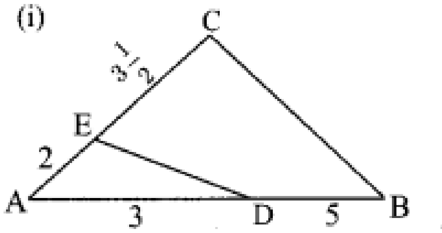
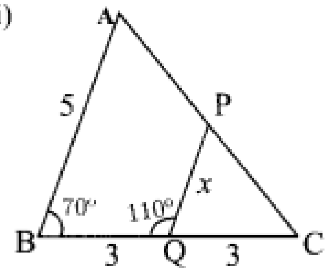
Geometry

Ex 4.1

Question 1.
Check whether the which triangles are similar and find the value of x.

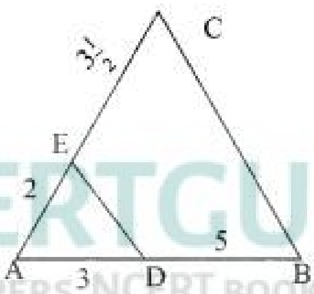
(i)


(ii)


Solution:

(i) $$\frac{AE}{AC} = \frac{AD}{AB} \text{ (for similar triangle)}$$

But here, $$\frac{2}{\frac{11}{2}} \neq \frac{3}{8}$$



$$2 \times \frac{2}{11} \neq \frac{3}{8}$$

$$\frac{4}{11} \neq \frac{3}{8}$$

$\therefore$ They are not similar triangles

(ii) In ΔABC, ΔPQC,

$$\angle ABC = \angle PQC = 70^\circ$$

$$\angle C = \angle C \text{ (common angles)}$$

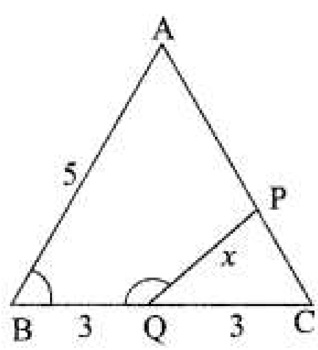
$$\therefore \angle A = \angle QPC (\therefore \text{ AAA criterion})$$

$\therefore \Delta ABC$ and ΔPQC are similar triangles

$$\frac{AB}{PQ} = \frac{BC}{QC}$$

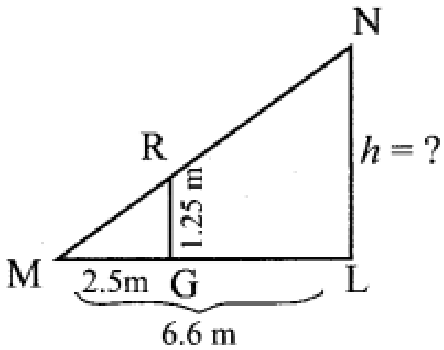
$$\frac{5}{x} = \frac{6}{3}$$

$$6x = 15$$



$x = \frac{15}{6} = 2.5$

Question 2.
A girl looks the reflection of the top of the lamp post on the mirror which is 6.6 m away from the foot of the lamppost. The girl whose height is 1.25 m is standing 2.5 m away from the mirror. Assuming the mirror is placed on the ground facing the sky and the girl, mirror and the lamppost are in a same line, find the height of the lamp post?
Solution:
In the picture ΔMLN, ΔMGRare similar triangles.



$$\therefore \quad \frac{GR}{LN} = \frac{MG}{ML}$$

$$\frac{1.25}{h} = \frac{2.5}{6.6}$$

$$1.25 \times 6.6 = 2.5 \times h$$

$$h = \frac{1.25 \times 6.6}{2.5}$$

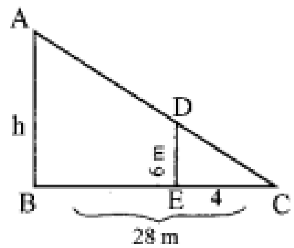
$$h = \frac{125}{100} \times \frac{66}{10} \times \frac{10}{25}$$

$$= \frac{33}{10} = 3.3 \text{ m}$$

∴ Height of the lamp post is 3.3 m.

Question 3.
A vertical stick of length 6 m casts a shadow 400 cm long on the ground and at the same time a tower casts a shadow 28 m long. Using similarity, find the height of the tower.
Solution:
In the picture ΔABC, ΔDEC are similar triangles.

$$\therefore \frac{AB}{DE} = \frac{BC}{EC}$$



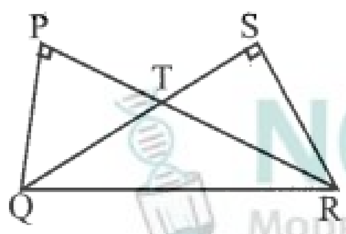
$$\frac{h}{6} = \frac{28}{4}$$

$$\cancel{4}h = \cancel{28}^{7} \times 6$$

$$h = 42 \text{ m}$$

Height of a tower = 42 m

Question 4.
Two triangles QPR and QSR, right angled at P and S respectively are drawn on the same base QR and on the same side of QR. If PR and SQ intersect at T, prove that PT × TR = ST × TQ.
Solution:
In ΔRPQ,
$RP^2 + PQ^2 = QR^2$
$\therefore PQ^2 = QR^2 – RP^2$ ………… (1)
In ΔTPQ,
$TP^2 + PQ^2 = QT^2$
$\therefore PQ2^2 = QT^2 – TP^2$ ………….. (2)
Equating (1) and (2) we get,
$QR^2 – RP^2 = QT^2 – TP^2$
RP = RT + TP
$\therefore QR^2 – (RT + TP)^2 = QT^2 – TP^2$
$\therefore QR^2 – RT^2 – TP^2 – 2RT.TP = QT^2 – TP^2$
$QR^2 = QT^2 + RT^2 + 2RT.TP$ …………. (5)



In ΔQSR,
$QS^2 + SR^2 = QR^2$
$SR^2 = QR^2 – SR^2$ ………..(3)
In ΔTSR,
$ST^2 + SR^2 = TR^2$

∴ $SR^2 = TR^2 – TS^2$ ………… (4)
Equating (3) and (4) we get
$QR^2 – SQ^2 = TR^2 – TS^2$
SQ = QT + TS
∴ $QR^2 – (2T + TS)^2 = TR^2 – TS^2$
$QR^2 – 2T^2 – TS^2 – 2QT.TS = TR^2 – TS^2$
∴ $2R^2 = TR^2 + QT^2 + 2QT.TS$ ………… (6)
Now equating (5) – (6), we get
$QT^2 +RT^2 + 2RT. TP = QT^2 + RT^2 + 2QT.TS$
∴ PT.TR = ST.TQ
Hence proved.

Question 5.
In the adjacent figure, ΔABC is right angled at C and DE⊥AB. Prove that ΔABC ~ ΔADE and hence find the lengths of AE and DE?

Solution:
In ΔABC & ΔADE
∠A is common & ∠C = ∠E = 90°
∴ by similarity
ΔABC ~ ΔADE

$$\therefore \frac{AB}{AD} = \frac{AC}{DE} = \frac{BC}{AE}$$

$$\frac{13}{3} = \frac{5}{DE} = \frac{12}{AE} \qquad (1)$$

$$13\,DE = 3 \times 5$$

$$DE = \frac{15}{13}$$

Since ΔABC is a right angled triangle.

$$AB^2 = BC^2 + AC^2$$

$$= 12^2 + 5^2$$

$$= 144 + 25$$

$$= 169$$

$$\Rightarrow \quad AB = 13$$

$$\frac{5}{\frac{5}{13}} = \frac{12}{AE}$$

$$5AE = \frac{15}{13} \times 12$$

$$AE = \frac{15}{13} \times \frac{12}{5}$$

$$AE = \frac{36}{13} = 2.7$$

$$DE = \frac{15}{13} = 1.1$$

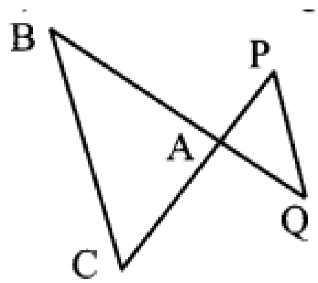
Substituting the values of DE and AE in (1)
we can prove that

$$\frac{AB}{AD} = \frac{AC}{DE} = \frac{BC}{AE}$$

$$\frac{13}{3} = \frac{5}{1 \cdot 1} = \frac{12}{2 \cdot 7} = 4 \cdot 3$$

It is proved that $\Delta ABC \sim \Delta ADE$.

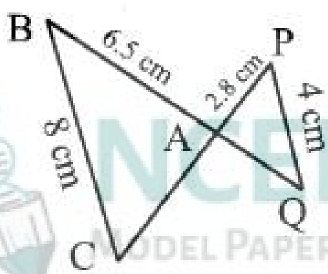
Question 6.
In the adjacent figure, $\Delta ACB \sim \Delta APQ$. If BC = 8 cm, PQ = 4 cm, BA = 6.5 cm and AP = 2.8 cm, find CA and AQ.



Solution:
ΔACB ~ ΔAPQ

$$\frac{AB}{AQ} = \frac{BC}{PQ} = \frac{CA}{AP}$$

$$\frac{6.5}{AQ} = \frac{8}{4} = \frac{CA}{2.8}$$



From (1)

$$\Rightarrow \quad 4CA = 8 \times 2.8$$

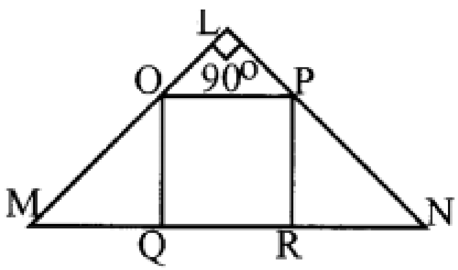
$$CA = \frac{22.4}{4} = 5.6 \text{ cm}$$

From (1)

$$\Rightarrow \quad 8AQ = 6.5 \times 4$$

$$AQ = \frac{26}{8} = 3.25 \text{ cm.}$$

Question 7.
In figure OPRQ is a square and MLN = 90°. Prove that
(i) ΔLOP ~ ΔQMO
(ii) ΔLOP ~ ΔRPN
(iii) ΔQMO ~ ΔRPN
(iv) QR^2 = MQ × RN.



Solution:
(i) In ΔLOP & ΔQMO, we have
∠OLP = ∠MQO (each equal to 90°)
and ∠LOP = ∠OMQ (corresponding angles)
ΔLOP ~ ΔQMO (by AA criterion of similarity)

(ii) In ΔLOP & ΔPRN, we have
∠PLO = ∠NRP (each equal to 90°)
∠LPO = ∠PNR (corresponding angles)
ΔLOP ~ ΔRPN

(iii) In ΔQMO & ΔRPN .
Since ΔLOP ~ ΔQMO and ΔLOP ~ ΔRPN
∠QMO ~ ΔRPN

(iv) We have
ΔQMO ~ ΔRPN (using (iii))
$\frac{MQ}{RP} = \frac{QO}{RN}$ (∵ PROQ is a square)
QR^2 = MQ × RN. [RP = QO, QO = QR]

Question 8.
If ΔABC ~ ΔDEF such that area of ΔABC is 9 cm^2 and the area of ΔDEF is 16 cm^2 and BC = 2.1 cm. Find the length of EF.
Solution:
Since the area of two similar triangles is equal to the ratio of the squares of any two corresponding

sides.

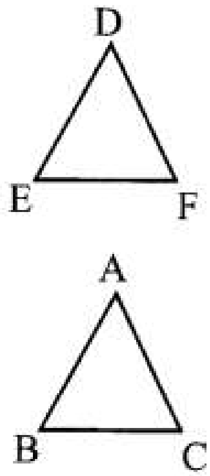
$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \frac{BC^2}{EF^2}$$

$$\frac{9}{16} = \frac{(2.1)^2}{EF^2}$$

$$\Rightarrow \quad \frac{3}{4} = \frac{2.1}{EF}$$

$$3EF = 8.4$$

$$EF = \frac{8.4}{3} = 2.8 \text{ cm.}$$



Note: Taking square root on both sides we get

$$EF = 2.8 \text{ cm.}$$

Question 9.
Two vertical poles of heights 6 m and 3 m are erected above a horizontal ground AC. Find the value of y.

Intranet

Solution:
ΔPAC, ΔQBC are similar 6 triangles

$$\therefore \quad \frac{PA}{QB} = \frac{AC}{BC} = \frac{PC}{QC}$$

$$\frac{6}{y} = \frac{AC}{BC}$$

$$\Rightarrow \quad (AC)y = 6BC \quad \ldots (1)$$

ΔACR & ΔABQ are similar triangles.

$$\frac{CR}{QB} = \frac{AC}{AB}$$

$$\frac{3}{y} = \frac{AC}{AB}$$

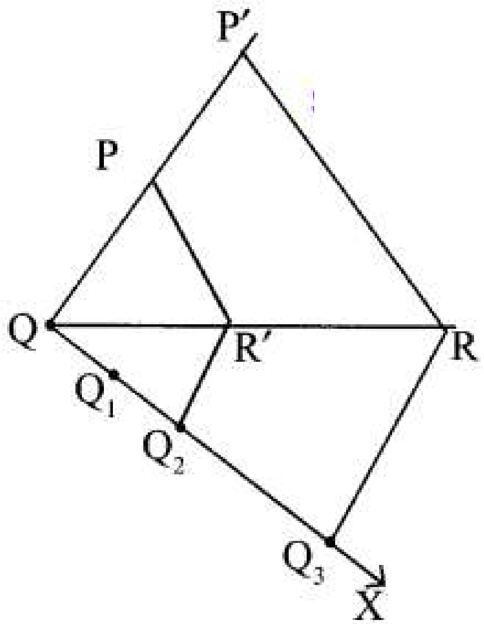
$$\Rightarrow \quad (AC)y = 3AB \quad \ldots (2)$$

$$(1) = (2) \Rightarrow 6BC = 3AB$$

$$2BC = AB$$

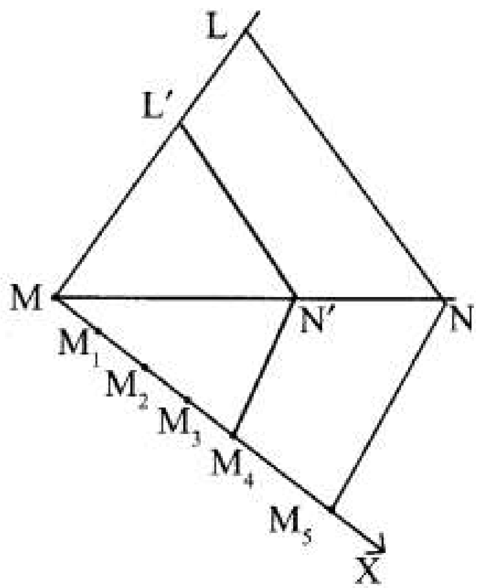
⇒ AC = AB + BC
= 2BC + BC
AC = 3BC
Substituting AC = 3BC in (1), we get
(AC)y = 6BC
3(BC)y = 6(BC)
y = $\frac{6}{3}$ = 2m

Question 10.
Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{2}{3}$ of the corresponding sides of the triangle PQR (scale factor $\frac{2}{3}$).
Solution:
Given a triangle PQR, we are required to construct another triangle whose sides are $\frac{3}{5}$ of the corresponding sides of the triangle PQR.



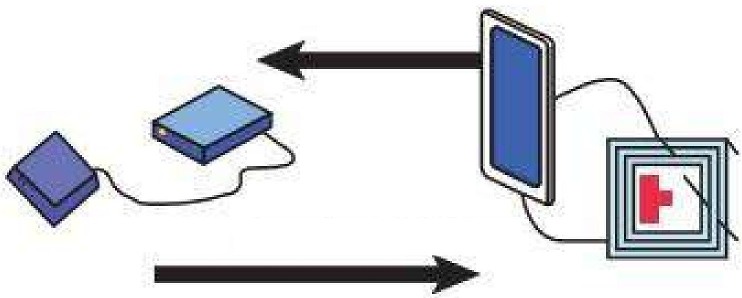
Steps of construction:
(1) Draw any ray QX making an acute angle with QR on the side opposite to the vertex P.
(2) Locate 3 (the greater of 2 and 3 in $\frac{2}{3}$) points. Q_1 Q_2, Q_2 on QX so that $QQ_1 = Q_1Q_2 = Q_2Q_3$
(3) Join Q_3R and draw a line through Q_2 (the second point, 2 being smaller of 2 and 3 in $\frac{2}{3}$) parallel to Q_3R to intersect QR at R'.
(4) Draw line through R' parallel to the line RP to intersect QP at P'.
The ΔP'QR' is the required triangle each of the whose sides is $\frac{2}{3}$ of the corresponding sides of 3 ΔPQR.

Question 11.
Construct a triangle similar to a given triangle LMN with its sides equal to $\frac{4}{5}$ of the corresponding sides of the triangle LMN
(scale factor $\frac{4}{5}$).
Solution:
Given a triangle LMN, we are required to construct another triangle whose sides are $\frac{4}{5}$ of the corresponding sides of the ΔLMN.



Steps of construction:
(1) Draw any ray making an acute angle to the vertex L.
(2) Locate 5 points (greater of 4 and 5 in $\frac{4}{5}$) M_1, M_2, M_3, M_4, and M_5 and MX so that $MM_1 = M_1M_2 = M_2M_3 = M_3M_4 = M_4M_5$
(3) Join M_5N and draw a line parallel to M_5N through M_4 (the fourth point, 4 being the smaller of 4 and 5 in $\frac{4}{5}$) to intersect MN atN'.
(4) Draw a line through N^1 parallel to the line NL to intersect ML and L'. Then ΔL'MN' is the required triangle each of the whose sides is $\frac{4}{5}$ of the corresponding sides of ΔLMN.

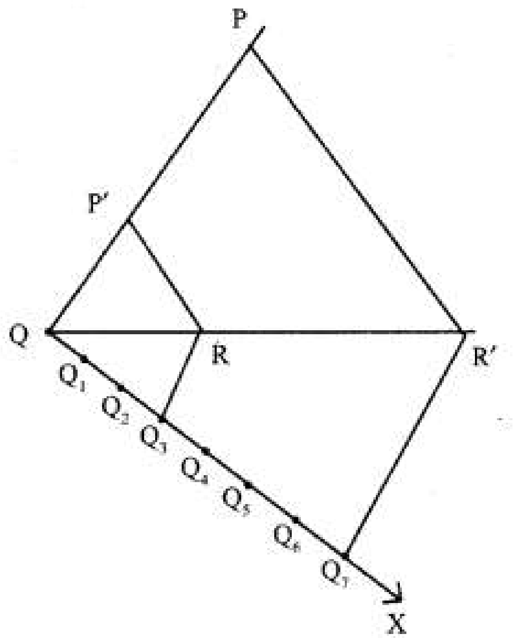
Question 12.
Construct a triangle similar to a given triangle ABC with its sides equal to $\frac{6}{5}$ of the corresponding sides of the triangle ABC
(scale factor $\frac{6}{4}$).
Solution:
ΔABC is the given triangle. We are required to construct another triangle whose sides are $\frac{6}{5}$ of the corresponding sides of the given triangle ABC
Steps of construction:



Passive RFID using EM-wave transmission

(1) Draw any ray BX making an acute angle with BC on the opposite side to the vertex A.
(2) Locate 6 points (the greater of 6 and 5 in $\frac{6}{5}$) B_1, B_2, B_3, B_4, B_5, B_6 so that $BB_1 = B_1B_2 = B_2B_3 = B_4B_5 = B_5B_6$.
(3) Join B_5 (the fifth point, 5 being smaller of 5 and 6 in $\frac{6}{5}$) to C and draw a live through B_6 parallel to B_5C intersecting the extended line segment BC at C^1.
(4) Draw a line through C' parallel to CA intersecting the extended line segment BA at A'.
Then ΔA'BC' is the required triangle each of whose sides is $\frac{6}{5}$ of the corresponding sides of the given triangle ABC.

Question 13.
Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{7}{3}$ of the corresponding sides of the triangle PQR (scale factor $\frac{7}{3}$).
Solution:
Given a triangle ΔPQR. We have to construct another triangle whose sides are $\frac{7}{3}$ of the corresponding sides of the given ΔPQR.



Steps of construction:

(1) Draw any ray QX making an acute angle with QR on the opposite side to the vertex P.

(2) Locate 7 points (the greater of 7 and 3 in $\frac{7}{3}$) Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, and Q_7 so that QQ_2 = $Q_1Q_2 = Q_2Q_3 = Q_3Q_4 = Q_4Q_5 = Q_5Q_6$
$= Q_6Q_7$

(3) Join Q_3 to R and draw a line segment through Q_7 parallel to Q_3R intersecting the extended line segment QR at R'.

(4) Draw a line segment through R' parallel to PR intersecting the extended line segment QP at P'.

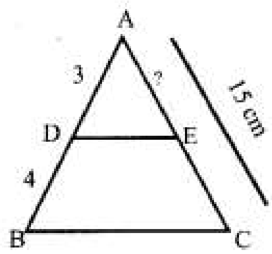
Then ΔP'QR' is the required triangle each of whose sides is $\frac{7}{3}$ of the corresponding sides of the given triangle.

Ex 4.2

Question 1.
In ΔABC, D and E are points on the sides AB and AC respectively such that DE || BC
(i) If $\frac{AD}{DB} = \frac{3}{4}$ and AC = 15 cm find AE.
(ii) If AD = 8x – 7, DB = 5x – 3, AE = 4x – 3 and EC = 3x – 1, find the value of x.
Solution:

(i) If $\frac{AD}{DB} = \frac{3}{4}$, AC = 15 cm, DE || BC, then by basic proportionality theorem.

$$\frac{AD}{AB} = \frac{AE}{AC}$$



$$\frac{3}{7} = \frac{AE}{15}$$

$$7\,AE = 3 \times 15$$

$$AE = \frac{45}{7} = 6.43 \text{ cm.}$$

(ii) By basic proportionality theorem.

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{8x-7}{5x-3} = \frac{4x-3}{3x-1}$$

Hint: -2; $\frac{-2}{2}$, $\frac{1}{2}$

$$(8x-7)(3x-1) = (5x-3)(4x-3)$$

$$24x^2 - 21x - 8x + 7 = 20x^2 - 12x - 15x + 9$$

$$24x^2 - 29x + 7 - 20x^2 + 27x - 9 = 0$$

$$4x^2 - 2x - 2 = 0$$

$$2x^2 - x - 1 = 0$$

$$(2x+1)(x-1) = 0$$

$x = 1, \frac{-1}{2} \Rightarrow x = 1$

Question 2.
ABCD is a trapezium in which AB || DC and P,Q are points on AD and BC respectively, such that PQ || DC if PD = 18 cm, BQ = 35 cm and QC = 15 cm, find AD.
Solution:
Any line parallel to the parallel sides of a trapezium dives the non-parallel sides proportionally.

∴ By thales theorem, In ΔACD, we have

$$\frac{AP}{PD} = \frac{AG}{GC} \Rightarrow \frac{x}{18} = \frac{AG}{GC} \quad ...(1)$$

In DABC, we have

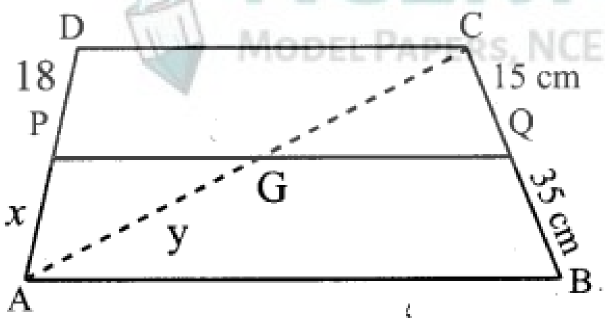
$$\frac{AG}{GC} = \frac{BQ}{QC} \Rightarrow \frac{AG}{GC} = \frac{35}{15} \quad ...(2)$$

From (1) and (2), we have

$$\frac{x}{18} = \frac{35^{7}}{15_{3}} \Rightarrow 3x = 126$$

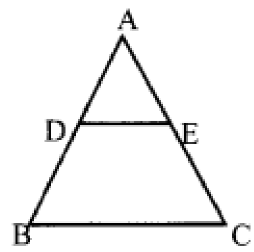
$$x = 42$$

$$AD = x + 18 = (42 + 18) = 60cm$$



Question 3.
In ΔABC, D and E are points on the sides AB and AC respectively. For each of the following cases show that DE || BC
(i) AB = 12 cm, AD = 8 cm, AE = 12 cm and AC = 18 cm.
(ii) AB = 5.6 cm, AD = 1.4 cm, AC = 7.2 cm and AE = 1.8cm.
Solution:

(i) In ΔABC, $AB = 12$ cm,



$AD = 8$ cm,

$AE = 12$ cm,

$AC = 18$ cm.

$$\text{If } \frac{AB}{AD} = \frac{AC}{AE} \Rightarrow \frac{12}{8} = \frac{18}{12}$$

$$\Rightarrow \quad \frac{3}{2} = \frac{3}{2}$$

∴ It is satisfied

∴ DE||BC

(ii) AB = 5.6 cm,
AD = 1.4 cm,
AC = 7.2 cm,
AE = 1.8 cm.

If $\frac{AB}{AD} = \frac{AC}{AE}$ is satisfied then BC || DE

$\frac{5.6}{1.4} = \frac{7.2}{1.8}$

$5.6 \times 1.8 = 1.4 \times 7.2$

$10.08 = 10.08$

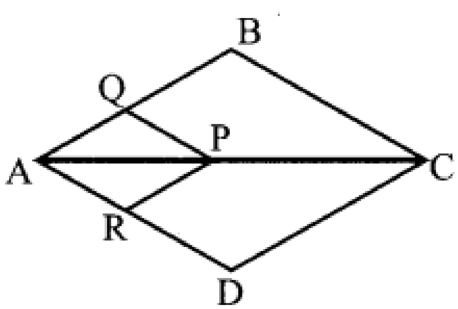
L.H.S = R.H.S

∴ It is satisfied

∴ DE||BC

Question 4.
In fig. if PQ || BC and PR ||CD prove that

$$\textbf{(i)} \ \frac{AR}{AD} = \frac{AQ}{AB} \ = \textbf{(ii)} \ \frac{QB}{AQ} = \frac{DR}{AR}.$$



Solution:

In the figure PQ || BC, PR ||CD.

(i) In ΔADC, by BPT $\frac{AR}{AD} = \frac{AP}{AC}$ ...(1)

In ΔACB, by BPT $\frac{AP}{AC} = \frac{AQ}{AB}$ ...(2)

From (1) and (2) we get

$$\frac{AR}{AD} = \frac{AP}{AC} = \frac{AQ}{AB}$$

$$\Rightarrow \quad \frac{AR}{AD} = \frac{AQ}{AB}$$

It is proved.

(ii) In ΔABC, $\frac{QB}{AQ} = \frac{PC}{AP}$ by BPT ...(1)

In ΔACD, $\frac{PC}{AP} = \frac{DR}{AR}$ by BPT. ...(2)

From (1) & (2)

$$\frac{QB}{AQ} = \frac{PC}{AP} = \frac{DR}{AR}$$

$$\therefore \frac{QB}{AQ} = \frac{DR}{AR}$$

It is proved.

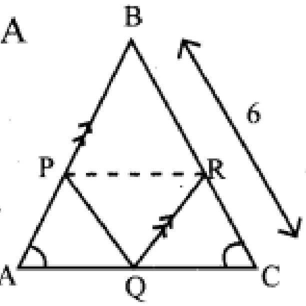
Question 5.
Rhombus PQRB is inscribed in ΔABC such that ∠B is one of its angle. P, Q and R lie on AB, AC and BC respectively. If AB = 12 cm and BC = 6 cm, find the sides PQ, RB of the rhombus.
Solution:
In ΔCRQ and ΔCBA
∠CRQ = ∠CBA (as RQ || AB)
∠CQR = ∠CAB (as RQ || AB)

$\therefore \Delta CRQ \cong \Delta CBA$

$\therefore \frac{CR}{CB} = \frac{RQ}{BA}$

Let side of Rhombus be 'a'.

$\therefore \frac{6-a}{6} = \frac{a}{12}$

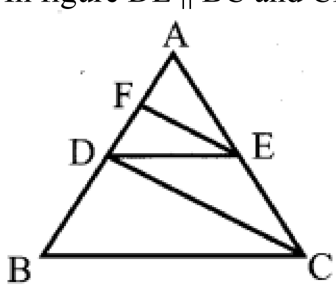


⇒ 72 – 12a = 6a
⇒ 18a = 72
a = 4
Side of rhombus PQ, RB = 4 cm, 4 cm.

Question 6.
In trapezium ABCD, AB || DC, E and F are points on non-parallel sides AD and BC respectively, such that EF || AB . Show that $\frac{AE}{ED} = \frac{BF}{FC}$
Solution:

Network Applications		
Applications of Internet.	**Applications of Intranet**	**Applications of Extranet**
Download programs and files	Sharing of company policies/ rules and regulations	Customer communications
Social media	Access employee database	Online education/ training
E-Banking	Distribution of circulars/Office Orders	Account status enquiry
E-Commerce	Access product and customer data	Inventory enquiry
E-mail	Submission of reports	Online discussion

Question 7.
In figure DE || BC and CD || EF . Prove that $AD^2 = AB \times AF$.



Solution:

$$TPT \Rightarrow \quad AD^2 = AB \times AF$$

$$\Delta AFE \cong \Delta ADC$$

$$\frac{AF}{AD} = \frac{AE}{AC} \quad (1)$$

$\Delta ADE \cong \Delta ABC$

$$\frac{AD}{AB} = \frac{AE}{AC} \quad (2)$$

Equating RHS of (1) and (2)

$$\frac{AF}{AD} = \frac{AD}{AB}$$

$$\Rightarrow \quad AD^2 = AF \times AB$$

It is proved.

Question 8.
In a ΔABC, AD is the bisector of ∠A meeting side BC at D, if AB = 10 cm, AC = 14 cm and BC = 6 cm, find BD and DC.
Solution:

$$\text{Let } \angle BAD = \angle CAD = \theta$$

$$\text{Assume } BD = y$$

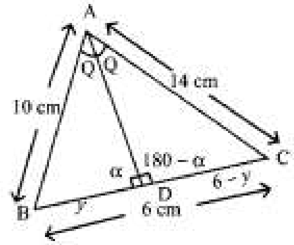
$$BC - CD = y$$

$$6 - CD = y$$

$$CD = 6 - y$$

$$\text{Assume } \angle ADB = \alpha$$

$$\angle ADC = 180 - \alpha$$



$$\text{In } \Delta ABD,\ \frac{BD}{\sin\theta} = \frac{AB}{\sin\alpha} \Rightarrow \frac{y}{\sin\theta} = \frac{10}{\sin\alpha}$$

$$\Rightarrow \sin\alpha = \frac{10}{y}\sin\theta \qquad ...(1)$$

$$\text{In } \Delta ACD,\ \frac{CD}{\sin\theta} = \frac{AC}{\sin(180-\alpha)}$$

$$\Rightarrow \frac{6-y}{\sin\theta} = \frac{14}{\sin\alpha} \qquad ...(2)$$

Substituting (1) in (2),

$$\frac{6-y}{\sin\theta} = \frac{14}{\frac{10}{y}\sin\theta} \Rightarrow 6 - y = \frac{14y}{10}$$

$$\Rightarrow y\left(1 + \frac{14}{10}\right) = 6 \Rightarrow y\left(\frac{24}{10}\right) = 6$$

$$\Rightarrow y\frac{60}{24} \Rightarrow y = 2.5$$

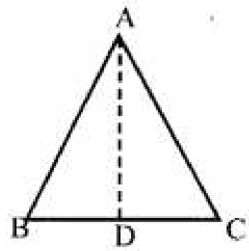
$$\therefore BD = 2.5 \text{ cm and } CD = 3.5 \text{ cm}$$

Question 9.
Check whether AD is bisector of ∠A of ΔABC in each of the following
(i) AB = 5 cm, AC = 10 cm, BD = 1.5 cm and CD = 3.5 cm.
(ii) AB = 4 cm, AC = 6 cm, BD = 1.6 cm and CD = 2.4 cm.
Solution:

AB = 5 cm,
AC = 10 cm,
BD = 1.5 cm,
CD = 3.5 cm,

By ABT, check whether $\frac{BD}{DC} = \frac{AB}{AC}$

$$\frac{1.5\times10}{3.5\times10} = \frac{5}{10}$$



$$\frac{15^{3}}{35_{7}} \neq \frac{5}{10_{2}}$$

$$\frac{3}{7} \neq \frac{1}{2}$$

$\therefore$ AD is not the bisector of ∠BAC.

(ii) AB = 4 cm,
AC = 6 cm,
BD = 1.6 cm,
CD = 2.4 cm.

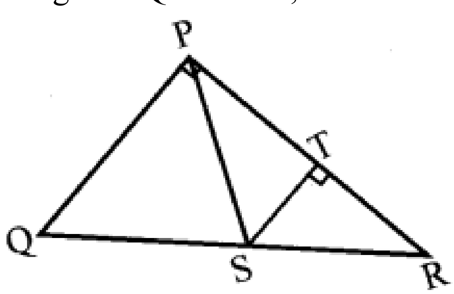
By ABT, check

$$\frac{BD}{DC} = \frac{1.6\times10}{2.4\times10} = \frac{16^{2}}{24_{3}} = \frac{2}{3}$$

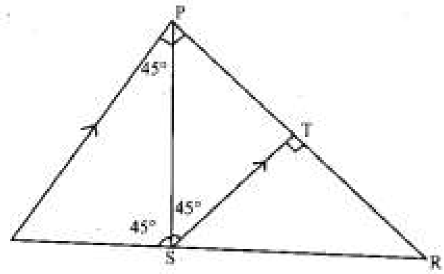
$$\frac{AB}{AC} = \frac{4^{2}}{6_{3}} = \frac{2}{3}$$

$\therefore$ AD is the bisector of ΔABC.

Question 10.
In figure ∠QPC = 90°, PS is its bisector. If ST⊥PR, prove that ST × (PQ + PR) = PQ × PR.



Solution:



In Δ PQR, since PS is angle bisector & applying angle.
bisector theorem $\frac{PR}{PQ} = \frac{SR}{SQ}$ ....(A)

Δ RTS ≋ Δ RPQ (similarity)

$$\frac{SR}{SQ} = \frac{TR}{TP} \quad ....(1)$$

Given ∠ PTS = 90°

∴ In Δ PTS, since ∠TPS = 45° (PS - angle bisector)

∠PST also = 45°

∴ ∠PTS is an isosceles Δ

$\Rightarrow$ PT = ST ....(2)

Using (2) in (1), we get $\frac{SR}{SQ} = \frac{TR}{ST}$ ....(3)

$$TR = PR - PT$$
$$= PR - ST$$

From (A) & (3), we get $\frac{PR}{PQ} = \frac{SR}{SQ} = \frac{TR}{ST}$

$$\therefore \quad PR \times ST = TR \times PQ$$
$$= (PR - ST) \times PQ$$
$$= PR \times PQ - ST \times PQ$$

$\therefore PR \times ST + ST \times PQ = PR \times PQ$

$$\Rightarrow ST(PR + PQ) = PR \times PQ$$

Hence proved.

Question 11.
ABCD is a quadrilateral in which AB = AD, the bisector of ∠BAC and ∠CAD intersect the sides BC and CD at the points E and F respectively. Prove that EF || BD.
Solution:
By angle bisector theorem in ΔABC,

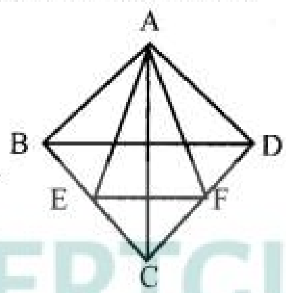
$$\frac{BE}{EC} = \frac{AB}{AC} \qquad (1)$$

By angle bisector theorem in ΔADC,

$$\frac{AD}{AC} = \frac{DF}{FC} \qquad (2)$$

Since AB = AD, equating (1) & (2)

$$\frac{BE}{EC} = \frac{DF}{FC}$$



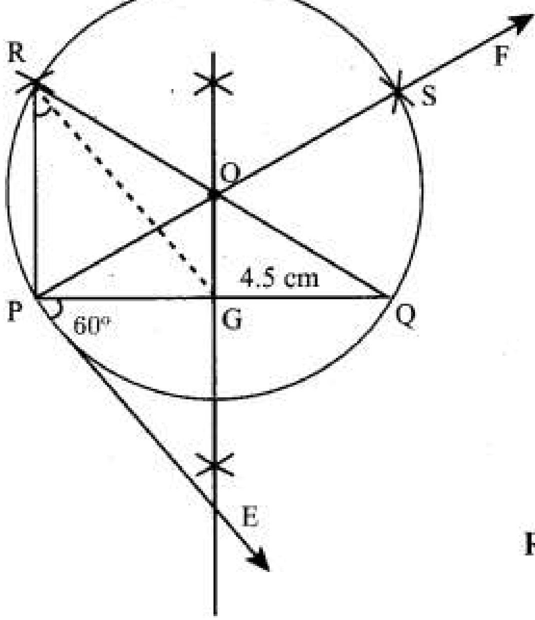
In ΔBDC, as EF is such that,

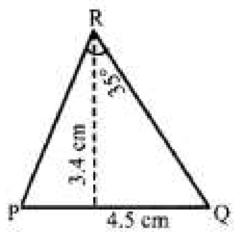
$$\frac{DF}{FC} = \frac{BE}{EC}$$

∴ EF || BD.

Question 12.
Construct a ΔPQR which the base PQ = 4.5 cm, ∠R=35° and the median from R to RG is 6 cm.
Solution:
Construction:
Step (1) Draw a line segment PQ = 4.5 cm
Step (2) At P, draw PE such that ∠QPE = 35°.
Step (3) At P, draw PF such that ∠EPF = 90°.
Step (4) Draw $\perp^r$ bisector to PQ which intersects PF at O.
Step (5) With O centre OP as raidus draw a circle.
Step (6) From G mark arcs of 6 cm on the circle.
Mark them as R and S.
Step (7) Join PR and RQ.

Step (8) PQR is the required triangle.

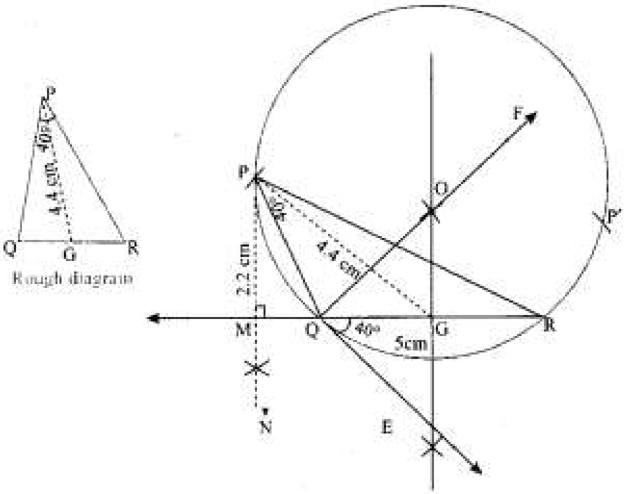




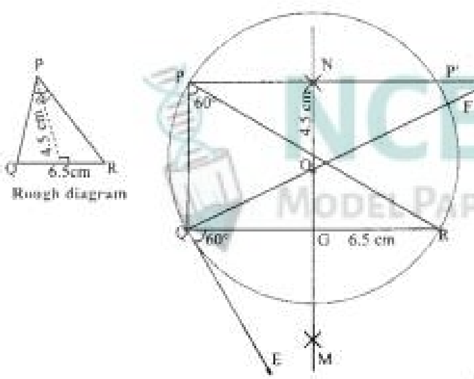
Rough diagram

Question 13.
Construct a ΔPQR in which QR = 5 cm, P = 40° and the median PG from P to QR is 4.4 cm. Find the length of the altitude from P to QR.
Solution:
Construction:
Step (1) Draw a line segment QR = 5 cm.
Step (2) At Q, draw QE such that ∠RQE = 40°.
Step (3) At Q, draw QF such that ∠EQF = 90°.
Step (4) Draw perpendicular bisector to QR, which intersects QF at O.
Step (5) With O as centre and OQ as raidus, draw a circle.
Step (6) From G mark arcs of radius 4.4 cm on the circle. Mark them as P and P'.
Step (7) Join PQ and PR.
Step (8) PQR is the required triangle.
Step(9) From P draw a line PN which is $\perp^r$ to LR. LR meets PN at M.

Step (10) The length of the altitude is PM = 2.2 cm.

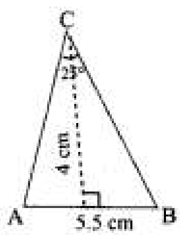


Question 14.
Construct a ΔPQR such that QR = 6.5 cm, ∠P = 60° and the altitude from P to QR is of length 4.5 cm.
Solution:

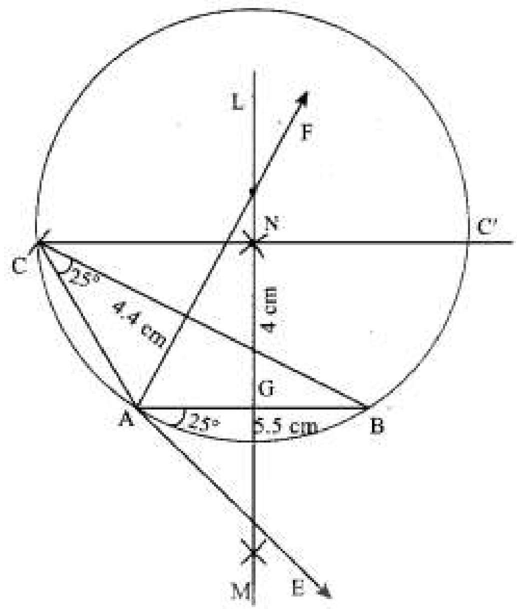


Construction:
Steps (1) Draw QR = 6.5 cm.
Steps (2) Draw ∠RQE = 60°.
Steps (3) Draw ∠FQE = 90°.
Steps (4) Draw $\perp^r$ bisector to QR.
Steps (5) The $\perp^r$ bisector meets QF at O.
Steps (6) Draw a circle with O as centre and OQ as raidus.
Steps (7) Mark an arc of 4.5 cm from G on the $\perp^r$ bisector. Such that it meets LM at N.
Steps (8) Draw PP' || QR through N.
Steps (9) It meets the circle at P, P'.
Steps (10) Join PQ and PR.
Steps (11) ΔPQR is the required triangle.

Question 15.
Construct a ΔABC such that AB = 5.5 cm, C = 25° and the altitude from C to AB is 4 cm.
Solution:

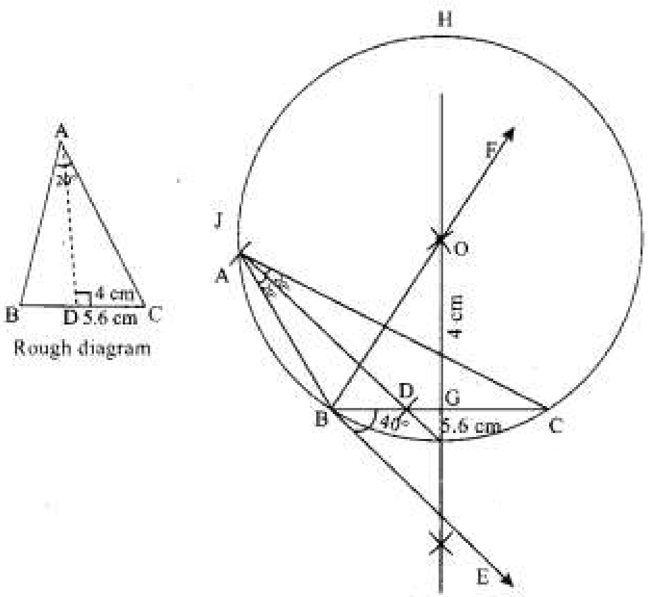


Rough diagram



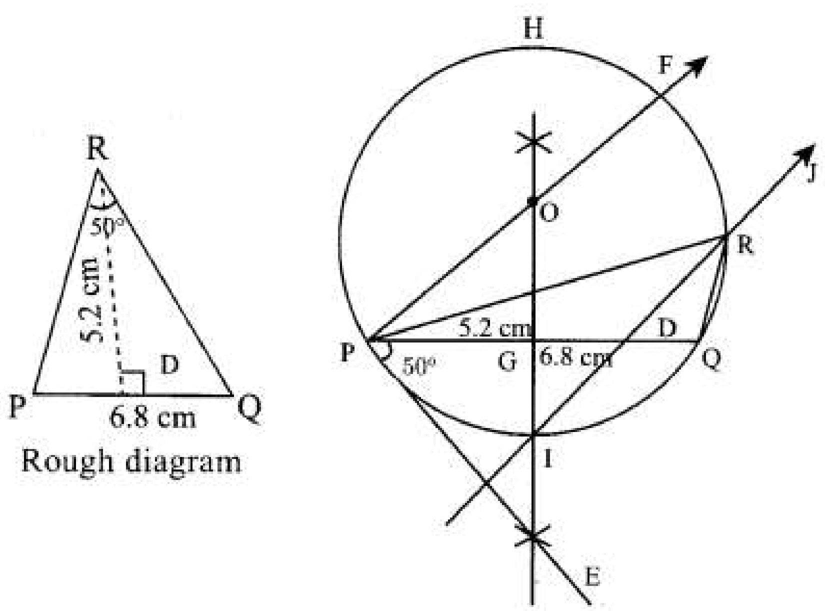
Construction:
Step (1) Draw $\overline{AB}$ = 5.5 cm
Step (2) Draw ∠BAE = 25°
Step (3) Draw ∠FAE = 90°
Step (4) Draw $\perp^r$ bisector to AB.
Step (5) The $\perp^r$ bisector meets AF at O.
Step (6) Draw a circle with O as centre and OA as radius.
Step (7) Mark an arc of length 4 cm from G on the $\perp^r$ bisector and name as N.
Step (8) Draw CC^1 || AB through N.
Step (9) Join AC & BC.
Step (10) ΔABC is the required triangle.

Question 16.
Draw a triangle ABC of base BC = 5.6 cm, ∠A=40° and the bisector of ∠A meets BC at D such that CD = 4 cm.
Solution:
Construction:
Steps (1) Draw a line segment BC = 5.6 cm.
Steps (2) At B, draw BE such that ∠CBE = 60°.
Steps (3) At B draw BF such that ∠EBF = 90°.



Steps (4) Draw $\perp^r$ bisector to BC, which intersects BF at 0.
Steps (5) With O as centre and OB as radius draw a circle.
Steps (6) From C, mark an arc of 4 cm on BC at D.
Steps (7) The $\perp^r$ bisector intersects the circle at I. Join ID.
Steps (8) ID produced meets the circle at A.
Now join AB and AC. ΔABC is the required triangle.

Question 1 7.
Draw ΔPQR such that PQ = 6.8 cm, vertical angle is 50° and the bisector of the vertical angle meets the base at D where PD = 5.2 cm.
Solution:

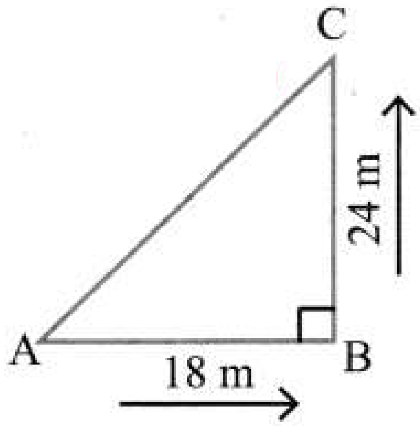


Steps (1) Draw a line segment PQ = 6.8 cm

Steps (2) At P, draw PE such that ∠QPE = 50°.
Steps (3) At P, draw PF such that ∠FPE = 90°.
Step (4) Draw $\perp^r$ bisector to PQ, which intersects PF at 0.
Step (5) With O as centre and OP as radius draw a circle.
Step (6) From P mark an arc of 5.2 cm on PQ at D.
Step (7) The $\perp^r$ bisector intersects the circle at I. Join ID.
Step (8) ID produced meets the circle at R. Now join PR & QR. ΔPQR is the required triangle.

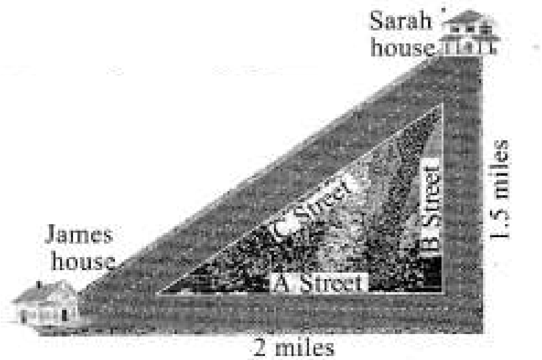
Ex 4.3

Question 1.
A man goes 18 m due east and then 24 m due north. Find the distance of his current position from the starting point?
Solution:
Using Pythagoras theorem



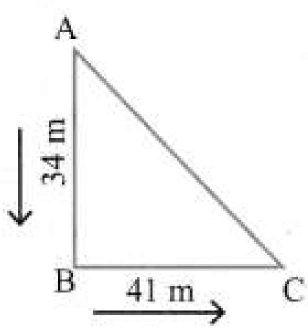
$AC^2 = AB^2 + BC^2$
$= (18)^2 + (24)^2$
$= 324 + 576$
$= 900$
$AC = \sqrt{900} = 30$ m
∴ The distance from the starting point is 30 m.

Question 2.
There are two paths that one can choose to go from Sarah's house to James house. One way is to take C street, and the other way requires to take A street and then B street. How much shorter is the direct path along C street? (Using figure).



Solution:
By using Pythagoras theorem
$AC^2 = AB^2 + BC^2$
$= 2^2 + (1.5)^2$
$= 4 + 2.25$
$= 6.25$
AC = 2.5 miles.
If one chooses C street the distance from James house to Sarah's house is 2.5 miles
If one chooses A street and B street he has to go 2 + 1.5 = 3.5 miles.
2.5 < 3.5, 3.5 – 2.5 = 1 Through C street is shorter by 1.0 miles.
∴ The direct path along C street is shorter by 1 mile.

Question 3.
To get from point A to point B you must avoid walking through a pond. You must walk 34 m south and 41 m east. To the nearest meter, how many meters would be saved if it were possible to make a way through the pond?
Solution:
By using Pythagoras



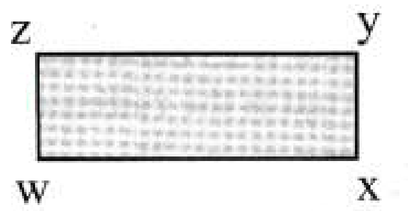
Hint:

	53.2
5	28,37.00
	25
103 × 3	337
	309
1062 × 2	2800
	2124
	676

$AC^2 = AB^2 + BC^2$
$= 34^2 + 41^2$
$= 1156 + 1681$
$= 2837$
AC = 53.26 m

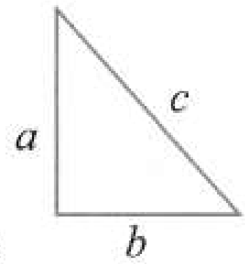
Through B one must walk 34 + 41 = 75 m walking through a pond one must comes only 53.2 m
∴ The difference is (75 – 53.26) m = 21.74 m
∴ To the nearest, one can save 21.74 m.

Question 4.
In the rectangle WXYZ, XY + YZ = 17 cm, and XZ + YW = 26 cm. Calculate the length and breadth of the rectangle?



Solution:
XY + YZ = 17 cm …………. (1)
XZ + YW = 26 cm ………… (2)
(2) ⇒ XZ = 13, YW = 13
(∵ In rectangle diagonals are equal).
(1) ⇒ XY = 5, YZ = 12 XY + YZ = 17
⇒ Using Pythagoras theorem
$5^2 + 12^2 = 25 + 144 = 169 = 13^2$
∴ In ΔXYZ = $13^2 = 5^2 + 12^2$ it is verified
∴ The length is 12 cm and the breadth is 5 cm.

Question 5.
The hypotenuse of a right triangle is 6 m more than twice of the shortest side. If the third side is 2 m less than the hypotenuse, find the sides of the triangle?
Solution:
Let a is the shortest side.
c is the hypotenuse
b is the third side.



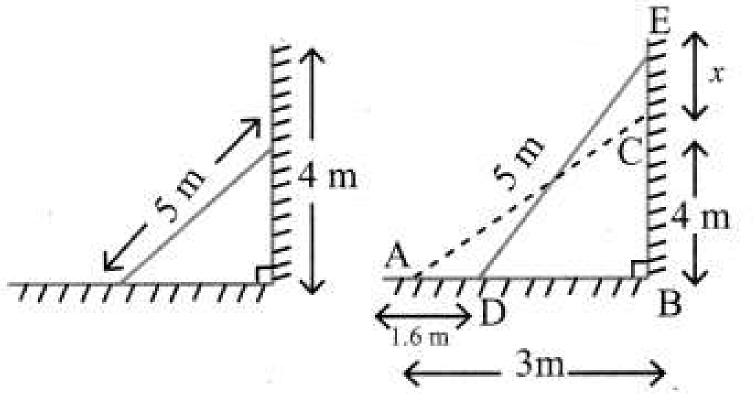
$$\begin{aligned}
c &= 2a + 6 \\
b &= c - 2 \\
&= 2a + 6 - 2 \\
&= 2a + 4 \\
c^2 &= a^2 + b^2 \\
&\quad \text{(Using pythagoras theorem)} \\
&= a^2 + (2a + 4)^2 \\
(2a + 6)^2 &= a^2 + (2a)^2 + 2(2a)4 + 4^2 \\
(2a)^2 + 2(2a)(6) + 6^2 &= a^2 + (2a)^2 + 16a + 16 \\
24a + 36 &= a^2 + 16a + 16 \\
a^2 + 16a - 24a + 16 - 36 &= 0 \\
a^2 - 8a - 20 &= 0 \\
(a - 10)(a + 2) &= 0 \\
a &= 10, -2. \\
b &= 2a + 4 = 2(10) + 4 = 24 \text{ m} \\
c &= 2a + 6 = 2(10) + 6 = 26 \text{ m}
\end{aligned}$$

Hint:
−20
−10   2

∴ The sides of the triangle are 10m, 24m, 26m.

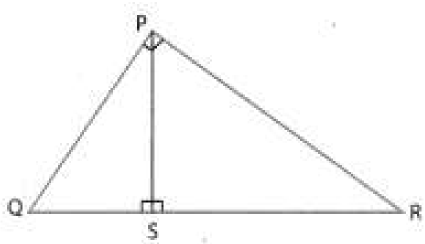
Verification $26^2 = 10^2 + 24^2$
676 = 100 + 576 = 676

Question 6.
5 m long ladder is placed leaning towards a vertical wall such that it reaches the wall at a point 4m high. If the foot of the ladder is moved 1.6 m towards the wall, then find the distance by which the top of the ladder would slide upwards on the wall.
Solution:
Let the distance by which top of the slide moves upwards be assumed as ‘x’.



From the diagram, DB = AB – AD
= 3 – 1.6 ⇒ DB = 1.4 m
also BE = BC + CE
= 4 + x
∴ DBE is a right angled triangle
$DB^2 + BE^2 = DE^2 \Rightarrow (1.4)^2 + (4 + x)^2 = 5^2$
$\Rightarrow (4 + x)^2 = 25 - 1.96 \Rightarrow (4 + x)^2 = 23.04$
$\Rightarrow 4 + x = \sqrt{23.04} = 4.8$
$\Rightarrow x = 4.8 - 4 \Rightarrow x = 0.8$ m

Question 7.
The perpendicular PS on the base QR of ΔPQR intersects QR at S, such that QS = 3 SR. Prove that $2PQ^2 = 2PR^2 + QR^2$.
Solution:



QS + SR = QR
QS = 3SR (given)
4SR = QR
$SR = \frac{QR}{4}$
& QS = 3SR
$= \frac{3QR}{4}$

In ΔPQS,
$PQ^2 = PS^2 + QS^2$ ………… (1)
In ΔPSR,
$PR^2 = PS^2 + SR^2$ ……….. (2)
(1) – (2) ⇒ $PQ^2 - PR^2 = QS^2 - SR^2$ …………. (3)

$$\therefore (3) \quad \Rightarrow PQ^2 - PR^2 = \frac{9}{16}QR^2 - \frac{QR^2}{16}$$

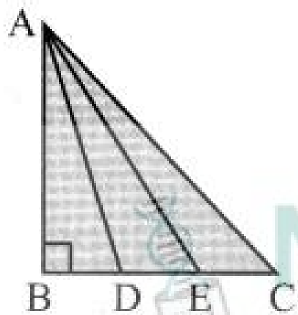
$$= \frac{8QR^2}{16} = \frac{QR^2}{2}$$

$$2PQ^2 - 2PR^2 = QR^2$$

$$2PQ^2 = QR^2 + 2PR^2$$

Hence it proved.

Question 8.
In the adjacent figure, ABC is a right-angled triangle with right angle at B and points D, E trisect BC. Prove that $8AE^2 = 3AC^2 + 5AD^2$.

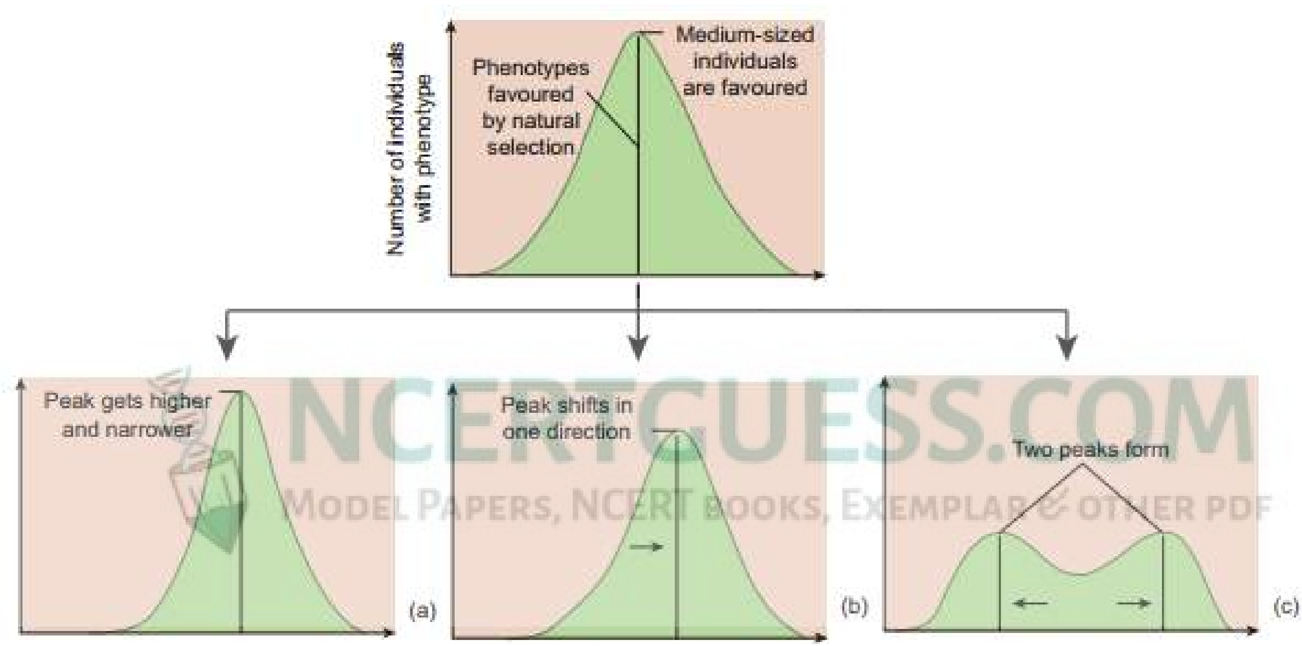


Solution:
Since D and E are the points of trisection of BC,
therefore BD = DE = CE
Let BD = DE = CE = x
Then BE = 2x and BC = 3x
In right triangles ABD, ABE and ABC, (using Pythagoras theorem)
We have $AD^2 = AB^2 + BD^2$
$\Rightarrow AD^2 = AB^2 + x^2$ ……………. (1)
$AE^2 = AB^2 + BE^2$
$\Rightarrow AB^2 + (2x)^2$
$\Rightarrow AE^2 = AB^2 + 4x^2$ ………… (2)
and $AC^2 = AB^2 + BC^2 = AB^2 + (3x)^2$
$AC^2 = AB^2 + 9x^2$
Now $8\ AE^2 - 3\ AC^2 - 5\ AD^2 = 8\ (AB^2 + 4x^2) - 3\ (AB^2 + 9x^2) - 5\ (AB^2 + x^2)$
$= 8AB^2 + 32x^2 - 3AB^2 - 27x^2 - 5AB^2 - 5x^2$
$= 0$
$\therefore 8\ AE^2 - 3\ AC^2 - 5\ AD^2 = 0$

$8 AE^2 = 3 AC^2 + 5 AD^2$.
Hence it is proved.

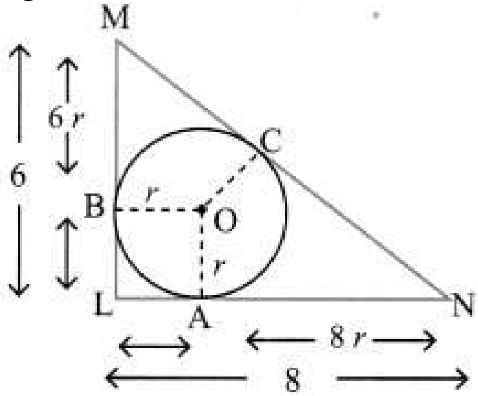
Ex 4.4

Question 1.
The length of the tangent to a circle from a point P, which is 25 cm away from the centre is 24 cm. What is the radius of the circle?
Solution:
$24^2 + r^2 = 25^2$
$576 + r^2 = 625$
$r^2 = 625 - 576$
$= 49$



Question 2.
ΔLMN is a right angled triangle with ∠L = 90°. A circle is inscribed in it. The lengths of the sides containing the right angle are 6 cm and 8 cm. Find the radius of the circle.
Solution:

ΔLMN,
By Pythagoras theorem,



$$MN^2 = LN^2 + LM^2$$
$$= 8^2 + 6^2 = 100$$
$$MN = 10$$

Now, Area of ΔLMN = Area of Δ OLM + Area of ΔOMN + Area of ΔONL.

$$\Rightarrow \frac{1}{2} \times LM \times LN = \frac{1}{2} LM \times r + \frac{1}{2} \times MN \times r + \frac{1}{2} \times NL \times r$$

$$\Rightarrow \quad \frac{1}{2} \times 6 \times 8 = \frac{1}{2} \times 6r + \frac{1}{2} \times 10r + \frac{1}{2} \times 8r$$

$$24 = 3r + 5r + 4r$$
$$12r = 24 \quad \Rightarrow r = 2\text{cm}$$

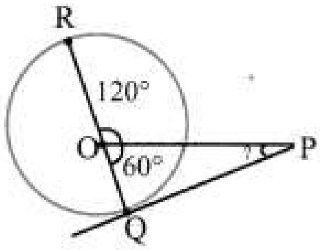
Question 3.
A circle is inscribed in ΔABC having sides 8 cm, 10 cm and 12 cm as shown in figure, Find AD, BE and CF.

Column I		Column II	
a)	Cambrian period	*i)*	Age of Reptiles
b)	Devoniar period	*ii)*	Age of fishes
c)	Cenozoic era	*iii)*	Age of inventebrates
d)	Mesozoic era	*iv)*	Age of mammals

(*a*) a - *iii* b - *ii* c - *iv* d - *i*
(*b*) a - *iv* b - *iii* c - *i* d - *ii*
(*c*) a - *iii* b - *iv* c - *i* d - *ii*
(*d*) a - *ii* b - *iii* c - *i* d - *iv*

Solution:
We know that the tangents drawn from are external point to a circle are equal.
Therefore AD = AF = x say.
BD = BE = y say and
CE = CF = z say
Now, AB = 12 cm, BC = 8 cm, and CA= 10 cm.
x + y = 12, y + z = 8 and z + x = 10
(x + y) + (y + z) + (z + x) = 12 + 8 + 10
2(x + y + z) = 30
x + y + z = 15
Now, x + y = 12 and x + y + z = 15
12 + z = 15 ⇒ z = 3
y + z = 8 and x + y + z = 15
x + 8 = 15 ⇒ x = 7
and z + x = 10 and x + y + z = 15
10 + y = 15 ⇒ y = 5
Hence, AD = x = 7 cm, BE = y = 5 cm
and CF = z = 3 cm.

Question 4.
PQ is a tangent drawn from a point P to a circle with centre O and QOR is a diameter of the circle such that ∠POR = 120° . Find ∠OPQ.
Solution:
∠POR + ∠POQ = 180° (straight angle = 180°)
∴ 120 +∠POQ = 180°
∠POQ = 60°
∠OQP = 90° (∵ radius is $\perp^r$ to the tangent at the point of contact)



∴ ∠POQ + ∠PQO + ∠OPQ = 180° (∵ sum of the 3 angles of a triangle is 180°)
∴ 60 + 90 + ∠OPQ = 80°
∠OPQ = 180° – 150° = 30°

Question 5.
A tangent ST to a circle touches it at B. AB is a chord such that ∠ABT = 65°. Find ∠AOB , where "O" is the centre of the circle.
Solution:
In the figure,
∠OBT = 90° (∵OB-radius, BT – Tangent)
= 115°
∴ ∠OBA = 90° – 65°
∠OAB = 25° (OA = OB)
∴ ∠AOB = 180° – 50°
= 130°

Evolutionery Human	Brain Capacity
(*A*) Homo sapienS	*i)* 900 cc
(*B*) Homo erectus	*ii)* 650 - 800 cc
(*C*) Homo habilis	*iii)* 350 - 450 cc
(*D*) Australopithecus	*iv)* 1300 - 1600 cc

(*a*) *a - iv* *b - i* *c - ii* *d - iii*
(*b*) *a - ii* *b - iv* *c - iii* *d - i*
(*c*) *a - ii* *b - iii* *c - iv* *d - i*
(*d*) *a - iii* *b - i* *c - ii* *d - iv*

Question 6.
In figure, O is the centre of the circle with radius 5 cm. T is a point such that OT = 13 cm and OT intersects the circle E, if AB is the tangent to the circle at E, find the length of AB.

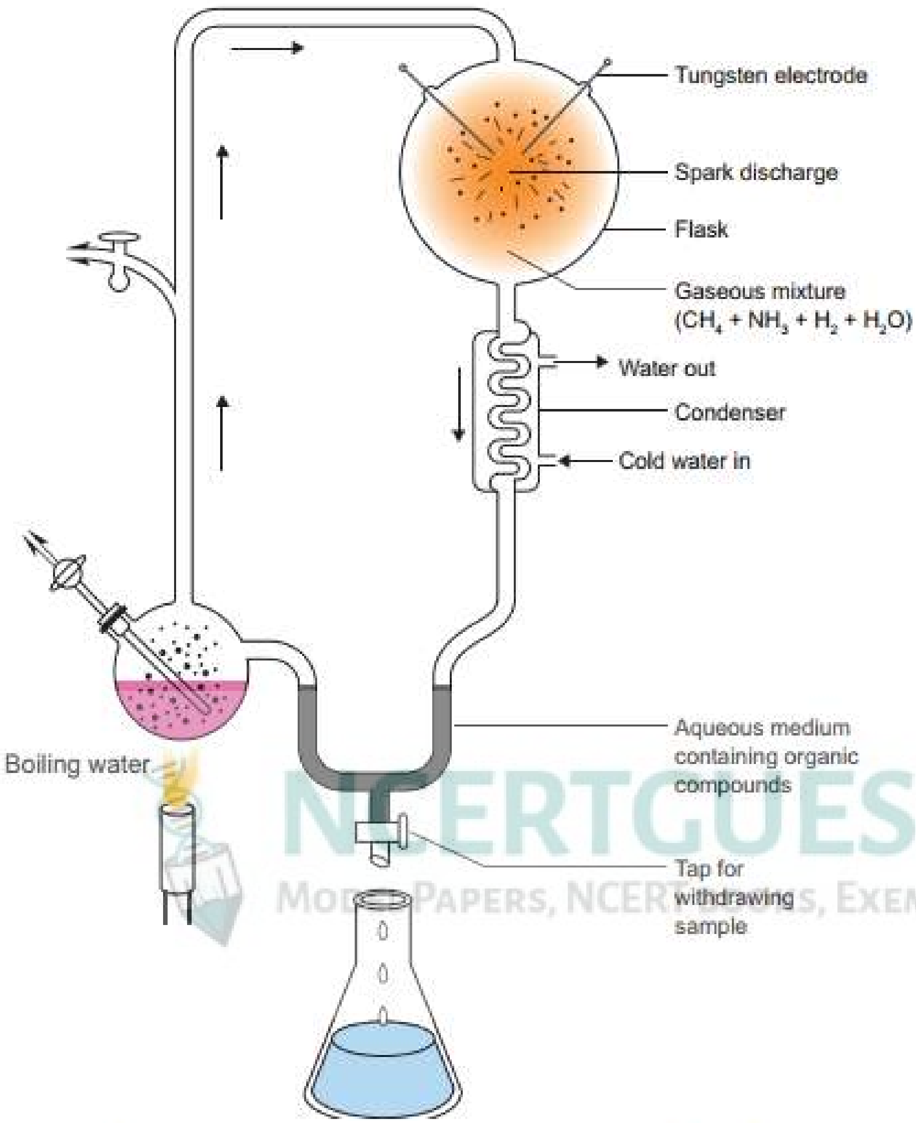


Fig. 6.1 Diagrammatic representation of Urey-Miller's experiment

i

Solution:

In ΔOPT, OP = r = 5 cm

OT = 13 cm

PT = 12 cm

In ΔOPA, $OA^2 = OP^2 + AP^2$ ………….. (1)

In ΔOAE, $OA^2 = OE^2 + AE^2$ …………. (2)

Equating (1) and (2),

$OP^2 + AP^2 = OE^2 + AE^2$ (∵ OP = OE = r)

∴ AP = AE

Parallel BQ = EB

In ΔAET, $AT^2 = AE^2 + ET^2$

$\therefore ET^2 = AT^2 - AE^2 = (AT + AE)(AT - AE)$

$\therefore ET^2 = (AT + AP)(AT - AE)$ ($\because AE = AP$)

$\therefore 8 \times 8 = 12 \times (AT - AE)$

$$\therefore \quad (AT - AE) = \frac{64}{12} = \frac{16}{3} \qquad ...(3)$$

$$AT + AE = AT + AP = PT = 12 \quad ...(4)$$

Adding (3) and (4),

$$2AT = \frac{16}{3} + 12$$

$$AT = \frac{8}{3} + \frac{18}{3} = \frac{26}{3}$$

$$AE = AT - AE$$

$$= \frac{26}{3} - \frac{16}{3} = \frac{10}{3}$$

Parallel $EB = \frac{10}{3}$

$$\therefore \quad AB = AE + EB = \frac{20}{3} \text{ cm.}$$

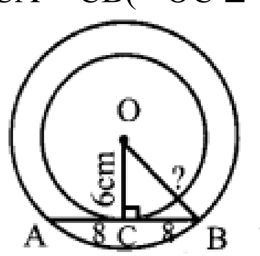
Question 7.

In two concentric circles, a chord of length 16 cm of larger circle becomes a tangent to the smaller circle whose radius is 6 cm. Find the radius of the larger circle.

Solution:

AB = 16 cm given

CA = CB($\because OC \perp^r AB$)



$OB^2 = OC^2 + BC^2$

$= 6^2 + 8^2$

= 36 + 64 = 100

OB = Radius of the larger circle = $\sqrt{100}$ = 10 cm.

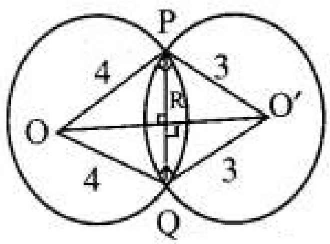
Question 8.

Two circles with centres O and O' of radii 3 cm and 4 cm respectively intersect at two points P and Q, such that OP and O'P are tangents to the two circles. Find the length of the common chord PQ.

Solution:

Given: OP = OQ = 4

O'P = O'Q = 3



OO' is the perpendicular bisector of chord PQ.

Let R be the point of intersection of PQ and OO'.

Assume PR = QR = x and OR = y

In OPO', $OP^2 + O'P^2 = (OO')^2 \Rightarrow OO'$

$= \sqrt{4^2 + 3^2} = 5$

OR = y ⇒ OR = 5 – y

In ΔOPR, $PR^2 + OR^2 = OP^2 \Rightarrow x^2 + y = 4^2$ ……….. (1)

In ΔO'PR, $PR^2 + O'R^2 = O'P^2 \Rightarrow x^2 + (5 - y)^2 = 9$ …………(2)

(1) – (2)=> $y^2 - (25 + y^2 - 10y) = 16 - 9$

$\Rightarrow y^2 - 25 - y^2 + 10y = 7$.

⇒ 10y = 25 + 7 ⇒ 10y =32

⇒ y =3.2

Substituting y = 3.2 in (1), we get x = $\sqrt{4^2 - 3.2^2}$

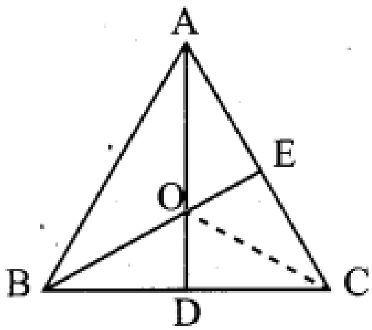
x = 2.4

PQ = 2x ⇒ PQ = 4.8 cm

Question 9.

Show that the angle bisectors of a triangle are concurrent.

Solution:



In ΔABC, let AD, BE are two angle bisectors.
They meet at the point 'O'
We have to prove that = $\frac{AC}{CD} = \frac{AO}{OD}$
Construct CO to meet the interesecting point O from C.

In Δ ABE, $\frac{AB}{AE} = \frac{BO}{OE}$ also $\frac{AB}{AC} = \frac{BD}{DC}$ (by angle bisector theorem)

$\therefore \frac{AB}{BD} = \frac{AC}{DC}$ ...(1)

In Δ ABD, $\frac{AB}{BD} = \frac{AO}{OD}$ ...(2)

From (1) & (2) we get $\frac{AC}{DC} = \frac{AO}{OD}$

Hence proved.

Question 10.
In ΔABC , with ∠B = 90° , BC = 6 cm and AB = 8 cm, D is a point on AC such that AD = 2 cm and E is the midpoint of AB. Join D to E and extend it to meet at F. Find BF.
In the figure ΔABC, ΔEBF are similar triangles.
Solution:
Consider DABC, Then D, E, F are respective points on the sides CA, AB and BC. By constrution

D, E, F are collinear.

By menelaus' theorem, $\frac{AE}{EB} \times \frac{BF}{FC} \times \frac{CD}{DA} = 1$ ..(1)

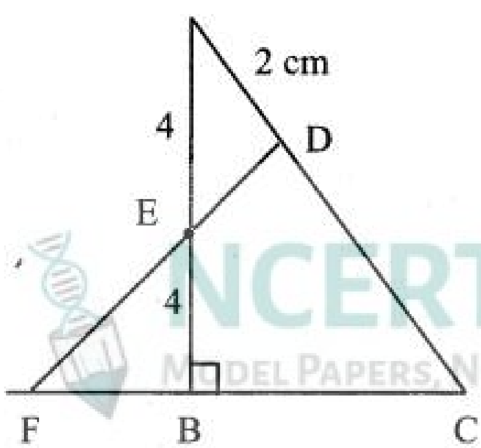
FC = FB + BC = BF + 6

By pythagoras therom $AC^2 = AB^2 + BC^2$

$= 64 + 36 = 100$

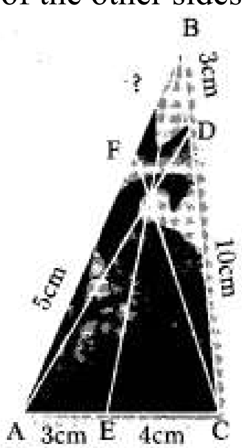
$\therefore$ AC = 10, CD = 8

$(1) \Rightarrow \frac{4}{4} \times \frac{BF}{BF + 6} \times \frac{8}{2} = 1$



4BF = BF + 6 ⇒ BF = 2cm

Question 11.
An artist has created a triangular stained glass window and has one strip of small length left before completing the window. She needs to figure out the length of left out portion based on the lengths of the other sides as shown in the figure.



Solution:
In the figure, let O be the concurrent point of the angle bisectors of the three angles.

$$\frac{BF}{FA} = \frac{OB}{OA} \quad ...(1)$$

$$\frac{CD}{DB} = \frac{OC}{OB} \quad ...(2)$$

$$\frac{AE}{EC} = \frac{OA}{OC} \quad ...(3)$$

Multiplying the corresponding sides of (1), (2) and (3) we get

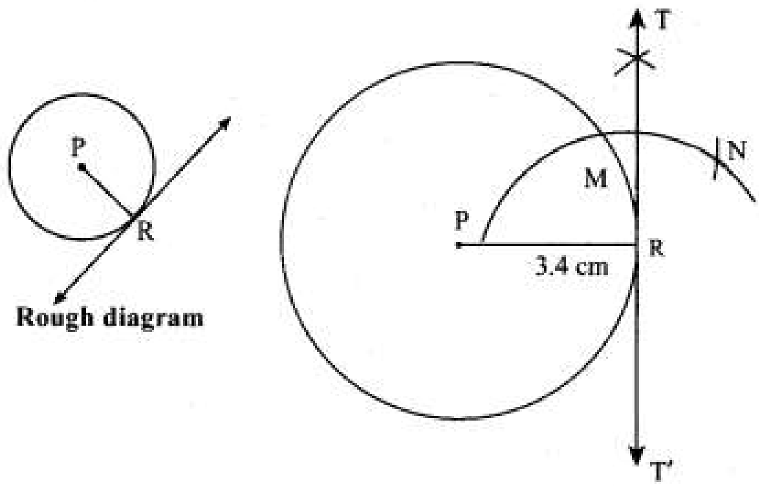
$$\frac{AF}{FA} \times \frac{CD}{DB} \times \frac{AE}{EC} = \frac{\cancel{OB}}{\cancel{OA}} \times \frac{\cancel{OC}}{\cancel{OB}} \times \frac{\cancel{OA}}{\cancel{OC}} = 1$$

$$\frac{x}{\cancel{5}} \times \frac{\cancel{10}^{2}}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{4}_{2}} = 1$$

$$\frac{x}{2} = 1$$

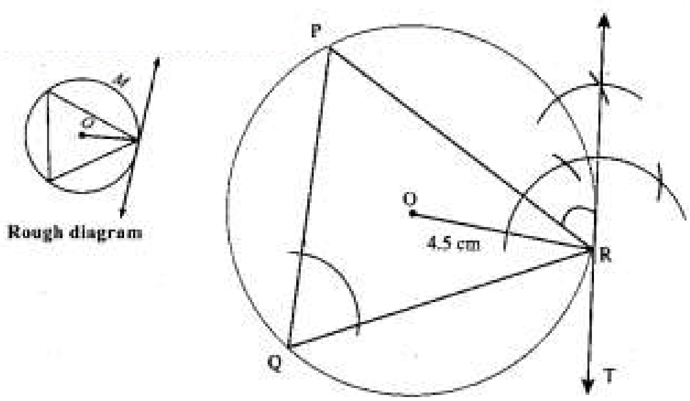
$$x = 2 \text{ cm}$$

Question 12.
Draw a tangent at any point R on the circle of radius 3.4 cm and centre at P ?
Solution:
Radius = 3.4 cm
Centre = P
Tangent at any point R.

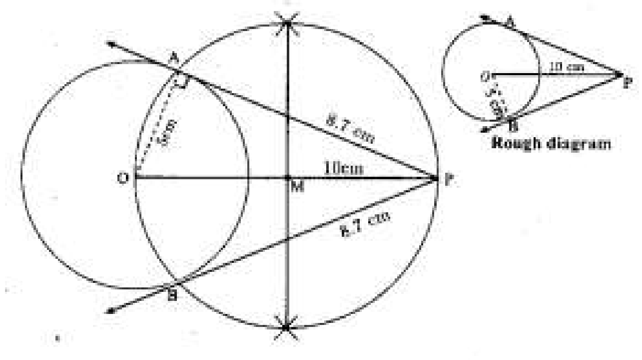


Construction:
Steps:
(1) Draw a circle with centre P of radius 3.4 cm.
(2) Take a point R on the circle. Join PR.
(3) Draw $\perp^r$ line TT^1 to PR. Which passes through R.
(4) TT1 is the required tangent.

Question 13.
Draw a circle of radius 4.5 cm. Take a point on the circle. Draw the tangent at that point using the alternate segment theorem.
Solution:
Construction:
Steps:
(1) With O as the centre, draw a circle of radius 4.5 cm.
(2) Take a point R on the circle. Through R draw any chord PR.
(3) Take a point Q distinct from P and R on the circle, so that P, Q, R are in anti-clockwise direction. Join PQ and QR.
(4) Through R drawn a tangent TT^1 such that ∠TRP = ∠PQR.
(5) TT^1 is the required tangent.



Question 14.
Draw the two tangents from a point which is 10 cm away from the centre of a circle of radius 5 cm. Also, measure the lengths of the tangents.
Solution:
Radius = 5 cm
The distance between the point from the centre is 10 cm.



Construction:
Steps:
(1) With O as centre, draw a circle of radius 5 cm.
(2) Draw a line OP =10 cm.
(3) Draw a perpendicular bisector of OP which cuts OP at M.
(4) With M as centre and MO as radius, draw a circle which cuts previous circle at A and B.
(5) Join AP and BP. AP and BP are the required tangents. Thus length of the tangents are PA and PB = 8.7 cm.

Verification:
In the right triangle ∠POA;

$$PA = \sqrt{OP^2 - OA^2}$$

$$PA = \sqrt{10^2 - 5^2}$$

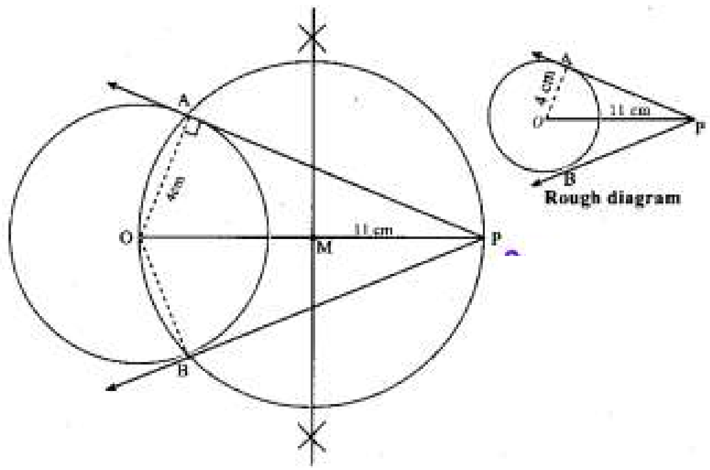
$$= \sqrt{100 - 25}$$

$$= \sqrt{75}$$

$$\cong 8.7 \text{ cm (approximately)}$$

Question 15.
Take a point which is 11 cm away from the centre of a circle of radius 4 cm and draw the two tangents to the circle from that point.
Solution:
Radius = 4 cm

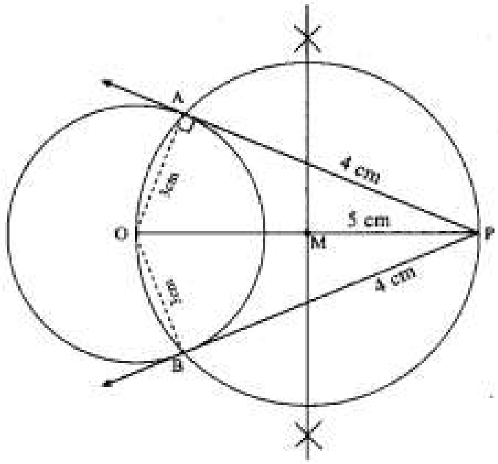
The distance of a point from the center =11 cm.

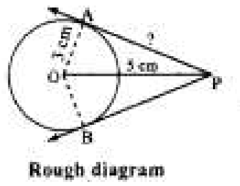


Construction:
Steps:
(1) With centre O, draw a circle of radius 4 cm.
(2) Draw a line OP = 11 cm.
(3) Draw a $\perp^r$ bisector of OP, which cuts atM.
(4) With M as centre and MO as radius, draw a circle which cuts previous circle at A and B.
(5) Join AP and BP. AP and BP are the required tangents. Thus length of the tangents are PA = PB = 10.2 cm.
Verification:
In the right triangle

$$\angle OPA,\ PA = \sqrt{OP^2 - OA^2}$$
$$= \sqrt{11^2 - 4^2}$$
$$= \sqrt{121 - 16}$$
$$= \sqrt{105}$$
$$\cong 10.2 \text{ cm (approximately)}$$

Question 16.
Draw the two tangents from a point which is 5 cm away from the centre of a circle of diameter 6 cm. Also, measure the lengths of the tangents.
Solution:
Diameter = 6 cm
Radius = $\frac{6}{2}$ = 3 cm.
The distance between the centre and the point is 5 cm.

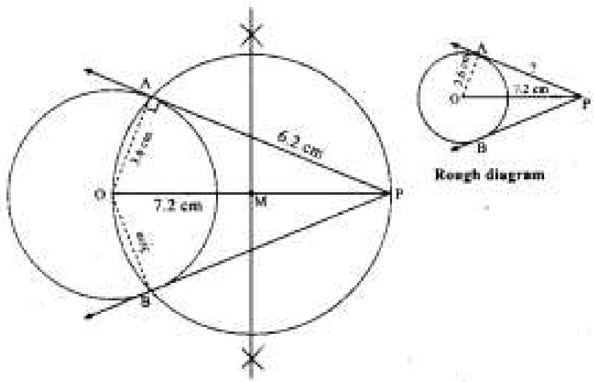




Construction:
Steps:
(1) With centre O, draw a circle of radius , 3cm.
(2) Draw a line OP = 5 cm.
(3) Draw a bisector of OP, which cuts OP and M.
(4) With M as centre and MO as radius draw a circle which cuts previous circle at A and B.
(5) Join AP and BP. AP and BP are the required tangents. Thus length of the tangents are PA = PB = 4 cm
Verification:
In the right triangle ΔOPA,

$$\begin{aligned} PA &= \sqrt{OP^2 - OA^2} \\ &= \sqrt{5^2 - 3^2} \\ &= \sqrt{25 - 9} = \sqrt{16} \\ &= 4 \text{ cm.} \end{aligned}$$

Question 17.
Draw a tangent to the circle from the point P having radius 3.6 cm, and centre at O. Point P is at a distance 7.2 cm from the centre.
Radius 3.6 cm.
Solution:
Distance from the centre to the point is 7.2 cm.

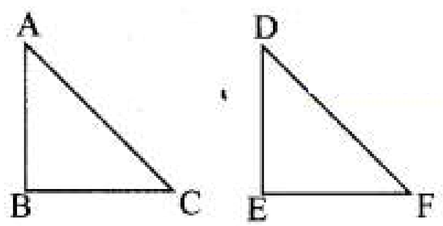


Construction:
Steps:
(1) Draw a circle of radius 3.6 cm with centre O.
(2) Draw a line OP = 7.2 cm.
(3) Draw a perpendicular bisector of OP, which cuts OP at M.
(4) With M as centre and MO as radius, draw a circle which cuts previous circle at A and B.
(5) Join AP and BP. AP and BP are the required tangents. Thus length of the tangents are PA = PB = 6.2 cm.
Verification:
In the right triangle.

$$OPA, PA = \sqrt{OP^2 - OA^2}$$
$$= \sqrt{7.2^2 - 3.6^2} = \sqrt{51.84 - 12.96}$$
$$= \sqrt{38.88}$$
$$\cong 6.2 \text{ cm (approximately)}$$

Ex 4.5

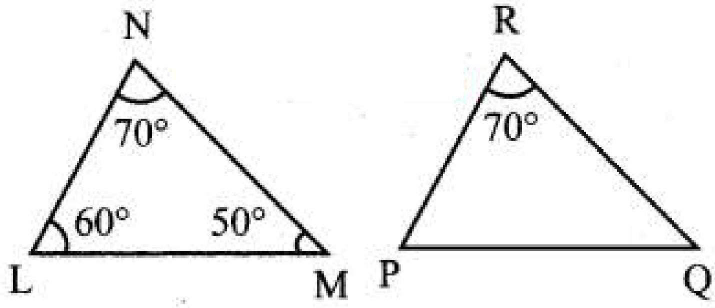
Multiple choice questions.
Question 1.
If in triangles ABC and EDF, $\frac{AB}{DE} = \frac{BC}{FD}$ then they will be similar, when
(1) ∠B = ∠E
(2) ∠A = ∠D
(3) ∠B = ∠D
(4) ∠A = ∠F
Solution:
(1) ∠B = ∠E
Hint:



$\frac{AB}{DE} = \frac{BC}{EF}$, then they will be similar when
∠B = ∠E

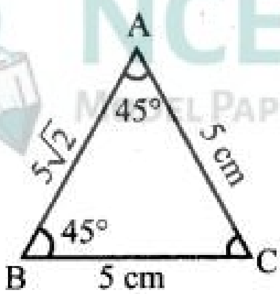
Question 2.
In, ΔLMN, ∠L = 60°, ∠M =50° . If ΔLMN ~ ΔPQR then the value of ∠R is
(1) 40°
(2) 70°
(3) 30°
(4) 110°
Solution:

(2) 70°



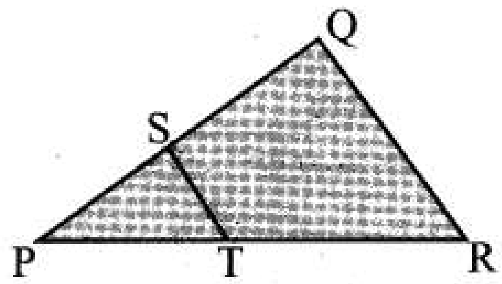
ΔLMN ~ ΔPQR, ∠R = 70°.

Question 3.
If ΔABC is an isosceles triangle with ∠C = 90° and AC = 5 cm, then AB is
(1) 2.5 cm
(2) 5 cm
(3) 10 cm
(4) $5\sqrt{2}$ cm
Solution:
(4) $5\sqrt{2}$cm
Hint:



$$AB = \sqrt{5^2 + 5^2} = \sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2} \text{ cm.}$$

Question 4.
In a given figure ST || QR, PS = 2 cm and SQ = 3 cm. Then the ratio of the area of ΔPQR to the area of ΔPST is



(1) 25 : 4
(2) 25 : 7
(3) 25 : 11
(4) 25 : 13
Solution:
(1) 25 : 4
Hint:
Ratio of the area of similar triangles is equal to the ratio of the square of their corresponding sides.
$\therefore 5^2 : 2^2 = 25 : 4$

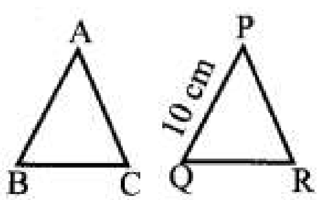
Question 5.
The perimeters of two similar triangles ΔABC and ΔPQR are 36 cm and 24 cm respectively. If PQ = 10 cm, then the length of AB is

(1) $6\frac{2}{3}$ cm (2) $\frac{10\sqrt{6}}{3}$ cm
(3) $66\frac{2}{3}$ cm (4) 15 cm

Solution:
(4) 15 cm
Hint:

$$\frac{AB}{PQ} = \frac{36}{24}$$

$$\frac{AB}{10\text{ cm}} = \frac{36}{24}$$

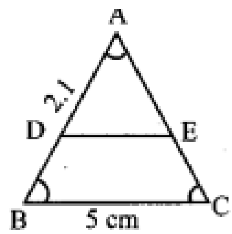


$$24\,AB = 360$$

$$AB = \frac{360^{30}}{24_2} = 15$$

Question 6.
If in ΔABC, DE || BC . AB = 3.6 cm, AC = 2.4 cm and AD = 2.1 cm then the length of AE is
(1) 1.4 cm
(2) 1.8 cm
(3) 1.2 cm
(4) 1.05 cm
Solution:
(1) 1.4 cm

$$\frac{AB}{AD} = \frac{AC}{AE}$$



$$\frac{3 \cdot 6}{2 \cdot 1} = \frac{2 \cdot 4}{A \cdot E}$$

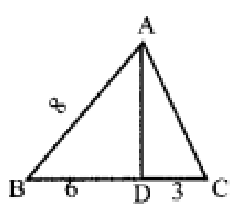
(3.6) (AE) = 2.1 × 2.4
AE = 1.4 cm

Question 7.
In a ΔABC , AD is the bisector of ∠BAC . If AB = 8 cm, BD = 6 cm and DC = 3 cm. The length of the side AC is
(1) 6 cm
(2) 4 cm
(3) 3 cm
(4) 8 cm
Solution:
(2) 4 cm
Hint:

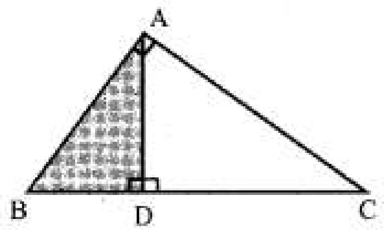
$$\frac{AB}{AC} = \frac{BD}{DC}$$

$$\frac{8}{x} = \frac{6}{3}$$

$$6x = 24 \Rightarrow x = 4$$



Question 8.
In the adjacent figure ∠BAC = 90° and AD ⊥ BC then



(1) BD.CD = BC^2
(2) AB.AC = BC^2
(3) BD.CD = AD^2
(4) AB.AC = AD^2
Solution:
(3) BD.CD = AD^2

(i) $BD \cdot CD = BC^2 \Rightarrow \frac{BD}{BC} = \frac{BC}{CD}$ ☒

(ii) $\frac{AB}{BC} = \frac{BC}{AC}$ ☒

(iii) $BD \cdot CD = AD^2 \Rightarrow \frac{BD}{AD} = \frac{AD}{CD}$ ☑

$\Rightarrow BD \cdot CD = AD^2$

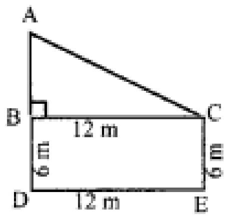
(iv) $AB \cdot AC = AD^2 \Rightarrow \frac{AB}{AD} = \frac{AD}{AC}$

$= BD \cdot CD = AD^2$ ☒

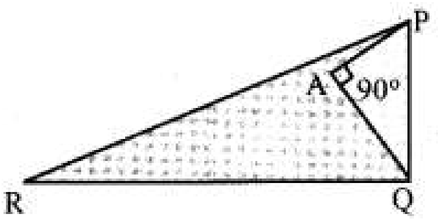
Question 9.
Two poles of heights 6 m and 11 m stand vertically on a plane ground. If the distance between their feet is 12 m, what is the distance between their tops?
(1) 13 cm
(2) 14 m
(3.) 15 m
(4) 12.8 m
Solution:
(1) 13 cm

Hint:

$$AC^2 = AB^2 + BC^2$$
$$= 5^2 + 12^2$$
$$= 25 + 144$$
$$= 169$$
$$AC = 13 \text{ m}$$



Question 10.
In the given figure, PR = 26 cm, QR = 24 cm, PAQ = 90° , PA = 6 cm and QA = 8 cm. Find ∠PQR

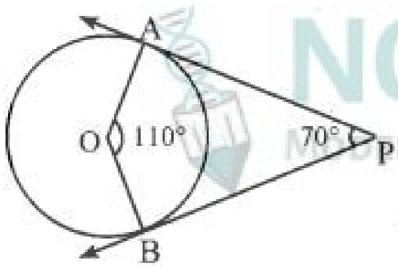


(1) 80°
(2) 85°
(3) 75°
(4) 90°
Solution:
(4) 90°
Hint:
PR = 26
QR = 24
∠PAQ = 90°
PQ = 10
PQ = $\sqrt{26^2 - 24^2} = \sqrt{100}$ = 10
∴ ∠PAQ = 90°

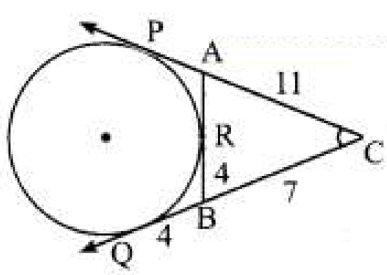
Question 11.
A tangent is perpendicular to the radius at the …………..
(1) centre
(2) point of contact
(3) infinity
(4) chord
Answer:
(2) point of contact

Question 12.
How many tangents can be drawn to the circle from an exterior point?
(1) one
(2) two
(3) infinite
(4) zero
Solution:
(2) two

Question 13.
The two tangents from an external points P to a circle with centre at O are PA and PB. If ∠APB = 70° then the value of ∠AOB is
(1) 100°
(2) 110°
(3) 120°
(4) 130°
Solution:
(2) 110°
Hint:

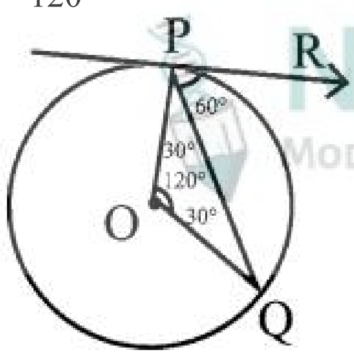


Question 14.
In figure CP and CQ are tangents to a circle T with centre at O. ARB is another tangent touching the circle at R. If CP = 11 cm and BC = 7 cm, then the length of BR is
(1) 6 cm
(2) 5 cm
(3) 8 cm
(4) 4 cm
Solution:
(4) 4 cm



BQ = BR
CP = CQ = 11
BC = 7, ∴ BQ = CQ – BC
= 11 – 7 = 4
BR = BQ = 4cm

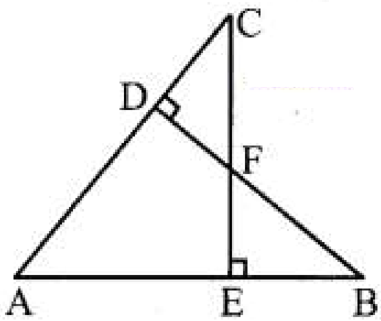
Question 15.
In figure if PR is tangent to the circle at P and O is the centre of the circle, then ∠POQ is
(1) 120°
(2) 100°
(3) 110°
(4) 90°
Solution:
(1) 120°
∠POQ = 180° -(30° + 30°)
= 180° – 60°
= 120°



Unit Exercise 4

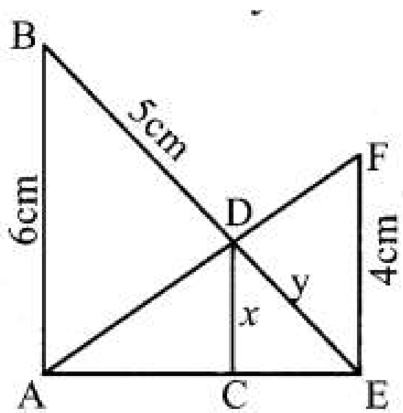
Question 1.
In the figure, if BD⊥AC and CE ∠ AB, prove that

(i) ΔAEC ~ ΔADB **(ii)** $\frac{CA}{AB} = \frac{CE}{DB}$



Solution:
In the figure's ΔAEC and ΔADB.
We have ∠AEC =∠ADB = 90 (∵ CE ∠AB and BD ∠AC)
and ∠EAC =∠DAB
[Each equal to ∠A]
Therefore by AA-criterion of similarity, we have ΔAEC ~ ΔADB
(ii) We have
ΔAEC ~ ΔADB [As proved above]
$\Rightarrow \frac{CA}{BA} = \frac{EC}{DB} \Rightarrow \frac{CA}{AB} = \frac{CE}{DB}$
Hence proved.

Question 2.
In the given figure AB||CD || EF . If AB = 6 cm, CD = x cm, EF = 4 cm, BD = 5 cm and DE = y cm. Find x and y.



Solution:
In the given figure, ΔAEF, and ΔACD are similar Δ[s].
∠AEF = ∠ACD = 90°
∠A = ∠A (common)
∴ ΔAEF ~ ΔACD (By AA criterion of similarity)

$$\frac{AE}{AC} = \frac{EF}{CD} = \frac{4}{x} \Rightarrow AC = \frac{AE \times CD}{EF}$$

$$AC = \frac{x \times AE}{4} \qquad ...(1)$$

In ΔEAB and ΔECD,
we have ∠ECD = ∠EAB = 90°.
∠E = ∠E (common)

∴ ΔECD ~ ΔEAB

$$\Rightarrow \frac{CE}{EA} = \frac{CD}{BA} = \frac{x}{6}$$

$$\frac{CE}{EA} = \frac{x}{6}$$

$$CE = \frac{x \times EA}{6} \qquad ...(2)$$

By BPT

$$\frac{CE}{EA} = \frac{y}{y+5}$$

$$\frac{x}{6} = \frac{y}{y+5} \qquad ...(3)$$

From (1) and (2), we have

$$AC + CE = \frac{x \times AE}{4} + \frac{x \times AE}{6}$$

$$\cancel{AE} = \cancel{AE} \times x\left[\frac{1}{4}+\frac{1}{6}\right]$$

$$1 = x\left(\frac{6+4}{24}\right) = \frac{10x}{24}$$

$$\therefore \qquad x = \frac{24}{10} = 2.4 \text{ cm} = \frac{12}{5}$$

Subtstituting x = 2.4 cm in (3)

We get, $\frac{2 \cdot 4}{6} = \frac{y}{y+5}$

$$6y = 2 \cdot 4y + 2 \cdot 4 \times 5$$

$$6y = 2 \cdot 4y + 12$$

$$6y - 2{\cdot}4y = 12$$

$$3{\cdot}6y = 12$$

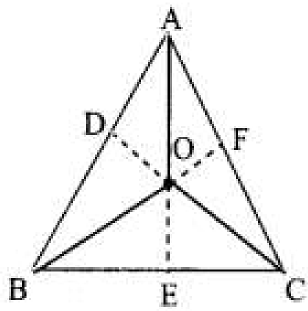
$$y = \frac{12\times 10}{3.6\times 10}$$

$$= \frac{\cancel{120}^{10}}{\cancel{36}_{3}} = 3.3 \text{ cm}$$

$$x = 2.4 \text{ cm}$$

$$y = 3.3 \text{ cm}$$

Question 3.
O is any point inside a triangle ABC. The bisector of ∠AOB , ∠BOC and ∠COA meet the sides AB, BC and CA in point D, E and F respectively.
Show that AD × BE × CF = DB × EC × FA
Solution:
In ΔAOB, OD is the bisector of ∠AOB.



$$\therefore \quad \frac{OA}{OB} = \frac{AD}{DB} \qquad \text{...(1)}$$

In ΔBOC, OE is the bisector of ∠BOC

$\therefore \frac{OB}{OC} = \frac{BE}{EC}$ ………… (2)

In ΔCOA, OF is the bisector of ∠COA.

$\therefore \frac{OC}{OA} = \frac{CF}{FA}$ ………….. (3)

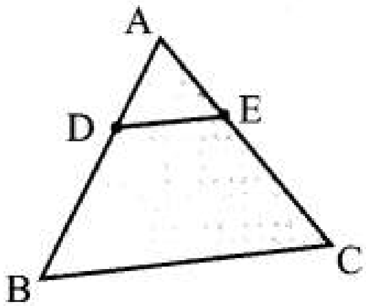
Multiplying the corresponding sides of (1), (2) and (3), we get

$$\frac{\cancel{OA}}{\cancel{OB}} \times \frac{\cancel{OB}}{\cancel{OC}} \times \frac{\cancel{OC}}{\cancel{OA}} = \frac{AD}{DB} \times \frac{BE}{EC} \times \frac{CF}{FA}$$

$$1 = \frac{AD}{DB} \times \frac{BE}{EC} \times \frac{CF}{FA}$$

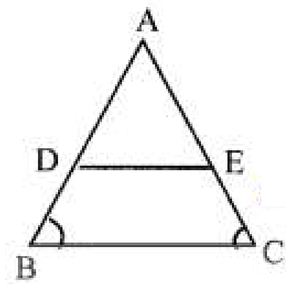
⇒ DB × EC × FA = AD × BE × CF
Hence proved.

Question 4.
In the figure, ABC is a triangle in which AB = AC . Points D and E are points on the side AB and AC respectively such that AD = AE . Show that the points B, C, E and D lie on a same circle.



Solution:

In order to prove that the points B, C, E and D are concyclic, it is sufficient to show that ∠ABC + ∠CED = 180° and ∠ACB + ∠BDE = 180°.



In ΔABC, we have AB = AC and AD = AE.

⇒ AB – AD = AC – AE

⇒ DB = EC

Thus we have AD = AE and DB = EC. (By the converse of Thale's theorem)

⇒ $\frac{AD}{DB} = \frac{AE}{EC}$ ⇒ DE||BC

∠ABC = ∠ADE (corresponding angles)

⇒ ∠ABC + ∠BDE = ∠ADE + ∠BDE (Adding ∠BDE on both sides)

⇒ ∠ABC + ∠BDE = 180°

⇒ ∠ACB + ∠BDE = 180° (∵ AB = AC ∴ ∠ABC = ∠ACB)

Again DE || BC

⇒ ∠ACB = ∠AED

⇒ ∠ACB + ∠CED = ∠AED + ∠CED (Adding ∠CED on both sides).

⇒ ∠ACB + ∠CED = 180° and

⇒ ∠ABC + ∠CED = 180° (∵ ∠ABC = ∠ACB)

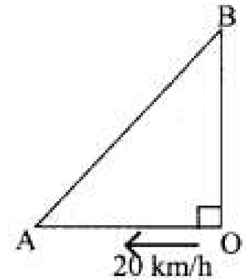
Thus BDEC is a quadrilateral such that

⇒ ∠ACB + ∠BDE = 180° and

⇒ ∠ABC + ∠CED = 180°

∴ BDEC is a cyclic quadrilateral. Hence B, C, E, and D are concyclic points.

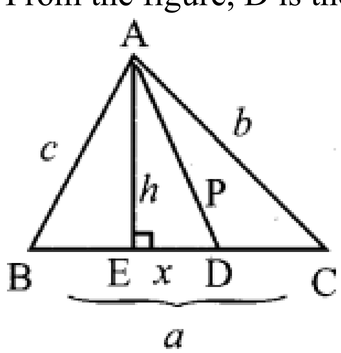
Question 5.
Two trains leave a railway station at the same time. The first train travels due west and the second train due north. The first train travels at a speed of 20 km/hr and the second train travels at 30 km/hr. After 2 hours, what is the distance between them?
Solution:
After 2 hours, let us assume that the first train is at A and the second is at B.



OA = speed × time
= 20 × 2 = 40 km
OB = speed × time
= 30 × 2 = 60 km
Distance between the trains after 2 hours,

$$AB = \sqrt{OA^2 + OB^2} \text{ (pythogoras theorem)}$$
$$= \sqrt{40^2 + 60^2}$$
$$= \sqrt{1600 + 3600} = \sqrt{5200} = \sqrt{400 \times 13}$$
$$= 20\sqrt{13}$$
$$AB = 72.11 \text{ km or } AB = 20\sqrt{13} \text{ km.}$$

Question 6.
D is the mid point of side BC and AE⊥BC. If i BC a, AC = b, AB = c, ED = x, AD = p and AE = h , prove that
(i) $b^2 = p^2 + ax + \frac{a^2}{4}$
(ii) $c^2 = p^2 - ax + \frac{a^2}{4}$
(iii) $b^2 + c^2 = 2p^2 + \frac{a^2}{2}$
Solution:
From the figure, D is the mid point of BC.



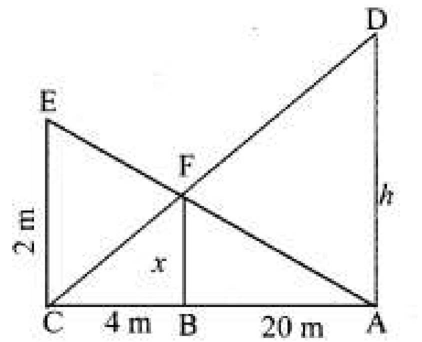
We have ∠AED = 90°
∴ ∠ADE < 90° and ∠ADC > 90°
i.e. ∠ADE is acute and ∠ADC is obtuse,

(i) In ΔADC, ∠ADC is obtuse angle.
$AC^2 = AD^2 + DC^2 + 2DC \times DE$
$\Rightarrow AC^2 = AD^2 + \frac{1}{2} BC^2 + 2 . \frac{1}{2} BC . DE$
$\Rightarrow AC^2 = AD^2 + \frac{1}{4} BC^2 + BC . DE$
$\Rightarrow AC^2 = AD^2 + BC . DE + \frac{1}{4} BC^2$
$\Rightarrow b^2 = p^2 + ax + \frac{1}{4} a^2$
Hence proved.

(ii) In ΔABD, ∠ADE is an acute angle.
$AB^2 = AD^2 + BD^2 - 2BD . DE$
$\Rightarrow AB^2 = AD^2 + \left(\frac{1}{2}BC\right)^2 - 2 \times \frac{1}{2} BC . DE$
$\Rightarrow AB^2 = AD^2 + \frac{1}{4} BC^2 - BC . DE$
$\Rightarrow AB^2 = AD^2 - BC . DE + \frac{1}{4} BC^2$
$\Rightarrow c^2 = p^2 - ax + \frac{1}{4} a^2$
Hence proved.

(iii) From (i) and (ii) we get .
$AB^2 + AC^2 = 2AD^2 + \frac{1}{2} BC^2$
i.e. $c^2 + b^2 = 2p^2 + \frac{a^2}{2}$
Hence it is proved.

Question 7.
A man whose eye-level is 2 m above the ground wishes to find the height of a tree. He places a mirror horizontally on the ground 20 m from the tree and finds that if he stands at a point C which is 4 m from the mirror B, he can see the reflection of the top of the tree. How height is the tree?
Solution:
From the figure; ΔDAC, ΔFBC are similar triangles and ΔACE & ΔABF are similar triangles.



∴ From ΔACD and ΔBCF

$$\frac{AD}{BF} = \frac{AC}{BC} \Rightarrow \frac{h}{x} = \frac{20+4}{4}$$

$$\frac{h}{x} = \frac{24}{4} = 6.$$

$$\Rightarrow \quad h = 6x. \qquad ...(1)$$

From ΔACE & ΔABF.

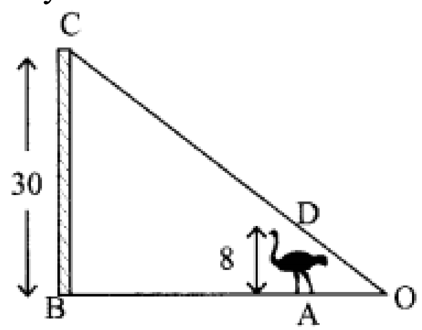
$$\frac{2}{x} = \frac{20+4}{20} \Rightarrow \frac{2}{x} = \frac{\cancel{24}^{6}}{\cancel{20}_{5}}$$

$$6x = 10 \qquad ...(2)$$

∴ height of the tree h = 6x = 10 m.

Question 8.
An emu which is 8 ft tall standing at the foot of a pillar which is 30 ft height. It walks away from the pillar. The shadow of the emu falls beyond emu. What is the relation between the length of the shadow and the distance from the emu to the pillar?
Solution:
Let OA (emu shadow) the x and AB = y.
⇒ pillar's shadow = OB = OA + AB

⇒ OB = x + y



From basic proportionality theorem,

$$\frac{OA}{OB} = \frac{AD}{BC}$$

$$\frac{x}{x+y} = \frac{8}{30}$$

Reciprocating on both sides, we get

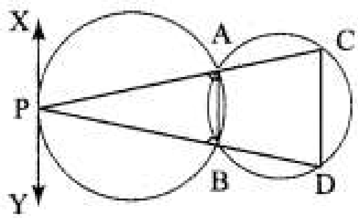
$$\Rightarrow \frac{x}{x+y} = \frac{30}{8} \Rightarrow 1 + \frac{y}{x} = \frac{30}{8} \Rightarrow \frac{y}{x} = \frac{30}{8} - 1$$

$$\Rightarrow \frac{y}{x} = \frac{30-8}{8} \Rightarrow \frac{y}{x} = \frac{22}{8} \Rightarrow \frac{y}{x} = \frac{11}{4}$$

$$\Rightarrow x = \frac{4}{11} \times y \Rightarrow \text{shadow} = \frac{4}{11} \times \text{distance}$$

Question 9.
Two circles intersect at A and B. From a point P on one of the circles lines PAC and PBD are drawn intersecting the second circle at C and D. Prove that CD is parallel to the tangent at P.
Solution:
Let XY be the tangent at P.
TPT: CD is || to XY.
Construction: Join AB.
ABCD is a cyclic quadilateral.
∠BAC + ∠BDC= 180° ………… (1)
∠BDC = 180° – ∠BAC …………. (2)
Equating (1) and (2)
we get ∠BDC = ∠PAB
Similarly we get ∠PBA = ∠ACD

as XY is tangent to the circle at ‘P’
∠BPY = ∠PAB (by alternate segment there)

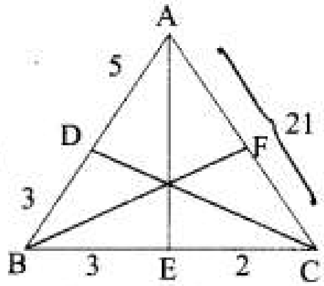


∴ ∠PAB = ∠PDC
∠BPY = ∠PDC
XY is parallel of CD.
Hence proved.

Question 10.
Let ABC be a triangle and D, E, F are points on the respective sides AB, BC, AC (or their extensions). Let AD: DB = 5 : 3, BE : EC = 3 : 2 and AC = 21. Find the length of the line segment CF.
Solution:

$\frac{AD}{DB} = \frac{5}{3}, \frac{BE}{EC} = \frac{3}{2},$

$AC = 21 \Rightarrow \frac{CF}{FA} = \frac{CF}{21 - CF}$



$\therefore$ By Ceva's theorem,

$$\frac{BE}{EC} \times \frac{CF}{FA} \times \frac{AD}{DB} = +1$$

$$\frac{\not{3}}{2} \times \frac{CF}{21 - CF} \times \frac{5}{\not{3}} = +1$$

$$\frac{CF}{21 - CF} = \frac{2}{5}$$

$$5CF = 42 - 2CF \quad ...(1)$$

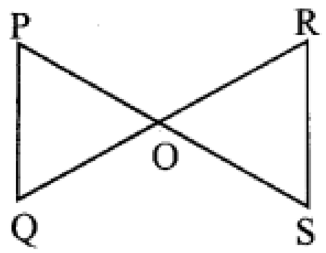
$$5CF + 2CF = 42 \quad ...(2)$$

$$7CF = 42$$

$$CF = \frac{42}{7} = 6 \text{ units}$$

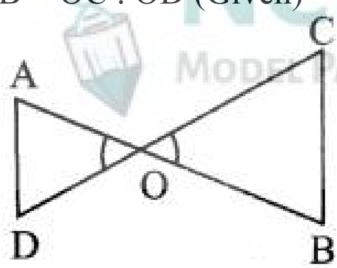
Additional Questions

Question 1.
In figure if PQ || RS, Prove that ΔPOQ ~ ΔSOR
Solution:
PQ || RS



So, ∠P = ∠S (A Hernate angles)
and ∠Q = ∠R
Also, ∠POQ = ∠SOR (vertically opposite angle)
∴ ΔPOQ ~ ΔSOR (AAA similarity criterion)

Question 2.
In figure OA . OB = OC . OD Show that ∠A = ∠C and ∠B = ∠D
Solution:
OA . OB = OC . OD (Given)

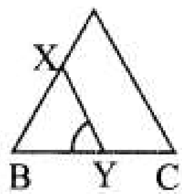


Also we have ∠AOD = ∠COB
(vertically opposite angles) …………. (2)

From (1) and (2)
∴ ΔAOD ~ ΔCOB (SAS similarity criterion)
So, ∠A = ∠C and ∠B = ∠D
(corresponding angles of similar triangles)

Question 3.
In figure the line segment XY is parallel to side AC of ΔABC and it divides the triangle into two parts of equal areas. Find the ratio $\frac{AX}{AB}$
Solution:

Given XY||AC



So, $\angle BXY = \angle A$ and

$\angle BYX = \angle C$ (corresponding angles)

$\therefore \Delta ABC \sim \Delta XBY$ (AAA similarity criterion)

So, $$\frac{ar(ABC)}{ar(XBY)} = \left(\frac{AB}{XB}\right)^2 \quad ...(1)$$

$$ar(ABC) = 2ar(XBY)$$

$$\frac{ar(ABC)}{ar(XBY)} = \frac{2}{1} \quad ...(2)$$

From (1) and (2),

$$\left(\frac{AB}{XB}\right)^2 = \frac{2}{1} \text{ i.e., } \frac{AB}{XB} = \frac{\sqrt{2}}{1}$$

$$\frac{XB}{AB} = \frac{1}{\sqrt{2}}$$

$$1-\frac{XB}{AB} = 1-\frac{1}{\sqrt{2}}$$

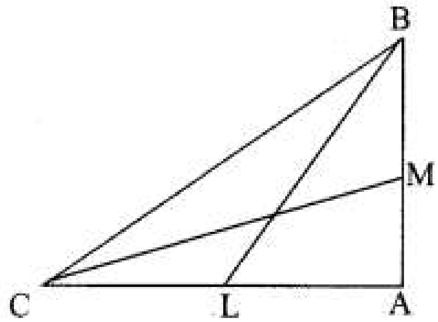
$$\frac{AB-XB}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}}$$

$$\frac{AX}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}} = \frac{2-\sqrt{2}}{2}$$

Question 4.
In AD⊥BC, prove that $AB^2 + CD^2 = BD^2 + AC^2$.
Solution:
From ΔADC, we have
$AC^2 = AD^2 + CD^2$ …………… (1)
(Pythagoras theorem)

From ΔADB, we have
$AB^2 = AD^2 + BD^2$ …………. (2)
(Pythagoras theorem)
Subtracting (1) from (2) we have,
$AB^2 – AC^2 = BD^2 – CD^2$
$AB^2 + CD^2 = BD^2 + AC^2$

Question 5.
BL and CM are medians of a triangle ABC right angled at A.
Prove that $4(BL^2 + CM^2) = 5BC^2$.
Solution:
BL and CM are medians at the ΔABC in which
A = ∠90°.
From ΔABC
$BC^2 = AB^2 + AC^2$ …………… (1)
(Pythagoras theorem)



From ΔABL,

$$BL^2 = AL^2 + AB^2 \quad ...(1)$$

$$BL^2 = \left(\frac{AC}{2}\right)^2 + AB^2$$

(L is the mid-point at AC)

$$BL^2 = \frac{AC^2}{4} + AB^2$$

$$4BL^2 = AC^2 + 4AB^2 \quad ...(2)$$

From ΔCMA,

$$CM^2 = AC^2 + AM^2$$

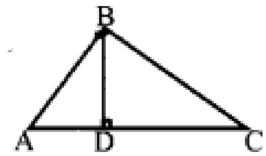
$$CM^2 = AC^2 + \left(\frac{AB}{2}\right)^2$$

(M is the mid-point at AB)

$$CM^2 = AC^2 + \frac{AB^2}{4}$$

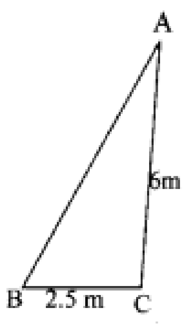
$4CM^2 = 4AC^2 + AB^2$ …………… (3)
Adding (2) and (3), we have
$4(BL^2 + CM^2) = 5(AC^2 + AB^2)$
$4(BL^2 + CM^2) = 5BC^2$ [From (1)]

Question 6.
Prove that in a right triangle, the square of the hypotenure is equal to the sum of the squares of the others two sides.
Solution:
Proof:



We are given a right triangle ABC right angled at B.
We need to prove that $AC^2 = AB^2 + BC^2$
Let us draw BD ⊥ AC
Now, ΔADB ~ ΔABC
$\frac{AD}{AB} = \frac{AB}{AC}$ (sides are proportional)
$AD . AC = AB^2$ …………. (1)
Also, ΔBDC ~ ΔABC
$\frac{CD}{BC} = \frac{BC}{AC}$
$CD . AC = BC^2$ …………. (2)
Adding (1) and (2)
$AD . AC + CD . AC = AB^2 + BC^2$
$AC(AD + CD) = AB^2 + BC^2$
$AC . AC = AB^2 + BC^2$
$AC^2 = AB^2 + BC^2$

Question 7.
A ladder is placed against a wall such that its foot is at a distance of 2.5 m from the wall and its top reaches a window 6 m above the ground. Find the length of the ladder.
Solution:
Let AB be the ladder and CA be the wall with the window at A.



Also, BC = 2.5 m and CA = 6 m
From Pythagoras theorem,
$AB^2 = BC^2 + CA^2$
$= (2.5)^2 + (6)^2$
= 42.25

AB = 6.5
Thus, length at the ladder is 6.5 m.

Question 8.
In figure O is any point inside a rectangle ABCD. Prove that $OB^2 + OD^2 = OA^2 + OC^2$.
Solution:
Through O, draw PQ||BC so that P lies on AB and Q lies on DC.
Now, PQ||BC
PQ⊥AB and PQ⊥DC (∵ ∠B = 90° and ∠C = 90°)
So, ∠BPQ = 90° and ∠CQP = 90°
Therefore BPQC and APQD are both rectangles.
Now from ΔOPB,
$OB^2 = BP^2 + OP^2$ ……………. (1)
Similarly from ΔOQD,
$OD^2 = OQ^2 + DQ^2$ …………. (2)
From ΔOQC, we have
$OC^2 = OQ^2 + CQ^2$ ……….. (3)
ΔOAP, we have
$OA^2 = AP^2 + OP^2$ …………. (4)
Adding (1) and (2)
$OB^2 + OD^2 = BP^2 + OP^2 + OQ^2 + DQ^2$ (As BP = CQ and DQ = AP)
$= CQ^2 + OP^2 + OQ^2 + AP^2$
$= CQ^2 + OQ^2 + OP^2 + AP^2$
$= OC^2 + OA^2$ [From (3) and (4)]

Question 9.
In ∠ACD = 90° and CD⊥AB. Prove that $\frac{BC^2}{AC^2} = \frac{BD}{AD}$
Solution:
ΔACD ~ ΔABC

So,

$$\frac{AC}{AB} = \frac{AD}{AC}$$

$$AC^2 = AB \cdot AD \qquad ...(1)$$

Similarly $\Delta BCD \sim \Delta BAC$

So,

$$\frac{BC}{BA} = \frac{BD}{BC}$$

$$BC^2 = BA \cdot BD \qquad ...(2)$$

From (1) and (2)

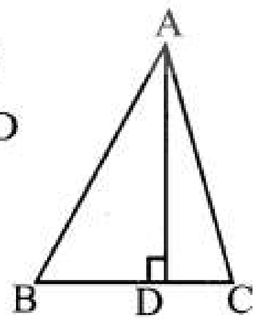
$$\frac{BC^2}{AC^2} = \frac{BA \cdot BD}{AB \cdot AD} = \frac{BD}{AD}$$

Question 10.

The perpendicular from A on side BC at a ΔABC intersects BC at D such that DB = 3 CD. Prove that $2AB^2 = 2AC^2 + BC^2$.

Solution:

We have DB = 3 CD



$$BC = BD + DC$$

$$BC = 3CD + CD$$

$$BC = 4CD$$

$$CD = \frac{1}{4}BC$$

$$CD = \frac{1}{4}BC \text{ and}$$

$$BD = 3CD = \frac{3}{4}BC$$

Since ΔABD is a right triangle (i) right angled at D

$AB^2 – AD^2 + BD^2$ ………… (ii)

By ΔACD is a right triangle right angled at D

$AC^2 = AD^2 + CD^2$ ………… (iii)
Subtracting equation (iii) from equation (ii),
we got

$$AB^2 - AC^2 = BD^2 - CD^2$$

$$\Rightarrow \quad AB^2 - AC^2 = \left(\frac{3}{4}BC\right)^2 - \left(\frac{1}{4}BC\right)^2$$

From (i)

$$CD = \frac{1}{4}\ BC, BD = \frac{3}{4}\ BC)$$

$$\Rightarrow \quad AB^2 - AC^2 = \frac{9}{16}BC^2 - \frac{1}{16}BC^2$$

$$\Rightarrow \quad AB^2 - AC^2 = \frac{1}{2}BC^2$$

$$\Rightarrow 2(AB^2 - AC^2) = BC^2$$

$$\Rightarrow \quad 2AB^2 = 2AC^2 + BC^2$$