cm is revolved about the sides containing the right angle in two ways. Find the difference in volumes of the two solids so formed.
Solution:
When the triangle ABC is rotated about AB, the



$$V_1 = \frac{1}{3} \times \frac{22}{7} \times 6 \times 6 \times 8$$
$$= \frac{2112}{7} = 301.71 \text{ cm}^3$$

When the Δ ABC i rotated about BC,
$r = 8$ cm, $h = 6$ cm
$$V_2 = \frac{1}{3} \times \frac{22}{7} \times 6 \times 8 \times 8 = \frac{2816}{7} = 402.29$$
$\therefore$ Difference in volume $= V_2 - V_1$
$$= 402.29 - 301.71$$
$$= 100.58 \text{ cm}^3$$

Question 6.
The volumes of two cones of same base radius are 3600 cm^3 and 5040 cm^3. Find the ratio of heights.

Solution:

$$V_1 = 3600 \text{ cm}^2, r_1 = r_2$$

$$V_2 = 5040 \text{ cm}^3$$

$$\frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \frac{3600}{5040}$$

$$\frac{h_1}{h_2} = \frac{90}{126} = \frac{45}{63} = \frac{15}{21} = \frac{5}{7}$$

$$\therefore h_1 : h_2 = 5 : 7$$

Question 7.
If the ratio of radii of two spheres is 4 : 7, find the ratio of their volumes.
Solution:

$$\frac{r_1}{r_2} = \frac{4}{7}$$

$$\therefore \text{Ratio of the volume of two spheres} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3}$$

$$= \frac{\cancel{\frac{4}{3}} \times \pi \times 4^3}{\cancel{\frac{4}{3}} \times \pi \times 4^3} = \frac{64}{343}$$

$$\therefore \quad V_1 : V_2 = 64 : 343$$

Question 8.
A solid sphere and a solid hemisphere have equal total surface area. Prove that the ratio of their volume is $3\sqrt{3} : 4$

Solution:

Sphere : Hemisphere

$4\pi r_1^2 : 3\pi r_2^2$

$4r_1^2 : 3r_2^2$

$\therefore \frac{r_1^2}{r_2} = \frac{3}{4} \Rightarrow \frac{r_1}{r_2} = \frac{\sqrt{3}}{2}$

$\therefore$ Their volumes $= \frac{4}{3}\pi r_1^3 : \frac{4}{3}\pi r_2^3$

$\Rightarrow \frac{\cancel{4}}{\cancel{3}}\pi(\sqrt{3})^3 : \frac{\cancel{4}}{\cancel{3}}\pi \times 2^3$

The ratio of their volume $= 3\sqrt{3} : 8$

Hence proved.

Question 9.

The outer and the inner surface areas of a spherical copper shell are 576π cm^2 and 324π cm^2 respectively. Find the volume of the material required to make the shell.

Solution:

$$4\pi R^2 = 576\pi \text{ cm}^2$$
$$4\pi r^2 = 324\pi \text{ cm}^2$$
$$4\pi R^2 = 576\pi$$
$$R^2 = 144 \text{ cm}^2 \Rightarrow R = 12 \text{ cm}$$
$$4\pi r^2 = 324\pi$$
$r^2 = 81 \text{ cm}^2 \Rightarrow r = 9$ cm

$\therefore$ Volume of the hollow sphere

$$= \frac{4}{3}\pi(R^3 - r^3) \text{ cu. units}$$
$$= \frac{4}{3}\times\frac{22}{7}\times(12^3 - 9^3)$$
$$= \frac{4}{3}\times\frac{22}{7}\times(1728 - 729)$$
$$= \frac{4}{3}\times\frac{22}{7}\times 999$$
$$= \frac{29304}{7}$$
$$= 4186.285$$
$$= 4186.29 \text{ cu. cm.}$$

$\therefore$ The volume of the material needed $= 4186.29 \text{ cm}^3$.

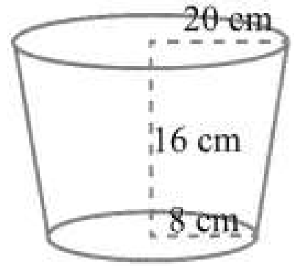
Question 10.
A container open at the top is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends are 8 cm and 20 cm respectively. Find the cost of milk which can completely fill a container at the rate of □ 40 per litre.
Solution:
Volume of the frustum

$$= \frac{1}{3}\pi(R^2 + Rr + r^2)h \text{ cubic units}$$

$$= \frac{1}{3} \times \frac{22}{7}(20^2 + 20 \times 8 + 8^2) \times 16$$

$$= \frac{1}{3} \times \frac{22}{7}(400 + 160 + 64) \times 16$$

$$= \frac{1}{3} \times \frac{22}{7} \times 624 \times 16$$



$$= \frac{73216}{7}$$

$= 10459.428 \text{ cm}^3$

$1000 \text{ cm}^3 = 1$ litre

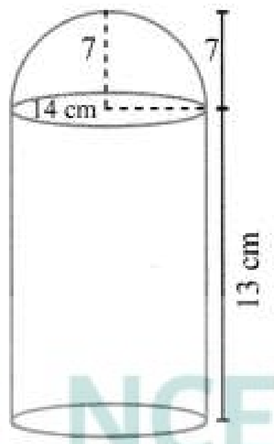
$\therefore 10459.428 \text{ cm}^3 = 10.459$ litres.

The cost of milk @ ₹ 40 per litre

= ₹ 418.36.

Ex 7.3

Question 1.
A vessel is in the form of a hemispherical bowl mounted by a hollow cylinder. The diameter is 14 cm and the height of the vessel is 13 cm. Find the capacity of the vessel.
Solution:
Diameter = 14 cm
Radius = 7 cm
Total height = 13 cm
Height of the cylindrical part = 13 – 7
= 6 cm
∴ Capacity of the vessel = Capacity of the cylinder + Capacity of the hemisphere.



Volume of the cylinder = $\pi r^2 h$

$$= \frac{22}{\cancel{7}} \times \cancel{7} \times 7 \times 6$$

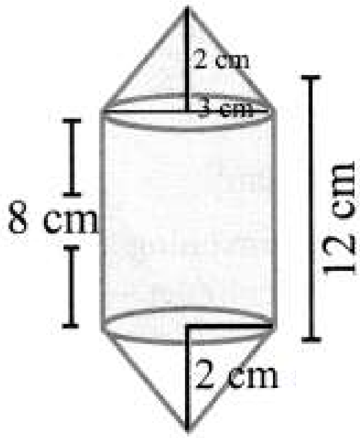
Volume of the hemisphere

$$= \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{\cancel{7}} \times \cancel{7} \times 7 \times 7$$

$$= \frac{2156}{3} = 718.67$$

∴ The total volume = 924 + 718.67
The capacity of the vessel = 1642.67 cm^3

Question 2.
Nathan, an engineering student was asked to make a model shaped like a cylinder with two cones attached at its two ends. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of the model that Nathan made.
Solution:
Volume of the model = Volume of the cylinder + Volume of 2 cones.



$$= \pi r^2 h + 2\frac{1}{3}\pi r^2 h$$

Volume of the cylinder part

$$= \frac{22}{7}\times\frac{3}{2}\times\frac{3}{2}\times 8$$

$$= \frac{396}{7} = 56.57 \text{ cm}^3$$

Volume of the conical parts

$$= \cancel{2}\times\frac{1}{\cancel{3}}\times\frac{22}{7}\times\frac{\cancel{3}}{\cancel{2}}\times\frac{3}{\cancel{2}}\times\cancel{2}$$

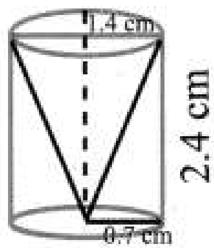
$$= 9.42 \text{ cm}^3$$

$\therefore$ Total volume $= 56.57 + 9.42$

$= 65.99 \text{ cm}^3$

The volume of the model that Nathan made = 66 cm^3

Question 3.
From a solid cylinder whose height is 2.4 cm and the diameter 1.4 cm, a cone of the same height and same diameter is carved out. Find the volume of the remaining solid to the nearest cm^3.
Solution:
Volume of the cylinder = $\pi r^2 h$ cu. units
Volume of the cone = $\frac{1}{3}\pi r^2 h$ cu. units



d = 1.4 cm, r = 0.7 cm = $\frac{7}{10}$
h = 2.4 cm = $\frac{24}{10}$
Volume of the cylinder:

$$= \frac{22}{\cancel{7}} \times \frac{\cancel{7}}{10} \times \frac{7}{10} \times \frac{24}{10}$$

$$= \frac{3696}{1000} = 3.696 \text{ cm}^3.$$

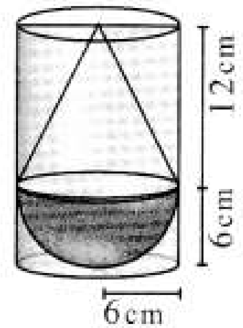
Volume of cone carved out

$$= \frac{1}{\cancel{3}} \times \frac{22}{\cancel{7}} \times \frac{\cancel{7}}{10} \times \frac{7}{10} \times \frac{\cancel{24}^{8}}{10}$$

$$= \frac{1232}{1000} = 1.232 \text{ cm}^3$$

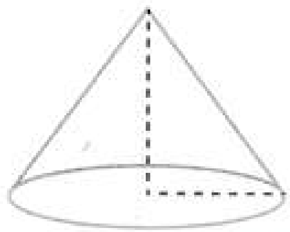
∴ Volume of the remaining solid = Volume of the cylinder – Volume of the cone
= 3.696 – 1.232
= 2.464
= 2.46 cm^3

Question 4.
A solid consisting of a right circular cone of height 12 cm and radius 6 cm standing on a hemisphere of radius 6 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of the water displaced out of the cylinder, if the radius of the cylinder is 6 cm and height is 18 cm.



Solution:
Volume of water displaced out = Volume of the solid immersed in.
Volume of the solid = Volume of the cone + Volume of the hemisphere

Cone



$r = 6$ cm
$h = 12$ cm

hemisphere

$r = 6$ cm

$\therefore$ Volume of the cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\times\frac{22}{7}\times6\times6\times12$$

$$= \frac{3168}{7} = 452.57 \text{ cm}^3 \text{ ...(1)}$$

Volume of the hemisphere $= \frac{2}{3}\pi r^3$

$$= \frac{2}{3}\times\frac{22}{7}\times6\times6\times6$$

$$= \frac{3168}{7} = 452.57 \text{ cm}^3$$

$\therefore$ The volume of water displaced out = Volume of the solid

= (1) + (2)
= 905.14 cm^3

Question 5.
A capsule is in the shape of a cylinder with two hemisphere stuck to each of its ends. If the length of the entire capsule is 12 mm and the diameter of the capsule is 3 mm, how much medicine it can hold?
Solution:
Volume of medicine the capsule can hold = Volume of the cylinder + 2 volume of hemisphere
Volume of the cylinder part

$$= \pi r^2 h = \frac{\cancel{22}^{11}}{7} \times \frac{3}{\cancel{2}} \times \frac{3}{\cancel{2}} \times \cancel{6}^{3}$$

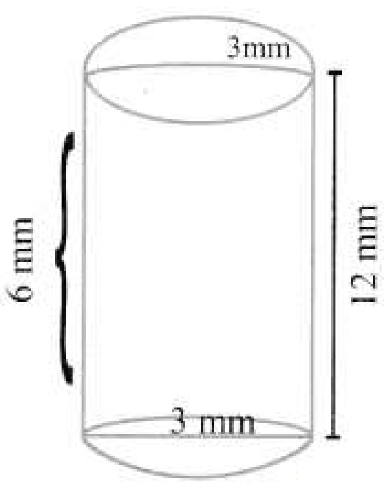
$$= \frac{297}{7} = 42.428 \text{ mm}^3$$

Volume of 2 hemispherical parts

$$= 2 \times \frac{2}{3} \times \pi \times r^3$$

$$= \cancel{2} \times \frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{22}^{11}}{7} \times \frac{\cancel{3}}{\cancel{2}} \times \frac{3}{\cancel{2}} \times \frac{3}{\cancel{2}}$$

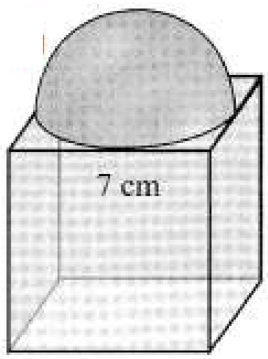
$$= 14.142 \text{ mm}^3$$



∴ The total volume = 56.571 mm^3

∴ The volume of the medicine the capsule can hold = 56.57 mm^3

Question 6.
As shown in figure a cubical block of side 7 cm is surmounted by a hemisphere. Find the surface area of the solid.



Solution:
Clearly, greatest diameter of the hemisphere is equal to the length of an edge of the cube is 7 cm.
Radius of the hemisphere = $\frac{7}{2}$ cm
Now, total surface area of the solid = Surface area of the cube + Curved surface area of the hemisphere – Area of the base of the hemisphere.

$$= \left(6\times 7^2 + 2\times\frac{22}{7}\times\left(\frac{7}{2}\right)^2 - \frac{22}{7}\times\left(\frac{7}{2}\right)^2\right)\text{cm}^2$$

$$= \left(294 + 77 - \frac{77}{2}\right)\text{cm}^2$$

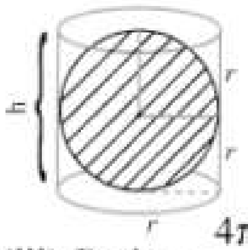
$$= \left(294 + \frac{77}{2}\right)\text{cm}^2$$

$$= 332.5 \text{ cm}^2$$

Question 7.
A right circular cylinder just enclose a sphere of radius r units.
Calculate
(i) the surface area of the sphere
(ii) the curved surface area of the cylinder
(iii) the ratio of the areas obtained in (i) and (ii).
Solution:

(i) Surface area of sphere = $4\pi r^2$ sq. units

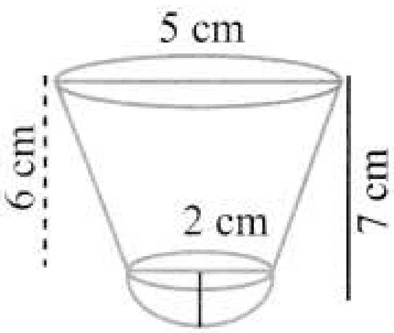
(ii) C.S.A of cylinder



$$= 2\pi r \times h$$
$$= 2\pi r \times 2r$$
$$= 4\pi r^2 \text{ sq. units}$$

(iii) Ratio = $\frac{4\pi r^2}{4\pi r^2} = 1:1$

Question 8.
A shuttle cock used for playing badminton has the shape of a frustum of a cone is mounted on a hemisphere. The diameters of the frustum are 5 cm and 2 cm. The height of the entire shuttle cock is 7 cm. Find its external surface area.
Solution:
External surface area of the cock = Surface area of frustum + CSA of hemisphere



CSA of frustum = $\pi(R + r)l$ sq. units.

$$\text{Here } R = \frac{5}{2} \text{ cm}$$

$$r = \frac{2}{2} = 1 \text{ cm.}$$

$$l = \sqrt{(R-r)^2 + h^2}$$

$$= \sqrt{(2.5-1)^2 + 6^2}$$

$$= \sqrt{1.5^2 + 36}$$

$$= \sqrt{2.25 + 36}$$

$$= \sqrt{38.25}$$

$$\cong 6.18$$

Hint:

```
            6.18
6        | 38.2500
         | 36
12 × 1   |   225
         |   121
1228 × 8 |  10400
         |   9824
         |    576
```

$\therefore$ CSA of the frustum

$$= \frac{22}{7} \times 3.5 \times 6.1 = \frac{469.7}{7}$$

$$= 67.1 \text{ cm}^2$$

CSA of hemisphere $= 2\,\pi r^2$

$$= 2 \times \frac{22}{7} \times 1 \times 1$$

$$= 6.28 \text{ cm}^2$$

$\therefore$ Total external surface area

$$= 67.1 + 6.28$$

$$= 73.38 \text{ cm}^2$$

$$= 73.39 \text{ cm}^2 \text{ (approx.)}$$

Ex 7.4

Question 1.
An aluminium sphere of radius 12 cm is melted to make a cylinder of radius 8 cm. Find the height of the cylinder.
Solution:
Sphere – Radius r_1 = 12 cm
Cylinder – Radius r_2 = 8 cm
h_2 = ?
Volume of cylinder = Volume of sphere melted

$$\pi r_2^2 h_2 = \frac{4}{3}\pi r_1^3$$

$$\frac{\cancel{22}}{\cancel{7}} \times 8 \times 8 \times h_2 = \frac{4}{3} \times \frac{\cancel{22}}{\cancel{7}} \times 12 \times 12 \times 12$$

$$h_2 = \frac{\cancel{4}}{\cancel{3}} \times \cancel{12}^{6} \times \cancel{12}^{4} \times \cancel{12}^{6} \times \frac{1}{\cancel{8}_{2}} \times \frac{1}{\cancel{8}_{2}}$$

∴ Height of the cylinder made = 36 cm.

Question 2.
Water is flowing at the rate of 15 km per hour through a pipe of diameter 14 cm into a rectangular tank which is 50 m long and 44 m wide. Find the time in which the level of water in the tanks will rise by 21 cm.
Solution:
In cylinder,
r = 7cm = 0.7m
l = 15 km = 15000 m
In tank
l = 50 m

b = 44 m
h = 0.21 m
Volume of water in tank = lbh
= 50 × 44 × 0.21
= 462 m^3

$$\text{Height of cylinderical pipe} = \frac{\text{Volume}}{\pi r^2}$$

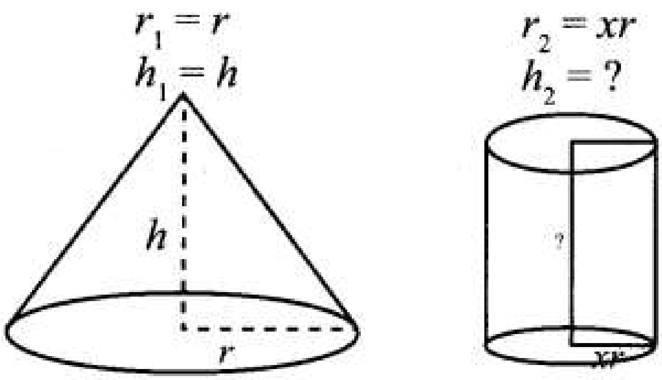
$$= \frac{462}{(0.07)^2\left(\frac{22}{7}\right)}$$

$$= \frac{462}{0.0154}$$

$$= 30000 \text{ m}$$

$$\text{Time} = \frac{30000}{15000} = 2 \text{ hours.}$$

Question 3.
A conical flask is full of water. The flask has base radius r units and height h units, the water poured into a cylindrical flask of base radius xr units. Find the height of water in the cylindrical flask.
Solution:
The volume of water poured = The volume of the water in the conical flask.



$$\pi r_2^2 h_2 = \frac{1}{3}\pi r_1^2 h_1$$

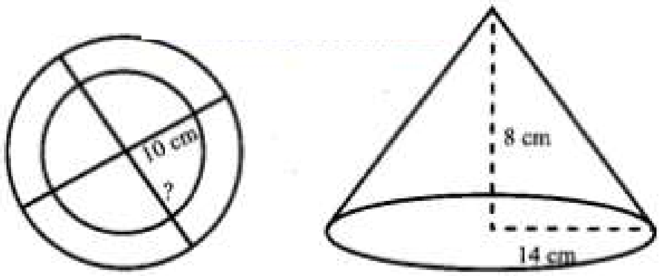
$$\frac{\cancel{22}}{\cancel{7}}\times(xr)(xr)h_2 = \frac{1}{3}\times\frac{22}{7}\times r\times h$$

$$x^2 h_2 = \frac{h}{3}$$

$$\therefore h_2 = \frac{h}{3x^2} \text{ units}$$

$\therefore$ The height of the water in the cylindrical flask = $\frac{h}{3x^2}$ units.

Question 4.
A solid right circular cone of diameter 14 cm and height 8 cm is melted to form a hollow sphere. If the external diameter of the sphere is 10 cm, find the internal diameter.
Solution:
Volume of the hollow sphere made = Volume of the cone melted.



$$\frac{4}{3}\pi(R^3 - r^3) = \frac{1}{3}\pi r^2 h$$

$$\frac{4}{3}\pi[5^3 - r^3] = \frac{1}{3}\pi \times 7 \times 7 \times 8$$

$$\frac{4}{3}\times\frac{22}{7}(125 - r^3) = \frac{1}{3}\times\frac{22}{7}\times 7 \times 7 \times 8$$

$$125 - r^3 = \cancel{392}^{98} \times \frac{1}{\cancel{4}}$$

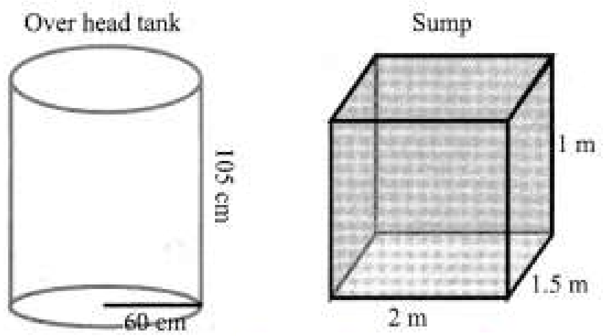
$$-r^3 = 98 - 125$$

$$r^3 = 27 \Rightarrow r = 3$$

The internal diameter $= 2r = 2 \times 3$

$= 6$ cm.

Question 5.
Seenu's house has an overhead tank in the shape of a cylinder. This is filled by pumping water from a sump (underground tank) which is in the shape of a cuboid. The sump has dimensions 2 m × 1.5 m × 1 m. The overhead tank has its radius of 60 cm and height 105 cm. Find the volume of the water left in the sump after the overhead tank has been completely filled with water from the sump which has been full, initially.
Solution:
Volume of water in the sump = lbh
$= 2 \times 1.5 \times 1 = 200 \times 150 \times 100 = 3000000$ cm^3

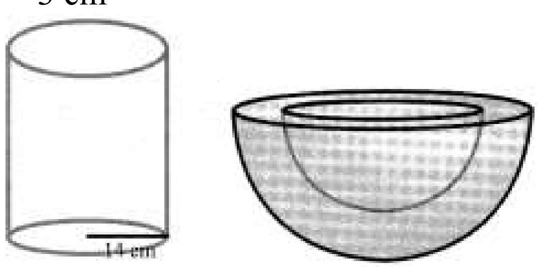


Volume of water in the overhead tank = $\pi r^2 h$

$$= \frac{22}{7} \times 60 \times 60 \times 105$$

= 1188000 cm^3
∴ The volume of water left in the sump = 3000000 – 1188000
= 1812000 cm^3

Question 6.
The internal and external diameter of a hollow hemispherical shell are 6 cm and 10 cm respectively. If it is melted and recast into a solid cylinder of diameter 14 cm, then find the height of the cylinder.
Solution:
D = 10 cm,
R = 5 cm
d = 6 cm,
r = 3 cm



Volume of the cylinder made = Volume of hemisphere melted.

$$\pi r^2 h = \frac{2}{3}\pi(R^3 - r^3) \text{ cubic units}$$

$$\frac{22}{\cancel{7}} \times 7 \times 7 \times h = \frac{2}{3} \times \frac{22}{\cancel{7}}(5^3 - 3^3)$$

$$h = \frac{2}{3}(125 - 27) \times \frac{1}{7} \times \frac{1}{7}$$

$$= \frac{2}{3} \times 98 \times \frac{1}{7} \times \frac{1}{7}$$

∴ Height of the cylinder made = 1.33 cm.

Question 7.
A solid sphere of radius 6 cm is melted into a hollow cylinder of uniform thickness. If the external radius of the base of the cylinder is 5 cm and its height is 32 cm, then find the thickness of the cylinder.
Solution:
Volume of the hollow cylinder made = Volume of the sphere melted.

$$\pi(R^2 - r^2)h = \frac{4}{3}\pi r^3$$

$$\frac{22}{7}(5^2 - r^2)32 = \frac{4}{3} \times \frac{22}{7} \times 6 \times 6 \times 6$$

$$25 - r^2 = \frac{4}{3} \times 6 \times 6 \times 6 \times \frac{1}{32}$$

$$25 - r^2 = 9$$

$$-r^2 = 9 - 25 = -16$$

$$r^2 = 16 \Rightarrow r = 4 \text{ cm.}$$

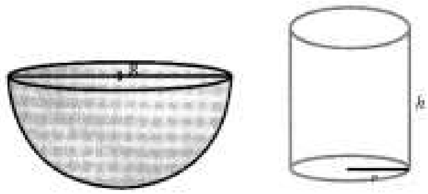
∴ The thickness = External radius – Internal radius
= 5 – 4 = 1

Question 8.
A hemispherical bowl is filled to the brim with juice. The juice is poured into a cylindrical vessel

whose radius is 50% more than its height. If the diameter is same for both the bowl and the cylinder then find the percentage of juice that can be transferred from the bowl into the cylindrical vessel.

Solution:

Diameter of the bowl = Diameter of the cylinder



$$\therefore \quad r_1 = r_2$$

$$1\frac{1}{2}h = r \Rightarrow \frac{3}{2}h = r$$

$$\left[\because r = 1 + .5 = 1.5 = \frac{3}{2}\right]$$

Volume of the hemispherical bowl $= \frac{2}{3}\pi r_1^3$

Volume of the cylindrical vessel

$$= \pi r_2^2 h \qquad (\because r_1 = r_2)$$

$$= \frac{22}{7} \times \left(\frac{3}{2}h\right)^2 \times h = \frac{22}{7} \times \frac{9}{4}h^3$$

$$= \frac{22}{7} \times \frac{9}{4}\left(\frac{2}{3}r\right)^3$$

$$= \frac{22}{7} \times \frac{\cancel{9}}{\cancel{4}} \times \frac{\cancel{8}^{2}}{\cancel{27}_{3}} r^3 = \frac{2}{3}\pi r_1^3$$

∴ Both volumes are equal.

∴ 100% of juice that can be transferred from the bowl into the cylindrical vessel.

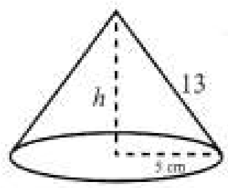
Ex 7.5

Multiple choice questions.
Question 1.
The curved surface area of a right circular cone of height 15 cm and base diameter 16 cm is ______
(1) 60π cm^2
(2) 68π cm^2
(3) 120π cm^2
(4) 136π cm^2
Answer:
(4) 136π cm^2
Hint:

$$l = \sqrt{h^2 + r^2} = \sqrt{15^2 + 8^2}$$
$$= \sqrt{225 + 64} = \sqrt{289} = 17 \text{ cm}$$
$$\pi r l = \frac{22}{7} \times 8 \times 17 = 136\pi \text{ cm}^2$$

Question 2.
If two solid hemispheres of same base radius r units are joined together along with their bases, then the curved surface area of this new solid is
(1) $4\pi r^2$ sq. units
(2) $67\pi r^2$ sq. units
(3) $3\pi r^2$ sq. units
(4) $8\pi r^2$ sq. units
Solution:
(1) $47\pi r^2$ sq. units]

Question 3.
The height of a right circular cone whose radius is 5 cm and slant height is 13 cm will be ___________
(1) 12 cm
(2) 10 cm
(3) 13 cm
(4) 5 cm
Answer:
(1) 12 cm
Hint:

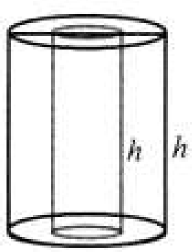


$$h = \sqrt{13^2 - 5^2} = \sqrt{169 - 25}$$
$$= \sqrt{144} = 12 \text{ cm.}$$

Question 4.
If the radius of the base of a right circular cylinder is halved keeping the same height, then the ratio of the volume of the cylinder thus obtained to the volume of original cylinder is
(1) 1 : 2
(2) 1 : 4
(3) 1 : 6
(4) 1 : 8
Solution:
(2) 1 : 4

Hint:

$$V_2 = \pi\left(\frac{r}{2}\right)^2 h = \frac{\pi r^2 h}{4}$$



$$V_1 = \pi r^2 h = \pi r^2 h$$

$$\frac{V_2}{V_1} = \frac{\frac{\pi r^2 h}{4}}{\pi r^2 h} = \frac{\cancel{\pi r^2 h}}{4} \times \frac{1}{\cancel{\pi r^2 h}}$$

$$= \frac{1}{4} = 1:4$$

Question 5.
The total surface area of a cylinder whose radius is $\frac{1}{3}$ of its height is

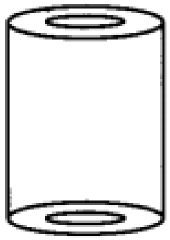
(1) $\frac{9\pi h^2}{8}$ sq.units (2) $24\pi h^2$ sq.units

(3) $\frac{8\pi h^2}{9}$ sq.units (4) $\frac{56\pi h^2}{9}$ sq.units

Solution:
(3) $\frac{8\pi h^2}{9}$ sq. units

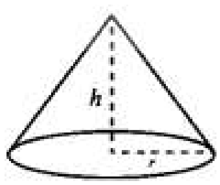
$$\begin{aligned}
\text{TSA of a cylinder} &= 2\pi r(h+r),\ r = \frac{1}{3}h \\
&= 2\pi \frac{1}{3}h\left(h + \frac{1}{3}h\right) \\
&= \frac{2}{3}\pi h^2 + \frac{2}{3}\pi \frac{h^2}{3} \\
&= \frac{2}{3}\pi h^2 + \frac{2}{3}\pi \frac{h^2}{3} \\
&= \frac{6\pi h^2 + 2\pi h^2}{9} = \frac{8\pi h^2}{9}
\end{aligned}$$

Question 6.
In a hollow cylinder, the sum of the external and internal radii is 14 cm and the width is 4 cm. If its height is 20 cm, the volume of the material in it is _______
(1) 560π cm^3
(2) 1120π cm^3
(3) 56π cm^3
(4) 360π cm^3
Answer:
(2) 1120π cm^3
Hint:
R + r = 14 cm
w = 4 cm
h = 90 cm



$$\begin{aligned} \text{Volume} &= \pi(R^2 - r^2)h \\ &= \pi(R + r)(R - r)h \\ &= \pi(14)4 \times 20 \\ &= 1120\pi \text{ cm}^3 \end{aligned}$$

Question 7.
If the radius of the base of a cone is tripled and the height is doubled then the volume is
(1) made 6 times
(2) made 18 times
(3) made 12 times
(4) unchanged
Solution:
(2) made 18 times
Hint:

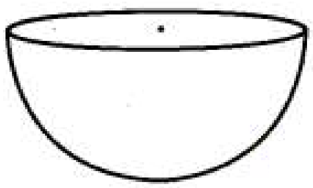


$$\begin{aligned} r &= 3r \\ h &= 2h \\ v &= \frac{1}{3}\pi(3r)^2 \times (2h) \\ &= \frac{1}{3} \times \pi 9r^2 \times 2h \\ &= 6\pi r^2 h = 18 \times \frac{1}{3}\pi r^2 h \\ &= \text{made 18 times.} \end{aligned}$$

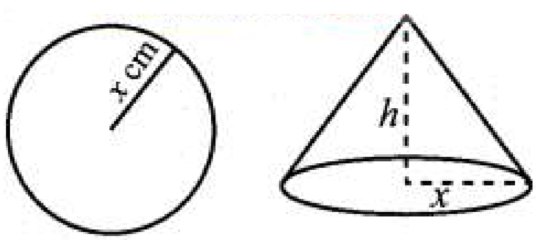
Question 8.
The total surface area of a hemisphere is how many times the square of its radius ______

(1) π
(2) 4π
(3) 3π
(4) 2π
Answer:
(3) 3π
Hint:

TSA = $3\pi r^2$



Question 9.
A solid sphere of radius x cm is melted and cast into a shape of a solid cone of same radius. The height of the cone is
(1) 3x cm
(2) x cm
(3) 4x cm
(4) 2x cm
Solution:
(3) 4x cm
Hint:



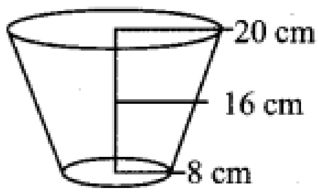
$$\frac{1}{3}\pi r^2 h = \frac{4}{3}\pi r^3$$

$$\frac{1}{\cancel{3}}\cancel{\pi}\, x^{\cancel{2}} h = \frac{4}{\cancel{3}}\cancel{\pi}\, \overset{x}{\cancel{x^3}}\, x$$

$$h = 4x \text{ cm}$$

Question 10.
A frustum of a right circular cone is of height 16 cm with radii of its ends as 8 cm and 20 cm.

Then, the volume of the frustum is _______
(1) 3328π cm3
(2) 3228π cm3
(3) 3240π cm3
(4) 3340π cm3
Answer:
(1) 3328π cm3Hint:

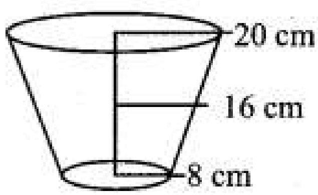


$$V = \frac{\pi h}{3}[R^2 + r^2 + Rr]$$
$$= \pi \times \frac{16}{3}[20^2 + 8^2 + 20 \times 8]$$
$$= \pi \times \frac{16}{3}[400 + 64 + 160]$$
$$= \pi \times \frac{16}{3} \times 624 = 3328\pi \text{ cm}^3$$

Question 11.
A shuttlecock used for playing badminton has the shape of the combination of
(1) a cylinder and a sphere
(2) a hemisphere and a cone
(3) a sphere and a cone
(4) frustum of a cone and a hemisphere
Solution:
(4) frustum of a cone and a hemisphere

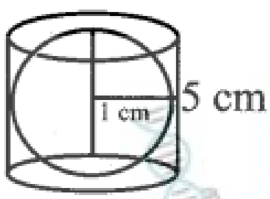
Question 12.
A spherical ball of radius r_1 units is melted to make 8 new identical balls each of radius r_2 units. Then $r_1 : r_2$ is
(1) 2 : 1
(2) 1 : 2
(3) 4 : 1
(4) 1 : 4
Solution:

Hint:



$$
\begin{aligned}
V &= \frac{\pi h}{3}[R^2 + r^2 + Rr] \\
&= \pi \times \frac{16}{3}[20^2 + 8^2 + 20 \times 8] \\
&= \pi \times \frac{16}{3}[400 + 64 + 160] \\
&= \pi \times \frac{16}{3} \times 624 = 3328\pi \text{ cm}^3
\end{aligned}
$$

Question 13.
The volume (in cm^3) of the greatest sphere that can be cut off from a cylindrical log of wood of base radius 1 cm and height 5 cm is



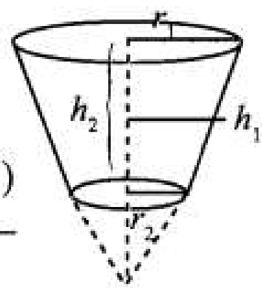
Solution:
(1) $\frac{4}{3}\pi$

$$\frac{4}{3}\pi \times 1 = \frac{4}{3}\pi$$

Question 14.
The height and radius of the cone of which the frustum is a part are h_1 units and r_1 units respectively. Height of the frustum is h_2 units and the radius of the smaller base is r_2 units. If $h_2 : h_1 = 1 : 2$ then $r_2 : r_1$ is
(1) 1 : 3
(2) 1 : 2
(3) 2 : 1
(4) 3 : 1
Solution:
(2) 1 : 2

$h_2 : h_1 = 1 : 2$

$r_2 : r_1$



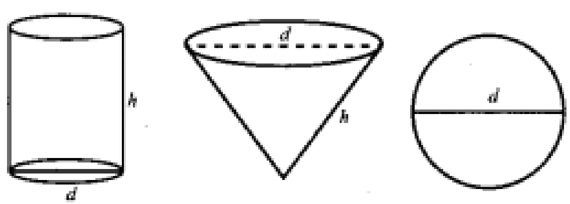
$$\frac{V_2}{V_1} = \frac{\cancel{\frac{1}{3}}\pi r_2^2 (h_1 - h_2)}{\cancel{\frac{1}{3}}\pi r_1^2 (h_1)}$$

$$\frac{V_2}{V_1} = \frac{r_2^2 h_1 - r_2^2 h_2}{r_1^2 h_1}$$

$$= \frac{r_2^2 2h_2 - r_2^2 h_2}{r_1^2 2h_2} = \frac{r_2^2 (2h_2 - h_2)}{r_1^2 2h_2}$$

$$= \frac{r_2^2 \cdot \cancel{h_2}}{r_1^2 2\cancel{h_2}} = \frac{r_2^2}{r_1^2} = \frac{1}{2}$$

Question 15.
The ratio of the volumes of a cylinder, a cone and a sphere, if each has the same diameter and same height is ____
(1) 1 : 2 : 3
(2) 2 : 1 : 3
(3) 1 : 3 : 2
(4) 3 : 1 : 2
Answer:
(4) 3 : 1 : 2
Hint:

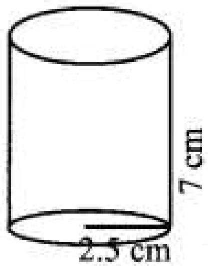


$$\cancel{\pi r^2} h : \frac{1}{3} \cancel{\pi r^2} h : \frac{4}{3} \cancel{\pi r^3} \qquad h : \frac{h}{3} : \frac{4}{3} r$$

Unit Exercise 7

Question 1.
The barrel of a fountain-pen cylindrical in shape, is 7 cm long and 5 mm in diameter. A full barrel of ink in the pen will be used for writing 330 words on an average. How many words can be written using a bottle of ink containing one fifth of a litre?
Solution:

Volume of the barrel = $\pi r^2 h$



$$r = \frac{5}{2} \text{ mm}$$
$$h = 7 \text{ cm} = 70 \text{ mm}$$
$$\therefore V = \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times 70$$

Volume of ink in the bottle = $\frac{1}{5}$ litre = $\frac{1000}{5}$ cm^3
= 200 cm^3

With 1.375 cm^3, no. of words can be written = 330 words.

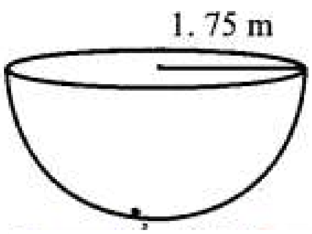
With 200 cm^3, No. of words can be written

$$= \frac{200 \times 330 \times 1000}{1.375 \times 1000} = \frac{200 \times 330 \times 1000}{1375}$$

$$= \frac{66000000}{1375} = 48000 \text{ words}$$

Question 2.
A hemi-spherical tank of radius 1.75 m is full of water. It is connected with a pipe which empties the tank at the rate of 7 litre per second. How much time will it take to empty the tank completely?
Solution:
Suppose the pipe takes x seconds to empty the tank. Then, volume of the water that flows out of the tank in x seconds = Volume of the hemispherical tank.
Volume of the water that flows out of the tank in x seconds.
= Volume of hemispherical shell of radius 175 cm.

$$7000x = \frac{2}{3} \times \frac{22}{7} \times \underline{175} \times 175 \times 175$$



$$x = \frac{2}{3} \times \frac{22}{7} \times \frac{175 \times 175 \times 175}{7000}$$

$$= 1604.16 \text{ seconds}$$

$$x = \frac{1604.16}{60} \text{ minutes}$$

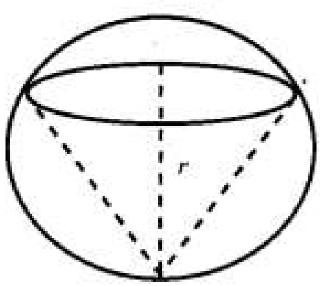
$$x = 26.73 \text{ minutes}$$

$$\cong 27 \text{ minutes}$$

Question 3.
Find the maximum volume of a cone that can be carved out of a solid hemisphere of radius r units.
Solution:
Radius of the base of cone = Radius of the hemisphere = r
Height of the cone = Radius of the hemisphere

$$\therefore \text{Volume of the cone} = \frac{1}{3}\pi r^2 \times r$$

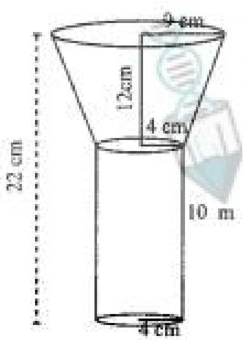
$$= \frac{1}{3}\pi r^3 \text{ cubic units}$$



Question 4.
An oil funnel of tin sheet consists of a cylindrical portion 10 cm long attached to a frustum of a cone. If the total height is 22 cm, the diameter of the cylindrical portion be 8cm and the diameter of the top of the funnel be 18 cm, then find the area of the tin sheet required to make the funnel.
Solution:
Slant height of the frustum

$$l = \sqrt{(R-r)^2 + h_1^2}$$
$$= \sqrt{(9-4)^2 + 12^2}$$
$$= \sqrt{5^2 + 12^2}$$
$$= \sqrt{25 + 144}$$
$$= \sqrt{169} = 13 \text{ cm}$$

Outer surface area $= 2\pi r h_2 + \pi(R + r)\, l$ sq. units



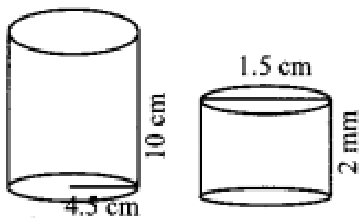
$$= \pi[2rh_2 + (R + r)l]$$
$$= \pi[2 \times 4 \times 10 + (9 + 4)13]$$
$$= \pi[80 + 169]$$
$$= \pi \times 249$$
$$= 782.57 \text{ cm}^2$$

$\therefore$ Area of the sheet required to make the funnel
$\cong 782.57 \text{ cm}^2$.

Question 5.
Find the number of coins, 1.5 cm in diameter and 2 mm thick, to be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm.
Solution:

No. of coins required .

$$= \frac{\text{Volume of the larger cylinder}}{\text{Volume of 1 coin}}$$



$$= \frac{\pi R^2 H}{\pi r^2 h}$$

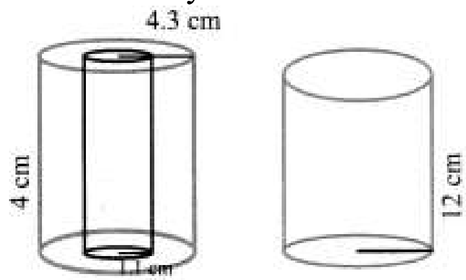
$$= \frac{\frac{22}{7} \times \frac{45}{20} \times \frac{45}{20} \times 10}{\frac{22}{7} \times \frac{15}{20} \times \frac{15}{20} \times \frac{2}{10}}$$

$$= \frac{22}{7} \times \frac{45}{20} \times \frac{45}{20} \times 10 \times \frac{7}{22} \times \frac{20}{15} \times \frac{20}{15} \times \frac{10}{2}$$

$$= 450 \text{ coins}$$

Question 6.
A hollow metallic cylinder whose external radius is 4.3 cm and internal radius is 1.1 cm and whole length is 4 cm is melted and recast into a solid cylinder of 12 cm long. Find the diameter of solid cylinder.
Solution:

Volume of the solid cylinder = Volume of the hollow cylinder melted.



$$\pi r_2^2 h = \pi(R^2 - r^2)h$$

$$\frac{22}{7} \times r_2^2 \times 12 = \frac{22}{7}\left((4.3)^2 - (1.1)^2\right)4$$

$$r_2^2 = \frac{\cancel{22}}{\cancel{7}}(18.49 - 1.21) \times \cancel{4} \times \frac{7}{\cancel{22}} \times \frac{1}{\cancel{12}_3}$$

$$= \frac{17.28}{3}$$

$$r_2^2 = 5.76$$

$$r_2 = 2.4$$

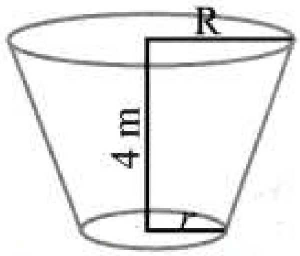
$\therefore$ The diameter of the solid cylinder

$$= 2 \times 2.4 \text{ cm}$$

$$= 4.8 \text{ cm}$$

Question 7.
The slant height of a frustum of a cone is 4 m and the perimeter of circular ends are 18 m and 16 m. Find the cost of painting its curved surface area at □ 100 per sq. m.

Solution:



$$2\pi R = 18$$

$$R = 18\times\frac{1}{2}\times\frac{7}{22} = 2.86 \text{ m}$$

$$2\pi r = 16 \text{ m}$$

$$r = 16\times\frac{1}{2}\times\frac{7}{22}$$

$$= 2.54 \text{ m}$$

$$l = 4 \text{ m}$$

$$\text{C.S.A of frustum} = \pi l(R + r)$$

$$= \frac{22}{7}\times 4\,(2.86 + 2.54)$$

$$= \frac{22}{7}\times 4\times 5.4$$

$$= 67.88\text{m}^2 \cong 68\text{m}^2$$

Cost of painting @ ₹ 100 per sq. m

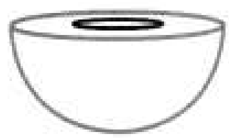
$$= 6.8 \times 100$$

$$= ₹6800$$

Question 8.
A hemi-spherical hollow bowl has material of volume $\frac{436\pi}{3}$ cubic cm. Its external diameter is 14 cm. Find its thickness.

Solution:

$$\begin{aligned}
\text{Volume} &= \frac{2}{3}\pi(R^3 - r^3) \\
D &= 14 \text{ cm}, R = 7 \text{ cm} \\
\frac{2}{\cancel{3}} \times \frac{\cancel{22}}{\cancel{7}} \times (7^3 - r^3) &= \frac{436\cancel{\pi}}{\cancel{3}} \\
(343 - r^3) &= \frac{436}{2} \\
-r^3 &= 218 - 343 \\
\cancel{-}r^3 &= \cancel{-}125 \\
r &= 5 \text{ cm}
\end{aligned}$$



Thickness of the bowl

$$\begin{aligned}
&= R - r = 7 - 5 \\
&= 2 \text{ cm}
\end{aligned}$$

Question 9.
The volume of a cone is $1005\frac{5}{7}$ cu. cm. The area of its base is $201\frac{1}{7}$ sq. cm. Find the slant height of the cone.

Solution:

$$\text{Volume of a cone} = 1005\frac{5}{7} \text{ cu. cm}$$

$$\frac{1}{3}\pi r^2 h = \frac{7040}{7}$$

Hint:
$\pi r^2 = \frac{1408}{7}$
$r^2 = \frac{1408}{\cancel{7}} \times \frac{\cancel{7}}{22}$
$= 64;\ r = 8$ cm

$$\frac{1}{3} \times \frac{1408}{\cancel{7}} \times h$$

$$= \frac{7040}{\cancel{7}} \times 3 = 15 \text{ cm}$$

$$h = \frac{7040}{1408} = 5 \text{ cm}$$

$\therefore$ The height of the cone = 15 cm

Hint:

	9.4
9	89.00
	81
184 × 4	800
	736

$$\text{Slant height } l = \sqrt{h^2 + r^2}$$

$$= \sqrt{15^2 + 8^2}$$

$$= \sqrt{225 + 64}$$

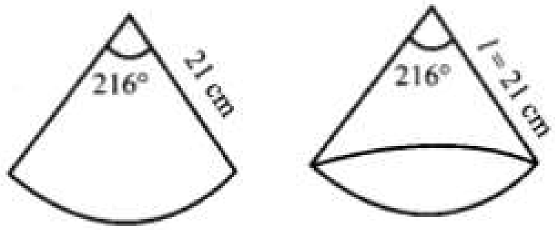
$$= \sqrt{289}$$

$$l = 17 \text{ cm}$$

Question 10.
A metallic sheet in the form of a sector of T a circle of radius 21 cm has central angle of 216°. The sector is made into a cone by bringing the bounding radii together. Find the volume of the cone formed.
Solution:

Length of the arc = $\frac{216}{360} \times 2 \times \frac{22}{7} \times 21$

$= 79.2$ cm



Base perimeter of the cone

$$= \text{length of the arc}$$

$$2\pi r = 79.2$$

$$r = 79.2 \times \frac{1}{2} \times \frac{7}{22}$$

$$= 12.6 \text{ cm}$$

$$l = 21 \text{ cm}$$

$$h = \sqrt{l^2 - r^2}$$

$$h = \sqrt{21^2 - 12.6^2}$$

$$= \sqrt{441 - 158.76}$$

$$= \sqrt{282.24}$$

$$h = 16.8 \text{ cm}$$

$\therefore$ Volume of the cone = $\frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 12.6 \times 12.6 \times 16.8$$

$$= 2794.18 \text{ cm}^3$$

Additional Questions

Question 1.
If the radii of the circular ends of a conical bucket which is 45 cm high are 28 cm and 7 cm, find the capacity of the bucket. (Use $\pi = \frac{22}{7}$)
Solution:
Clearly bucket forms frustum of a cone such that the radii of its circular ends are r_1 = 28 cm, r_2 = 7 cm, h = 45 cm.
Capacity of the bucket = volume of the frustum

$$\Rightarrow \frac{1}{3} \times \pi h[r_1^2 + r_2^2 + r_1 r_2]$$

$$\Rightarrow \frac{1}{3} \times \frac{22}{7} \times 45\,[28^2 + 7^2 + 28 \times 7)]$$

$$\Rightarrow 22 \times 15 \times (28 \times 4 + 7 + 28)$$

$$\Rightarrow 330 \times 147\ \text{cm}^3 \Rightarrow 48510\ \text{cm}^3$$

Question 2.
Find the depth of a cylindrical tank of radius 28 m, if its capacity is equal to that of a rectangular tank of size 28 m × 16 m × 11 m.
Solution:

Volume of the cylindrical tank = Volume of the rectangle tank

$$\pi r^2 h = 28 \times 16 \times 11 \text{ m}^3$$

$$\frac{22}{\cancel{7}} \times \cancel{28}^4 \times \cancel{28} \times h = \cancel{28} \times 16 \times 11$$

$$h = \frac{16 \times 11}{88} = 2 \text{ m}$$

Question 3.
What is the ratio of the volume of a cylinder, a cone, and a sphere. If each has the same diameter and same height?
Solution:

Volume of a cylinder $= \pi r^2 h$

Volume of a cone $= \frac{1}{3}\pi r^2 h$

Volume of a sphere $= \frac{4}{3}\pi r^3$

Their ratio $V_1 : V_2 : V_3$

$$\cancel{\pi} \cancel{r^2} h : \frac{1}{3}\cancel{\pi} \cancel{r^2} h : \frac{4}{3}\cancel{\pi} \cancel{r^3}$$

$$h : \frac{h}{3} : \frac{4r}{3}$$

$3h : h : 4r$

$3h : h : 2(2r)$ (where $2r = h$)

$\therefore V_1 : V_2 : V_3 = 3 : 1 : 2$

Question 4.
Find the number of coins, 1.5 cm is diameter and 0.2 cm thick, to be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm.

Solution:

$$\text{No. of coins required}$$

$$= \frac{\text{Volume of the cylinder}}{\text{Volume of 1 coin}}$$

$$= \frac{\pi r_1^2 h_1}{\pi r_2^2 h_2} = \frac{\pi \times \frac{45}{20} \times \frac{45}{20} \times 10}{\pi \times \frac{15}{20} \times \frac{15}{20} \times \frac{2}{10}}$$

$$= \frac{\cancel{45} \times \cancel{45} \times \cancel{10}}{\cancel{20} \times \cancel{20}} \times \frac{\cancel{20}^{10}}{\cancel{15}} \times \frac{\cancel{20}^{10}}{\cancel{15}} \times \frac{\cancel{10}^{5}}{\cancel{2}}$$

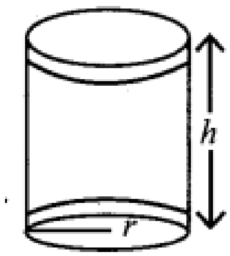
$$= 450$$

Question 5.
A spherical ball of iron has been melted and made into small balls. If the raidus of each smaller ball is one-fourth of the radius of the original one, how many such balls can be made?
Solution:

$$\text{No. of balls required} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi \times \left(\frac{r}{4}\right)^3}$$

$$= r^3 \times \frac{4}{r} \times \frac{4}{r} \times \frac{4}{r} = 64$$

Question 6.
A wooden article was made by scooping out a hemisphere from each end of a cylinder as shown in figure. If the height of the cylinder is 10cm and its base is of radius 3.5 cm find the total surface area of the article.



Solution:
Radius of the cylinder be r Height of the cylinder be h Total surface area of the article = CSA of cylinder + CSA of 2 hemispheres = $2\pi rh + 2\pi r^2 = 2\pi r(h + 2r)$

$$= 2\times\frac{22}{7}\times3.5\times(10+2\times3.5)$$

$$= 22 \times 17 = 374 \text{ cm}^2$$