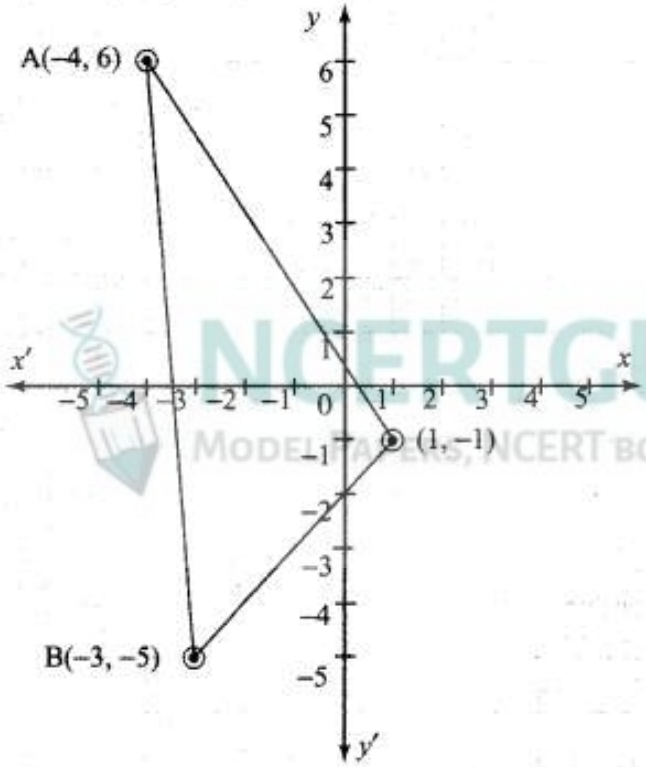
Coordinate Geometry

Ex 5.1

Question 1.
Find the area of the triangle formed by the points
(i) (1, -1), (-4, 6) and (-3, -5)
(ii) (-10, -4), (-8, -1) and (-3, -5)
Solution:

(i) (1, –1), (–4, 6), and (–3, –5,)



$$\begin{array}{lll} A(-4, 6), & B(-3, -5), & C(1, -1) \\ \downarrow & \downarrow & \downarrow \\ (x_1, y_1) & (x_2, y_2) & (x_3, y_3) \end{array}$$

$\therefore$ Area of the ΔABC =

$$\frac{1}{2}[(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3)] \text{ sq. units}$$

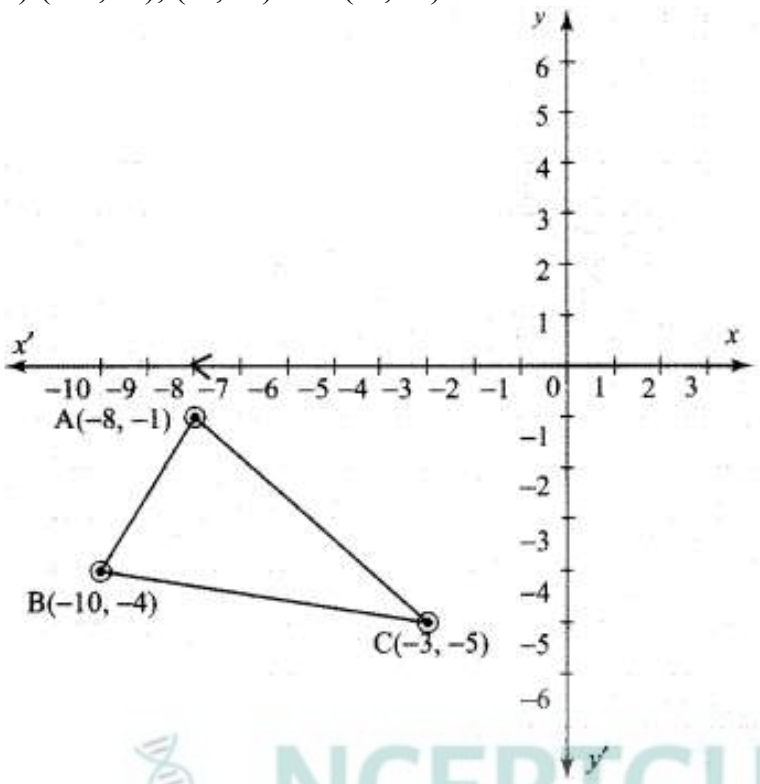
$$= \frac{1}{2}\begin{bmatrix} ((-4\times -5) + (-3\times -1) + (1\times 6)) \\ -((-3\times 6) + (1\times -5) + (-4\times -1)) \end{bmatrix}$$

$$= \frac{1}{2}[(20+3+6) - (-18-5+4)]$$

$$= \frac{1}{2}[29 - (-19)] = \frac{1}{2}[29+19] = \frac{1}{2}\times 48$$

= 24 sq. units

(ii) (-10, -4), (-8, -1) and (-3, -5)



$$\begin{array}{lll} A(-8,-1), & B(-10,-4) & C(-3,-5) \\ \downarrow & \downarrow & \downarrow \\ (x_1, y_1) & (x_2, y_2) & (x_3, y_3) \end{array}$$

$\therefore$ Area of the ΔABC

$$= \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix} \text{ sq. units}$$

$$= \frac{1}{2}\begin{vmatrix} -8 & -10 & -3 & -8 \\ -1 & -4 & -5 & -1 \end{vmatrix}$$

$$= \frac{1}{2}\left[\begin{array}{l} ((-8\times-4)+(-10\times-5)+(-3\times-1)) \\ -((-1\times-10)+(-4\times-3)+(-5\times-8)) \end{array}\right]$$

$$= \frac{1}{2}[(32+50+3)-(10+12+40)]$$

$$= \frac{1}{2}[85-62] = \frac{1}{2}\times 23 = 11.5 \text{ sq. units}$$

Question 2.
Detemine whether the sets of points are collinear ?

(i) $\left(-\frac{1}{2}, 3\right)$**, (–5, 6) and (–8, 8)**

(ii) $(a, b + c)$**,** $(b, c + a)$ **and** $(c, a + b)$

Solution:
(i)

$A\left(\overset{x_1}{-\frac{1}{2}}, \overset{y_2}{3}\right)$, $B(\overset{x_1}{-5}, \overset{y_1}{6})$ and $C(\overset{x_1}{-8}, \overset{y_1}{8})$

Area of $\Delta ABC = \frac{1}{2}\left|\begin{matrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{matrix}\right|$

$= \frac{1}{2}\left|\begin{matrix} -\frac{1}{2} & -5 & -8 & -\frac{1}{2} \\ 3 & 6 & 8 & 3 \end{matrix}\right|$

$= \frac{1}{2}\left[\begin{matrix} \left(\left(-\frac{1}{2}\times 6\right)+(-5\times 8)+(-8\times 3)\right) \\ -\left((3\times -5)+(6\times -8)+\left(8\times\frac{-1}{2}\right)\right) \end{matrix}\right]$

$= \frac{1}{2}[(-3-40-24)-(-15-48-4)]$

$= \frac{1}{2}[-67-(-67)]$

$= \frac{1}{2}[-67 + 67] = 0$ sq. units

$\therefore$ The given points are collinear.

(ii) A $(a, b + c)$, B $(b, c + a)$ and C $(c, a + b)$

$x_1 \quad y_1 \qquad x_2 \quad y_2 \qquad x_3 \quad y_3$

Area of the ΔABC

$$= \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix} \text{ sq.units}$$

$$= \frac{1}{2}\begin{vmatrix} a & b & c & a \\ b+c & c+a & a+b & b+c \end{vmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} (ac + a^2 + ab + b^2 + bc + c^2) \\ -(b^2 + bc + c^2 + ac + a^2 + ab) \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} \cancel{ac} + \cancel{a^2} + \cancel{ab} + \cancel{b^2} + \cancel{bc} + \cancel{c^2} \\ -\cancel{b^2} - \cancel{bc} - \cancel{c^2} - \cancel{ac} - \cancel{a^2} - \cancel{ab} \end{bmatrix}$$

$$= \frac{1}{2}(0) = 0 \text{ sq. units}$$

$\therefore$ The given points are collinear

Question 3.

Vertices of given triangles are taken in order and their areas are provided aside. In each case, find the value of 'p'.

S.No.	Vertices	Area (sq.units)
(i)	(0,0), $(p, 8)$, (6, 2)	20
(ii)	(p, p), (5,6), (5, –2)	32

Solution:

Area 20 sq. units.

$x_1 \quad y_1 \quad x_2 \quad y_2 \quad x_3 \quad y_2$
$A(0 \quad 0)' B(p, \quad 8)' \; C(6 \quad 2)$

Area 20 sq. units.

(i) Area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} 0 & P & 6 & 0 \\ 0 & 8 & 2 & 0 \end{vmatrix} = 20$

$\Rightarrow [(0 + 2p + 0) - (0 + 48 + 0)] = 40$

$2p - 48 = 40$

$2p = 40 + 48 = 88$

$p = 44.$

(ii) A (p, p), B (5, 6), C (5, –2) Area

$x_1 \; y_1 \quad x_2 \; y_2 \quad x_3 \; y_3 \quad 32$

Area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} p & 5 & 5 & p \\ p & 6 & -2 & p \end{vmatrix} = 32$

$\Rightarrow [(6p - 10 + 5p) - (5p + 30 - 2p)] = 64$

$\Rightarrow (11p - 10 - 3p - 30) = 64$

$\Rightarrow 8p - 40 = 64$

8p = 104
p = 13

Question 4.
In each of the following, find the value of 'a' for which the given points are collinear.
(i) (2, 3), (4, a) and (6, -3)
(ii) (a, 2 – 2a), (-a + 1, 2a) and(-4 – a,6 – 2a)
Solution:

(i) A(2, 3), B(4, a), and C(6, –3)

$x_1\ y_1 \qquad x_2\ y_2 \qquad x_3\ \ y_3$

Area of the triangle ΔABC

$$= \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix} \text{ sq. units}$$

$$\Rightarrow \quad \frac{1}{2}\begin{vmatrix} 2 & 4 & 6 & 2 \\ 3 & a & -3 & 3 \end{vmatrix} = 0$$

($\because$ the points are collinear).

$$\begin{aligned} [(2a - 12 + 18) - (12 + 6a - 6)] &= 0 \\ (2a + 6) - (6 + 6a) &= 0 \\ 2a + 6 - 6 - 6a &= 0 \\ -4a &= 0 \\ a &= 0 \end{aligned}$$

(ii) A (a, 2 – 2a), B (–a + 1, 2a), and C (–4 – a, 6 – 2a)

$x_1 \quad y_1 \qquad x_2 \quad y_2 \qquad\qquad x_3 \quad y_3$

Area of the ΔABC

$$= \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix} \text{ sq. units}$$

$$= \frac{1}{2}\begin{vmatrix} a & -a+1 & -4-a & a \\ 2-2a & 2a & 6-2a & 2-2a \end{vmatrix} = 0$$

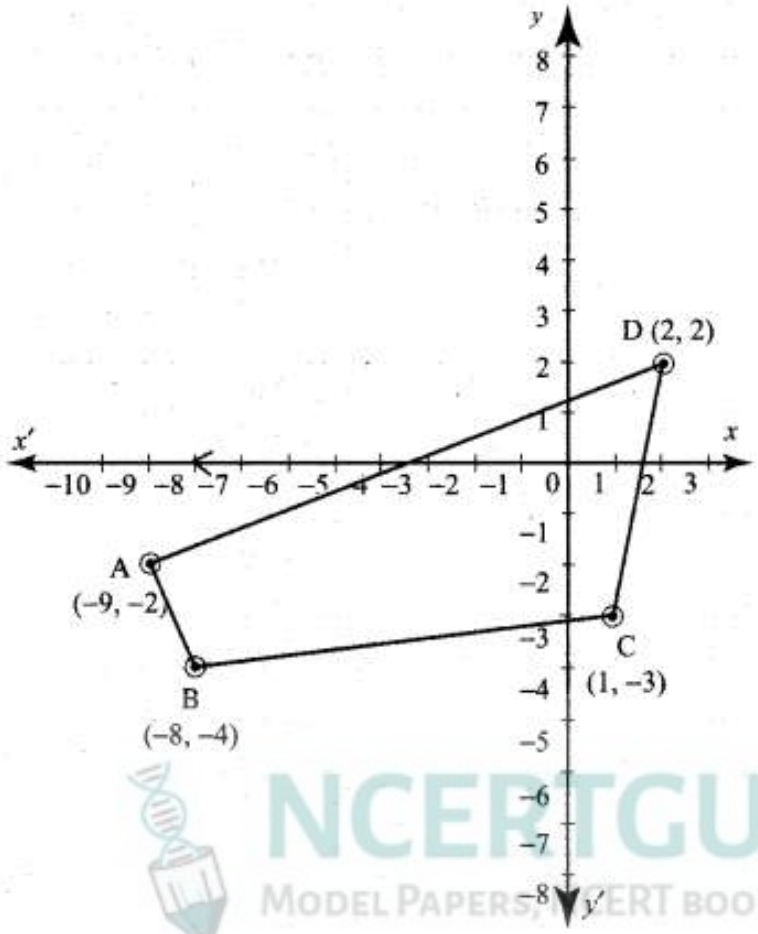
$$[(2a^2 + (-a + 1) \times (6 - 2a) + (-4 - a) \times (2 - 2a))$$
$$- ((2 - 2a)(-a + 1) + 2a(-4 - a) + a(6 - 2a))] = 0$$

$$\Rightarrow \quad 2a^2 - 6a + 6 + 2a^2 - 2a + (-8) - 2a + 8a + 2a^2$$
$$-[-2a + 2a^2 + 2 - 2a - 8a - 2a^2 + 6a - 2a^2] = 0$$

$$\Rightarrow \cancel{2a^2} - 6a + 6 + 2a^2 - \cancel{2a} - 8 - \cancel{2a} + 8a + 2a^2 + \cancel{2a}$$
$$- \cancel{2a^2} - 2 + \cancel{2a} + 8a + 2a^2 - 6a + 2a^2 = 0$$

$$8a^2 + 4a - 4 = 0$$
$$2a^2 + a - 1 = 0$$
$$2a^2 + 2a - a - 1 = 0$$
$$2a(a + 1) - 1(a + 1) = 0$$
$$\Rightarrow \quad (2a - 1)(a + 1) = 0$$
$$2a - 1 = 0$$
$$\Rightarrow \quad a = \frac{1}{2},$$
$$a + 1 = 0$$
$$\Rightarrow \quad a = -1$$

Question 5.
Find the area of the quadrilateral whose vertices are at
(i) (-9, -2), (-8, -4), (2, 2) and (1, -3)
(ii) (-9, 0), (-8, 6), (-1, -2) and (-6, -3)
Solution:
(i) (-9, -2), (-8, -4), (2, 2), and (1, -3)



$\underset{x_1\ \ y_1}{A(-9,\ -2)}$, $\underset{x_2\ \ y_2}{B(-8,\ -4)}$, $\underset{x_3\ \ y_3}{C(1,\ -3)}$,
$\underset{x_4\ \ y_4}{D(2,\ 2)}$

Area of the quadrilateral

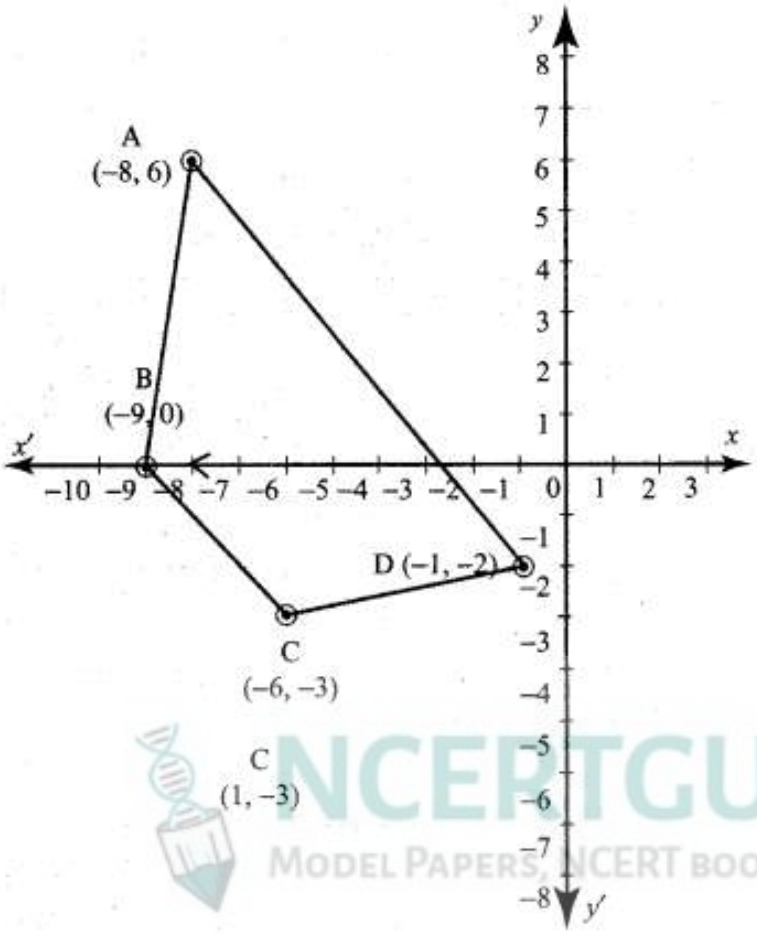
$$= \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{vmatrix} \text{ sq. units}$$

$$= \frac{1}{2}\begin{vmatrix} -9 & -8 & 1 & 2 & -9 \\ -2 & -4 & -3 & 2 & -2 \end{vmatrix} \text{ sq. units.}$$

$$= \frac{1}{2}[(36+24+2-4)-(16-4-6-18)]$$

$$= \frac{1}{2}[58-(-12)] = \frac{1}{2}(70) = 35 \text{ sq. units}$$

(ii) (-9, 0), (-8, 6), (-1, -2) and (-6, -3)



$$\overset{x_1}{A}(-8,\ \overset{y_1}{6}),\ \overset{x_2}{B}(-9\ \overset{y_2}{0}),\ \overset{x_3}{C}(-6,\ \overset{y_3}{-3})$$

$$\overset{x_4}{D}(-1\ \overset{y_4}{-2})$$

Area of the quadrilateral ABCD

$$= \frac{1}{2}\begin{vmatrix} -8 & -9 & -6 & -1 & -8 \\ \times & \times & \times & \times & \\ 6 & 0 & -3 & -2 & 6 \end{vmatrix} \text{ sq. units}$$

$$= \frac{1}{2}[(0+27+12-6)-(-54+0+3+16)]\text{sq. units}$$

$$= \frac{1}{2}(33-(-35)) = \frac{1}{2}(33+35) = \frac{68}{2} = 34 \text{ sq. units}$$

Question 6.
Find the value of k, if the area ofa quadrilateral is 28 sq.units, whose vertices are (-4, -2), (-3, k), (3, -2) and (2, 3)
Solution:

$$\begin{array}{cccccc} (-4,-2), & (-3,k) & (3,-2) & \text{and} & (2,3) \\ \downarrow & \downarrow & \downarrow & & \downarrow \\ (x_1,y_1) & (x_2,y_2) & (x_3,y_3) & & (x_4,y_4) \end{array}$$

Area of the quadrilateral

$$= \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{vmatrix} \text{ sq. units}$$

$$= \frac{1}{2}\begin{vmatrix} -4 & -3 & 3 & 2 & -4 \\ -2 & k & -2 & 3 & -2 \end{vmatrix} = 28 \text{ sq. units}$$

$$\Rightarrow [(-4k + 6 + 9 - 4) - (6 + 3k - 4 - 12)] = 56$$

$$\Rightarrow 11 - 4k - (-10 + 3k) = 56$$

$$\Rightarrow -7k + 21 = 56$$

$$-7k = 56 - 21 = 35$$

$$k = -5$$

Question 7.
If the points A(-3, 9) , B(a, b) and C(4,-5) are collinear and if a + b = 1, then find a and b.
Solution:

$$\begin{array}{lll} A(-3, 9), & B(a, b) & C(4, -5), \\ \downarrow & \downarrow & \downarrow \\ (x_1, y_1) & (x_2, y_2) & (x_3, y_3) \end{array}$$

are collinear points, $a + b = 1$ (given)

$\therefore$ Area of the Δ

$$= \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix} \text{ sq. units}$$

$$= \frac{1}{2}\begin{vmatrix} -3 & a & 4 & -3 \\ 9 & b & -5 & 9 \end{vmatrix} = 0$$

($\because$ points are collinear)

$$\begin{aligned} (-3b - 5a + 36) - (9a + 4b + 15) &= 0 \\ (-3b - 4b) + (-5a - 9a) + (36 - 15) &= 0 \\ -7b - 14a &= -21 \\ -7(b + 2a) &= -21 \\ b + 2a &= 3 \\ (b + a) + a &= 3 \\ 1 + a &= 3 \\ a = 2 \Rightarrow b = 1 - 2 &= -1 \end{aligned}$$

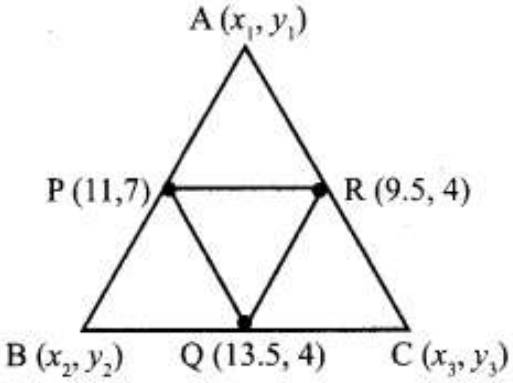
$$\boxed{\begin{aligned} a &= 2 \\ b &= -1 \end{aligned}}$$

Question 8.

Let P(11, 7), Q(13.5, 4) and R(9.5, 4) be the mid-points of the sides AB, BC and AC respectively of ΔABC . Find the coordinates of the vertices A, B and C. Hence find the area of ΔABC and compare this with area of ΔPQR.

Solution:

p (11, 7), Q (13.5, 4), and R (9.5, 4) are the mid points of the sides of a ΔABC.



$$\text{Mid point } (x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\therefore \text{ Mid point of AB } = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = (11, 7)$$

$$\Rightarrow \quad \frac{x_1 + x_2}{2} = 11$$

$$\Rightarrow \quad x_1 + x_2 = 22 \quad \ldots(1),$$

$$\frac{y_1 + y_2}{2} = 7$$

$\Rightarrow$ $y_1 + y_2 = 14$ ...(2)

$$\text{Mid point of BC} = \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$$

$$= (13.5, 4)$$

$\Rightarrow$ $\frac{x_2 + x_3}{2} = 13.5$

$\Rightarrow$ $x_2 + x_3 = 27.0$ ...(3)

$\Rightarrow$ $\frac{y_2 + y_3}{2} = 4$

$\Rightarrow$ $y_2 + y_3 = 8$ ...(4)

$$\text{Mid point AC} = \left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2}\right)$$

$$= (9.5, 4)$$

$\Rightarrow$ $\frac{x_1 + x_3}{2} = 9.5$

$x_1 + x_3 = 19.0$ ...(5)

$$\frac{y_1 + y_3}{2} = 4$$

$y_1 + y_3 = 8$ ...(6)

(1) + (3) + (5) $\rightarrow 2(x_1 + x_2 + x_3) = 68$

$x_1 + x_2 + x_3 = 34$ ...(7)

(7) – (1) $\Rightarrow$ $x_3 = 34 - 22 = 12$

(7) – (3) $\Rightarrow$ $x_1 = 34 - 27 = 7$

(7) – (5) $\Rightarrow$ $x_2 = 34 - 19 = 15$

(2) + (4) + (6) $\rightarrow 2(y_1 + y_2 + y_3) = 30$

$\Rightarrow \quad y_1 + y_2 + y_3 = 15 \qquad \ldots(8)$

$(8) - (2) \rightarrow y_3 = 15 - 14 = 1$

$(8) - (4) \rightarrow y_1 = 15 - 8 = 7$

$(8) - (6) \rightarrow y_2 = 15 - 8 = 7$

$\therefore$ The vertices of the ΔABC are

A(7, 7) B (15, 7) C(12, 1) $\Rightarrow$ A(7, 7) B (12, 1)
C (15, 7)

$$\therefore \quad \text{Area of } \Delta ABC = \frac{1}{2}\begin{vmatrix} 7 & 12 & 15 & 7 \\ 7 & 1 & 7 & 7 \end{vmatrix}$$

$$= \frac{1}{2}\left[(7+84+105)-(84+15+49)\right]$$

$$= \frac{1}{2}[196-148]$$

$= \frac{1}{2}[48] = 24$ sq. units.

Area of $\Delta PQR = \frac{1}{2}\begin{vmatrix} 9.5 & 13.5 & 11 & 9.5 \\ 4 & 4 & 7 & 4 \end{vmatrix}$

$= \frac{1}{2}\left[(38+94.5+44)-(54+44+66.5)\right]$

$= \frac{1}{2}[176.5-164.5] = \frac{1}{2}[12] = 6$ sq. units

Area of $\Delta ABC = 4 \times$ Area of ΔPQR.

$\therefore$ The vertices of the ΔABC are

$A(x_1, y_1) = (6, 7)$

$B(x_2, y_2) = (14, 7)$

$C(x_3, y_3) = (13, 1)$

$\therefore$ Area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} 6 & 14 & 13 & 6 \\ 7 & 7 & 1 & 7 \end{vmatrix}$ sq. units

$= \frac{1}{2}[(42+14+91)-(98+91+6)]$

$= \frac{1}{2}(147-195)$

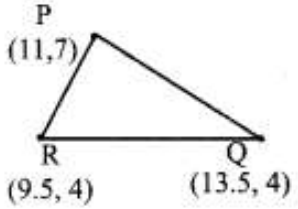
$= \frac{1}{2}(-48) = -24 = 24$ sq. units

(Area cannot be (–ve). Area is always (+ve))

$\therefore$ Area $= 24$ sq. units.

Area of ΔPRQ

$= \frac{1}{2}\begin{vmatrix} 11 & 9.5 & 13.5 & 11 \\ 7 & 4 & 4 & 7 \end{vmatrix}$



$$= \frac{1}{2}[(44+38+94.5)-(66.5+54+44)]$$

$$= \frac{1}{2}(176.5-164.5)$$

$$= \frac{1}{2}\times(-12)-6=6 \text{ sq. units}$$

(Area cannot be (–ve). Area is always (+ve))

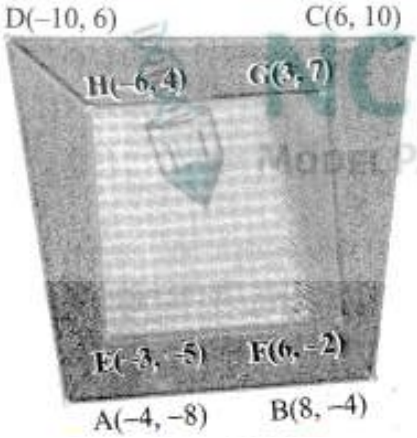
$\therefore$ Area of ΔPRQ = 6 sq. units

Area of ΔABC = 24 sq. units

$\therefore$ Area of ΔABC = 4 × Area of ΔPRQ

Question 9.

In the figure, the quadrilateral swimming pool shown is surrounded by concrete patio. Find the area of the patio.



Solution:

Area of the patio = Area of the quadrilateral ABCD – Area of the swimming pool EFGFI.

Area of the quadrilateral ABCD

$$= \frac{1}{2}\begin{vmatrix} -4 & 8 & 6 & -10 & -4 \\ -8 & -4 & 10 & 6 & -8 \end{vmatrix} \text{ sq. units}$$

$$= \frac{1}{2}[(16+80+36+80)-(-64-24-100-24)]$$

$$= \frac{1}{2}[212+212\} = 212$$

Area of the swimming pool EFGH

$$= \frac{1}{2}\begin{vmatrix} -3 & 6 & 3 & -6 & -3 \\ -5 & -2 & 7 & 4 & -5 \end{vmatrix}$$

$$= \frac{1}{2}[(6+42+12+30)-(-30-6-42-12)]$$

$$= \frac{1}{2}(90-(-90)) = 90$$

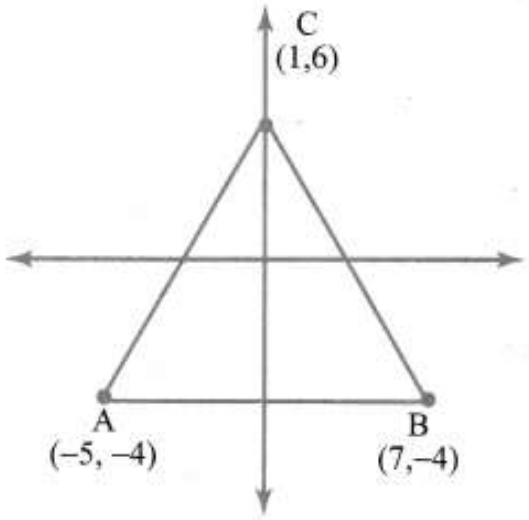
$\therefore$ Area of the patio $= 212 - 90$

$= 122$ sq. units

Question 10.

A triangular shaped glass with vertices at A(-5, -4), B(1, 6) and C(7, -4) has to be painted.
If one bucket of paint covers 6 square feet, how many buckets of paint will be required to paint the whole glass, if only one coat of paint is applied.

Solution:



$$\text{Area of } \Delta ABC = \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix} \text{ sq. units}$$

$$= \frac{1}{2}\begin{vmatrix} -5 & 7 & 1 & -5 \\ -4 & -4 & 6 & -4 \end{vmatrix}$$

$$= \frac{1}{2}[(20+42-4)-(-28-4-30)]$$

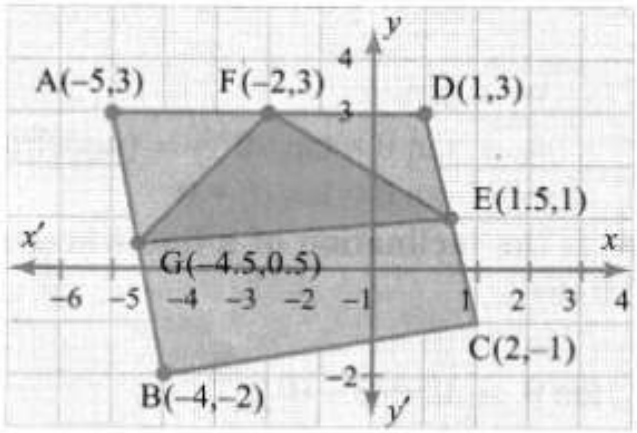
$$= \frac{1}{2}[(62-4)-(-62)] = \frac{1}{2}\times(58+62) = \frac{1}{2}\times 120$$

$$= 60 \text{ sq. units}$$

No. of paint cans required = 10

$$\frac{\text{Area of the } \Delta \text{ given}}{\text{Area of the paints cans}} = \frac{60}{6} = 10 \text{ cans}$$

Question 11.
In the figure, find the area of
(i) triangle AGF
(ii) triangle FED
(iii) quadrilateral BCEG
Solution:



(i) $A(-5, 3),\ G(-4.5, 0.5),\ F(-2, 3)$

$\downarrow \qquad \downarrow \qquad \downarrow$

$(x_1, y_1) \quad (x_2, y_2) \quad (x_3, y_3)$

$$\text{Area of } \Delta AGF = \frac{1}{2}\begin{vmatrix} -5 & -4.5 & -2 & -5 \\ 3 & 0.5 & 3 & 3 \end{vmatrix}$$

$$= \frac{1}{2}[(-2.5 - 13.5 - 6) - (-13.5 - 1.0 - 15)]$$

$$= \frac{1}{2}[(-22) - (-29.5)] = \frac{1}{2}(-22 + 29.5)$$

$$= \frac{1}{2}(7.5) = 3.75 \text{ sq. units}$$

(ii) Area of ΔFED

$$= \frac{1}{2}\begin{vmatrix} -2 & 1.5 & 1 & -2 \\ 3 & 1 & 3 & 3 \end{vmatrix}$$

$$= \frac{1}{2}[(-2 + 4.5 + 3) - (4.5 + 1 - 6)$$

$$= \frac{1}{2}(5.5 + 0.5) = \frac{1}{2} \times 6 \text{ sq. units}$$

$$= 3 \text{ sq. units}$$

(iii) Area of quadrilateral BCEG.

$$= \frac{1}{2}\begin{vmatrix} -4 & 2 & 1.5 & -4.5 & -4 \\ -2 & -1 & 1 & 0.5 & -2 \end{vmatrix}$$

$$= \frac{1}{2}[(4+2+0.75+9)-(-4-1.5-4.5-2.0)]$$

$$= \frac{1}{2}[15.75-(-12)] = \frac{1}{2}[15.75+12]$$

$$= \frac{1}{2} \times 27.75$$

$$= 13.88 \text{ sq. units}$$

Ex 5.2

Question 1.
What is the slope of a line whose inclination with positive direction of x -axis is
(i) 90°
(ii) 0°
Solution:
(i) $\theta = 90°$
$m = \tan\theta = \tan 90° = \propto$ (undefined)
(ii) $m = \tan\theta = \tan 0° = 0$

Question 2. What is the inclination of a line whose
slope is (i) 0 Solution:
(i) Slope = 0
$\tan\theta = 0$
$\tan 0 = 0$
$\therefore \theta = 0°$

(ii) Slope = 1
$\tan\theta = 1$
$\tan 45° = 1$
$\therefore \theta = 45°$
angle of inclination is 45°

Question 3.
Find the slope of a line joining the points
(i) $(5, \sqrt{5})$ with origin
(ii) (sin θ, -cos θ) and (-sin θ, cos θ)
(i) $(5, \sqrt{5})$) with origin (0, 0)

Solution:

If two points are given, slope $m = \frac{y_2 - y_1}{x_2 - x_1}$

$(5, \sqrt{5})$, $(0, 0)$ $\quad \therefore m = \frac{0-\sqrt{5}}{0-5} = \frac{\sqrt{5}}{5} = \frac{1}{\sqrt{5}}$

$x_1 \quad y_1 \quad x_2 \quad y_2$

(ii) $(\sin\theta, -\cos\theta)$ and $(-\sin\theta, \cos\theta)$

$\downarrow \qquad\qquad \downarrow$

$(x_1 \ , \ y_1) \qquad (x_2 \ , \ y_2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\cos\theta - (-\cos\theta)}{-\sin\theta - \sin\theta}$$

$$= \frac{2\cos\theta}{-2\sin\theta} = -\cot\theta$$

Question 4.

What is the slope of a line perpendicular to the line joining A(5, 1) and P where P is the mid-point of the segment joining (4, 2) and (-6,4).

Solution:

P is the mid point of the segment joining (4, 2) and (-6, 4)

$$P(x, y) = \left(\frac{4+(-6)}{2}, \frac{2+4}{2}\right) = (-1, 3)$$

$x_1 \ y_1 \qquad x_2 \ y_2$

A (5, 1), P (–1, 3).

$$\text{Slope of AP} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-1}{-1-5} = \frac{2}{-6} = \frac{-1}{3}$$

$$\text{Slope of the line } \perp^r \text{ to AP} = \frac{-1}{\text{slope of AP}} = \frac{-1}{\frac{-1}{3}} = 3$$

Question 5.

Show that the given points are collinear (-3, -4), (7, 2) and (12, 5)

Solution:

The verticles are A(-3, -4), B(7, 2) and C (12, 5)

$$\text{Slope of AB} = \frac{2-(-4)}{7-(-3)} = \frac{6}{10} = \frac{3}{5}$$

$$\text{Slope of BC} = \frac{5-2}{12-7} = \frac{3}{5}$$

Slope of AB = Slope of BC
∴ The points A, B and C lie on the same line.
∴ They are collinear.

Question 6.
If the three points (3, -1), (a, 3), (1, -3) are collinear, find the value of a.
Solution:
Slope of AB = slope of BC.

$$\frac{3-(-1)}{a-3} = \frac{-3-3}{1-a}$$

$$\frac{4}{a-3} = \frac{-6}{1-a} \Rightarrow 4(1-a) = (a-3)(-6)$$

$$4 - 4a = -6a + 18$$

$$2a = 18 - 4 = 14$$

$$a = 7$$

Question 7.
The line through the points (-2, a) and (9, 3) has slope $-\frac{1}{2}$. Find the value of a.
Solution:
A line joining the points (-2, a) and (9, 3) has slope m = $-\frac{1}{2}$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-a}{9-(-2)} = \frac{-1}{2}$$

$$2(3-a) = -1(11) \Rightarrow -2a = -11 - 6 = -17$$

$$a = \frac{17}{2}.$$

Question 8.
The line through the points (-2, 6) and (4, 8) is perpendicular to the line through the points (8, 12) and (x, 24) . Find the value of x.

Solution:

The line through the points A (–2, 6), and B (4, 8)

$$\text{Slope of AB } (m_1) = \frac{8-6}{4-(-2)} = \frac{2}{6} = \frac{1}{3}$$

The line through the points C(8, 12) and D(x, 24)

$$\text{Slope of CD } (m_2) = \frac{24-12}{x-8} = \frac{12}{x-8}$$

$$\text{AB} \perp^r \text{CD} \Rightarrow m_1 \times m_2 = -1$$

$$\Rightarrow \quad \frac{1}{3} \times \frac{12^4}{x-8} = -1$$

$$\Rightarrow \quad 4 = -1 \times (x-8)$$

$$\Rightarrow \quad 4 = 8 - x$$

$$\Rightarrow \quad x = 8 - 4 \Rightarrow x = 4$$

Question 9.

Show that the given points form a right angled triangle and check whether they satisfies pythagoras theorem

(i) A(1, -4), B(2, -3) and C(4, -7)

(ii) L(0, 5), M(9, 12) and N(3, 14)

Solution:

Slope of AB $= \frac{-3-(-4)}{2-1} = \frac{1}{1} = 1$

Slope of BC $= \frac{-7-(-3)}{4-2} = \frac{-3}{2}$

Slope of AC $= \frac{-7-(-4)}{4-1} = \frac{-7+4}{3} = \frac{-3}{3} = -1$

Slope of AB × slope of AC = $1 \times -1 = -1$

∴ **Sol :** yes. AB $\perp^r$ to AC

By Pythagoras theorem

$AB^2 + AC^2 = BC^2$.

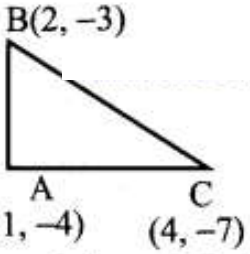
$AB = \sqrt{(-3-(-4))^2 + (2-1)^2} = \sqrt{1^2+1^2} = \sqrt{2}$

$AC = \sqrt{(-7-(-4))^2 + (4-(1))^2} = \sqrt{(-3)^2+3^2}$

$= \sqrt{9+9} = \sqrt{18}$

$BC = \sqrt{(-7-(-3))^2 + (4-2)^2} = \sqrt{-4^2+2^2}$

$= \sqrt{16+4} = \sqrt{20}$



By Pythagoras theorem

$BC^2 = AB^2 + AC^2$

$\sqrt{20}^2 = \sqrt{2}^2 + \sqrt{18}^2$

$20 = 2 + 18 = 20$ ∴ Hence it is satisfied

(ii) Slope of LM $= \frac{12-5}{9-0} = \frac{7}{9}$

Slope of MN $= \frac{14-12}{3-9} = \frac{2}{-6} = \frac{1}{-3}$

Slope of LN $= \frac{14-5}{3-0} = \frac{\cancel{9}^{3}}{\cancel{3}} = 3$

Slope of MN × slope of LN $= \frac{-1}{3} \times 3 = -1$

Yes; MN $\perp^r$ to LN.

$\therefore$ L, M, N form a right angled triangle.

$LM = \sqrt{(12-5)^2 + (9-0)^2} = \sqrt{7^2 + 9^2} = \sqrt{49+81} = \sqrt{130}$

$MN = \sqrt{(14-12)^2 + (3-9)^2} = \sqrt{2^2 + (-6)^2} = \sqrt{4+36} = \sqrt{40}$

$LN = \sqrt{(14-5)^2 + (3-0)^2} = \sqrt{9^2 + 3^2} = \sqrt{81+9} = \sqrt{90}$

By Pythagoras theorem

$LM^2 = MN^2 + LN^2$

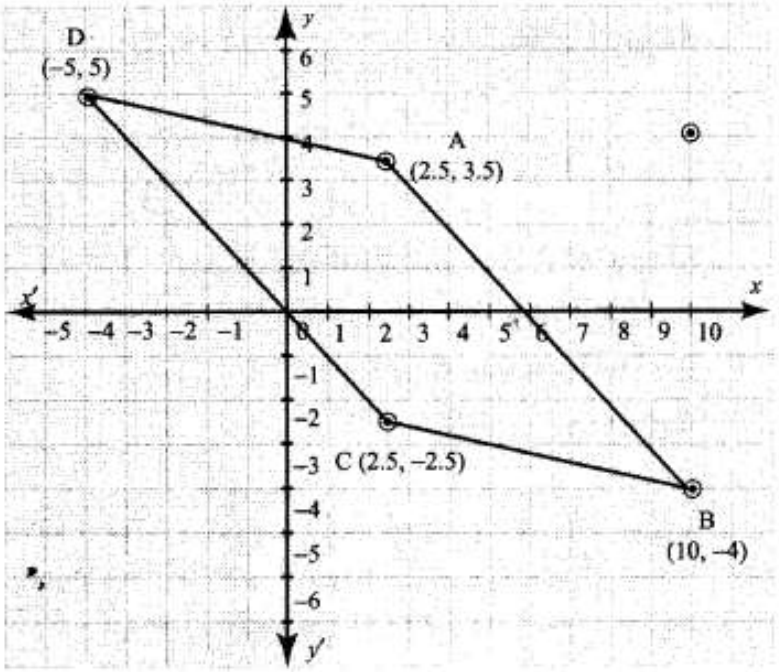
$\sqrt{130}^2 = \sqrt{40}^2 + \sqrt{90}^2 \Rightarrow 130 = 40 + 90.$

Hence it is satisfied.

Question 10.

Show that the given points form a parallelogram : A(2.5, 3.5), B(10, -4), C(2.5, -2.5) and D(-5, 5)

Solution:



$$\text{Slope of AD}(m_1) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 3.5}{-5 - 2.5} = \frac{1.5}{-7.5}$$

$$\text{A}(2.5, 3.5)\quad \text{D}(5,5) \quad = \frac{0.1 \times 10}{0.5 \times 10} = \frac{-1}{5}$$

B (10, –4)

C (2.5, – 2.5)

$\therefore m_1 = m_2 \quad \therefore$ AD || BC ...(1)

Slope of AB $(m_3) = \dfrac{-4-3.5}{10-2.5} = \dfrac{-7.5}{7.5} = -1.$

A (2.5, 3.5)

B (10, –4)

Slope of CD $(m_4) = \dfrac{5-(-2.5)}{-5-2.5} = \dfrac{7.5}{-7.5} = -1$

C (2.5, –2.5)

D (–5, 5)

$m_3 = m_4. \quad \therefore$ AB || CD. ...(2)

From (1) and (2), the opposite sides of the quadrilateral are parallel to each other.

Mid point of AC $= \left(\dfrac{2.5+2.5}{2}, \dfrac{3.5-2.5}{2}\right)$

$= (2.5, .5)$

& mid point of BD $= \left(\dfrac{10-5}{2}, \dfrac{-4+5}{2}\right)$

$= (2.5, .5)$ [$\because$ mid point of AC = mid point of BD]

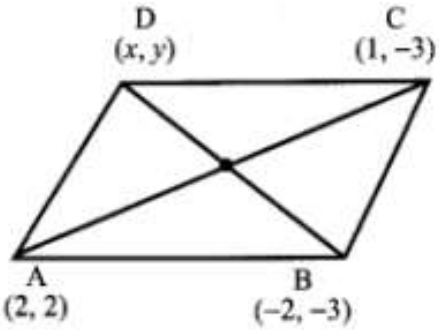
$\therefore$ The given points form a parallelogram.

Question 11.

If the points A(2, 2), B(-2, -3), C(1, -3) and D(x, y) form a parallelogram then find the value of x and y.

Solution:

A(2, 2), B(-2, -3), C(1, -3), D(x, y)



Since ABCD forms a parallelogram, slope of opposite sides are equal and diagonals bisect each other.

Mid point of BD = Mid point of AC

$$\left(\frac{x+(-2)}{2}, \frac{y+(-3)}{2}\right) = \left(\frac{2+1}{2}, \frac{2+(-3)}{2}\right)$$

$$\frac{x-2}{2} = \frac{3}{2} \qquad \frac{y-3}{2} = \frac{2-3}{2}$$

$$x-2 = 3 \qquad y-3 = -1$$

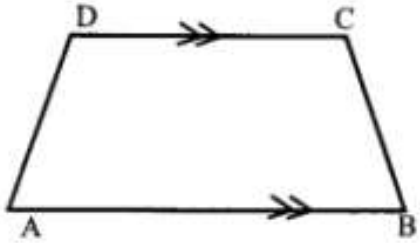
$$\boxed{x = 5} \qquad \boxed{y = 2}$$

Question 12.

Let A(3, -4), B(9, -4), C(5, -7) and D(7, -7). Show that ABCD is a trapezium.

Solution:

A (3, -4), B (9, -4), C (5, -7) and D (7, -7)



If only one pair of opposite sides of a quadrilateral are parallel, then it is said to be a trapezium.

$$\therefore \text{Slope of AB } (m_1) = \frac{-4-(-4)}{9-3} = \frac{0}{6} = 0$$

$$\text{Slope of CD } (m_2) = \frac{-7-(-7)}{7-5} = \frac{-7+7}{+2} = \frac{0}{+2} = 0$$

$$\text{Slope of BC } (m_3) = \frac{-7-(-4)}{5-9} = \frac{-7+4}{-4} = \frac{\not{-}3}{\not{-}4}$$

$$= \frac{3}{4}$$

$$\text{Slope of AD } (m_4) = \frac{-7-(-4)}{7-3} = \frac{-7+4}{4} = \frac{-3}{4}$$

$$m_1 = m_2$$

$$m_3 \neq m_4$$

∴ One pair of opposite sides are parallel.

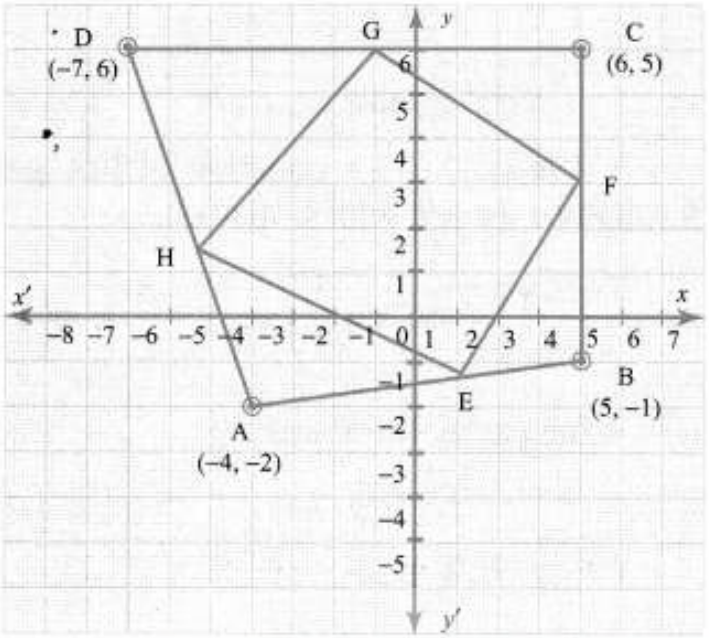
∴ ABCD is a trapezium.

Question 13.

A quadrilateral has vertices at A(- 4, -2), B(5, -1) , C(6, 5) and D(-7, 6). Show that the mid-points of its sides form a parallelogram

Solution:

Mid point of AB= $\left(\frac{-4+5}{2}, \frac{-2-1}{2}\right) = E\left(\frac{1}{2}, \frac{-3}{2}\right)$



Mid point of BC = $\left(\frac{5+6}{2}, \frac{-1+5}{2}\right) = F\left(\frac{11}{2}, \frac{4}{2}\right)$

Mid point of CD = $\left(\frac{6+(-7)}{2}, \frac{5+6}{2}\right) = G\left(\frac{-1}{2}, \frac{11}{2}\right)$

Mid point of AD = $\left(\frac{-4-7}{2}, \frac{-2+6}{2}\right) = H\left(\frac{-11}{2}, \frac{4}{2}\right)$

$\therefore$ Slope of EF = $\left(\frac{\frac{4}{2}-\left(\frac{-3}{2}\right)}{\frac{11}{2}-\frac{1}{2}}\right) = \frac{\frac{4+3}{2}}{\frac{10}{2}} = \frac{7}{10}$

Slope of FG = $\left(\frac{\frac{11}{2}-\frac{2}{1}}{\frac{-1}{2}-\frac{11}{2}}\right) = \left(\frac{\frac{11-4}{2}}{\frac{-12}{2}}\right) = \frac{7}{-12}$

$$\text{Slope of GH} = \left(\frac{2-\frac{11}{2}}{\frac{-11}{2}-\left(\frac{-1}{2}\right)}\right) = \left(\frac{\frac{4-11}{2}}{\frac{-10}{2}}\right)$$

$$= \frac{+7}{10}$$

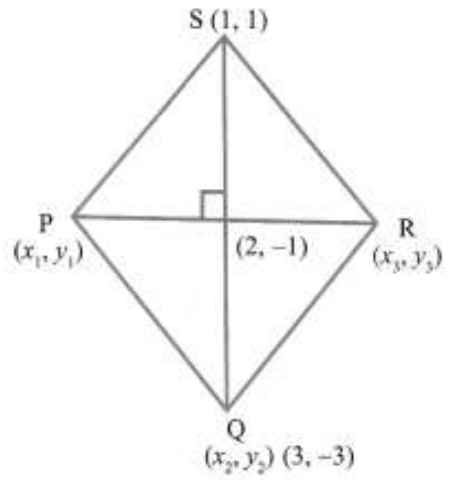
$$\text{Slope of HE} = \left(\frac{2-\left(\frac{-3}{2}\right)}{\frac{-11}{2}-\frac{1}{2}}\right) = \left(\frac{\frac{4+3}{2}}{\frac{-12}{2}}\right) = \frac{7}{-12}$$

$$\left(\frac{\frac{1}{2}+\left(\frac{-1}{2}\right)}{2}, \frac{\frac{-3}{2}+\frac{11}{2}}{2}\right) = \left(\frac{\frac{-11}{2}+\frac{11}{2}}{2}, \frac{\frac{4}{2}+\frac{4}{2}}{2}\right)$$

$$\left(0, \frac{8}{2}\right) = (0, 4)$$

In a parallelogram diagonals bisect each other. Opposite sides are parallel as their slopes are equal the mid points of the diagonals are the same.
∴ Mid points of the sides of a quadrilateral form a parallelogram.

Question 14.
PQRS is a rhombus. Its diagonals PR and QS intersect at the point M and satisfy QS =2PR. If the coordinates of S and M are (1, 1) and (2, -1) respectively, find the coordinates of P.
Solution:



M is the mid point of QS.

$$\frac{x_2 + 1}{2} = 2 \Rightarrow x_2 = 3$$

$$\frac{y_2 + 1}{2} = -1 \Rightarrow y_2 = -3$$

$$\Rightarrow \quad Q = (3, -3)$$

$$\therefore \frac{x_1 + x_3}{2} = 2 \Rightarrow x_1 + x_3 = 4 \qquad ...(1)$$

$$\frac{y_1 + y_3}{2} = -1 \Rightarrow y_1 + y_3 = -2 \qquad ...(2)$$

$$\left(\frac{y_3 - y_1}{x_3 - x_1}\right) \times \left(\frac{-3-1}{3-1}\right) = -1$$

$$(\because m_1 \times m_2 = -1\}$$

$$\Rightarrow \frac{y_3 - y_1}{x_3 - x_1} = \frac{1}{2} \Rightarrow x_3 - x_1 = 2(y_3 - y_1) \qquad ...(3)$$

$$\therefore \text{QS} = 2\text{PR}$$

$$\text{QS} = \sqrt{(-3-1)^2 + (3-1)^2}$$

$$= \sqrt{(-4)^2 + (2)^2} = \sqrt{20}$$

$$\Rightarrow \text{PR} = \frac{\sqrt{20}}{2}$$

$$\Rightarrow \quad \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2} = \frac{\sqrt{20}}{2}$$

from (3),

$$\Rightarrow \sqrt{[2(y_3 - y_1)]^2 + (y_3 - y_1)^2} = \frac{\sqrt{20}}{2}$$

$$\Rightarrow \sqrt{5(y_3 - y_1)^2} = \frac{\sqrt{20}}{2}$$

$$\Rightarrow (y_3 - y_1) \times \sqrt{5} = \frac{\sqrt{5} \times \sqrt{4}}{2}$$

$$\Rightarrow y_3 - y_1 = 1 \rightarrow (4)$$

$$\therefore x_3 - x_1 = 2 \rightarrow (5)$$

Solving (4) and (2),

$(4) + (2) \Rightarrow 2y_3 = -1 \Rightarrow y_3 = \frac{-1}{2}$

$(4) - (2) \Rightarrow -2y_1 = 3 \Rightarrow y_1 = \frac{-3}{2}$

Solving (5) and (1),

$(5) + (1) \Rightarrow 2x_3 = 6 \Rightarrow x_3 = 3$

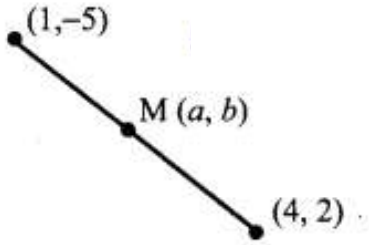
$(5) - (1) \Rightarrow -2x_1 = -2 \Rightarrow x_1 = +1$

$\therefore \mathrm{P} = \left(+1, \frac{-3}{2}\right)$ or

If $y_3 - y_1 = -1$ from (4), we get $p = (3, \frac{-1}{2})$

Ex 5.3

Question 1.
Find the equation of a straight line passing through the mid-point of a line segment joining the points(1, -5) (4, 2) and parallel to
(i) X axis
(ii) Y axis
Solution:



$$\therefore a = \frac{1+4}{2} \qquad b = \frac{-5+2}{2}$$

$$\Rightarrow \quad a = \frac{5}{2} \text{ and } \quad b = \frac{-3}{2}$$

$y - b = m\,(x - a) \rightarrow$ equation of line passing through (a, b) and with slope 'm'.

$$\Rightarrow y-\left(\frac{-3}{2}\right) = m\left(x-\frac{5}{2}\right) \Rightarrow y+\frac{3}{2} = m\left(x-\frac{5}{2}\right)$$

(i) Line parallel to x axis:

$$\text{is } y = c$$

$$y + \frac{3}{2} = 0 \Rightarrow 2x + 3 = 0$$

(ii) Line parallel to y axis

$$\text{is } \quad x = c$$

$$x - \frac{5}{2} = 0$$

$$\Rightarrow \quad 2x - 5 = 0$$

Question 2.
The equation of a straight line is 2(x – y) + 5 = 0. Find its slope, inclination and intercept on the Y axis.
Solution:
2(x – y) + 5 = 0
⇒ 2x – 2y + 5 =

$\Rightarrow 2y = 2x + 5$

$\Rightarrow \quad y = x + \frac{5}{2} \qquad \therefore$ slope $= 1$

Inclination, $m = 1$

$\Rightarrow \quad \tan\theta = 1$

$\theta = 45°$

$\Rightarrow y$ intercept $\Rightarrow y = \frac{5}{2}$

Question 3.

Find the equation of a line whose inclination is 30° and making an intercept – 3 on the Y axis.

Solution:

$\theta = 30°$

y intercept $(x = 0) = -3$ (i.e) when $x = 0, y = -3$

let equation of line be : $y = mx + c$

$m = \tan\theta$

$\Rightarrow \quad m = \tan 30° = \frac{1}{\sqrt{3}}$

$\therefore y = \frac{1}{\sqrt{3}} x + c$

When $x = 0, y = -3$

$\Rightarrow \quad -3 = 0 + c \Rightarrow c = -3$

$\therefore$ Equation of line: $\boxed{y = \frac{x}{\sqrt{3}} - 3}$

$\Rightarrow \quad x - \sqrt{3}y - 3\sqrt{3} = 0$

Question 4.

Find the slope and y intercept of $\sqrt{3}x + (1 - \sqrt{3})y = 3$.

Solution:

$$x\sqrt{3}+(1-\sqrt{3})y = 3$$

$$\Rightarrow \quad (1-\sqrt{3})y = -x\sqrt{3}+3$$

$$\Rightarrow \quad y = x\underbrace{\left(\frac{\sqrt{3}}{\sqrt{3}-1}\right)}_{m}+\underbrace{\left(\frac{3}{1-\sqrt{3}}\right)}_{c}$$

$$\boxed{\text{slope} = \frac{\sqrt{3}}{\sqrt{3}-1}} = \frac{\sqrt{3}}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{\sqrt{3}\left(\sqrt{3}+1\right)}{\left(\sqrt{3}\right)^2-1} = \frac{3+\sqrt{3}}{2}$$

when $x = 0$,

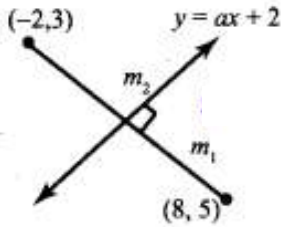
$$\boxed{y = \frac{3}{1-\sqrt{3}}}$$

$$\rightarrow y \text{ intercept} = \frac{3}{1-\sqrt{3}}\times\frac{1+\sqrt{3}}{1+\sqrt{3}} = \frac{3+3\sqrt{3}}{-2}$$

Question 5.

Find the value of 'a', if the line through (-2, 3) and (8, 5) is perpendicular to y = ax = + 2

Solution:



$$m_1 = \frac{5-3}{8-(-2)} = \frac{\cancel{2}}{\cancel{10}_{5}} = \frac{1}{5}$$

$$m_2 = a.$$

$\because$ lines are perpendicular $\Rightarrow m_1 \times m_2 = -1$

$$\Rightarrow \frac{1}{5}\times a = -1 \Rightarrow \boxed{a = -5}$$

Question 6.
The hill in the form of a right triangle has its foot at (19, 3)The inclination of the hill to the ground is 45°. Find the equation of the hill joining the foot and top.
Solution:
$\theta = 45°$
Coordinate of foot of hill = (19, 3) let equation of line be $y = mx + c$
$m = \tan\theta = \tan 45° = 1$
$\Rightarrow y = x + c$
Substituting $y = 3$ & $x = 19$, $3 = 19 + c \Rightarrow c = -16$
$\therefore$ Equation of line: $\boxed{y = x - 16} = x - y - 16 = 0$

Question 7.
Find the equation of a line through the given pair of points
(i) $\underset{x_1}{\left(2, \underset{y_1}{\frac{2}{3}}\right)}$ **and** $\underset{x_2}{\left(\frac{-1}{2}, \underset{y_2}{-2}\right)}$
(ii) (2, 3) and (-7, -1)
Solution:
(i) Equation of the line in two point form is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - \frac{2}{3}}{-2 - \frac{2}{3}} = \frac{x - 2}{-\frac{1}{2} - 2} \quad [\because (x_1, y_1) \text{ is } (2, \tfrac{2}{3}),\ (x_2, y_2) \text{ is } -\tfrac{1}{2}, -2)]$$

$$\frac{\frac{3y-2}{3}}{\frac{-6-2}{3}} = \frac{x-2}{\frac{-1-4}{2}} \Rightarrow \frac{3y-2}{-8} = \frac{2(x-2)}{-5}$$

$$\Rightarrow -15y + 10 = -16x + 32$$
$$\Rightarrow 16x - 15y + 10 - 32 = 0$$
$$\Rightarrow 16x - 15y - 22 = 0$$

(ii) (x_1, y_1) is (2, 3) (x_2, y_2) is (–7, – 1)

$$\therefore \text{ Equation is } \frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\Rightarrow \frac{y-3}{-1-3} = \frac{x-2}{-7-2} \Rightarrow \frac{y-3}{-4} = \frac{x-2}{-9}$$

$$\Rightarrow \frac{y-3}{4} = \frac{x-2}{9}$$

$\Rightarrow 9y - 27 = 4x - 8$
$\Rightarrow 4x - 9y - 8 + 27 = 0$
$\Rightarrow 4x - 9y + 19 = 0$

Question 8.
A cat is located at the point(-6, -4) in xy plane. A bottle of milk is kept at (5, 11). The cat wish to consume the milk traveling through shortest possible distance. Find the equation of the path it needs to take its milk.
Solution:
A = (x_1, y_1) = (-6, -4)
B = (x_2, y_2) = (5, 11)
Shortest path between A and B is a line joining A and B.

$$m = \frac{y_2-y_1}{x_2-x_1} = \frac{11-(-4)}{5-(-6)} = \frac{15}{11}$$

$$y - y_1 = m(x - x_1)$$

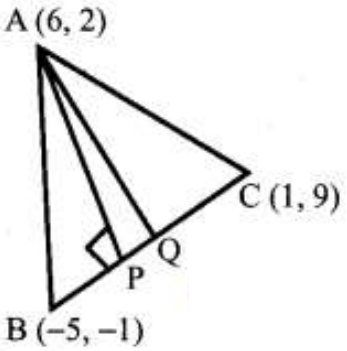
$$\Rightarrow \quad y-(-4) = \frac{15}{11}(x-(-6))$$

$$\Rightarrow \quad (y+4) \times 11 = 15 \times (x-(-6))$$

$$\Rightarrow \quad 11y + 44 = 15x + 90$$

$$\Rightarrow \quad 15x - 11y + 46 = 0$$

Question 9.
Find the equation of the median and altitude of ΔABC through A where the vertices are A(6, 2) B(-5, -1) and C(1, 9)
Solution:



Let AP be the altitude

$\Rightarrow AP \perp BC$

$$\text{Slope of BC} = \frac{9-(-1)}{1-(-5)}$$

$$= \frac{10}{6} = \frac{5}{3}$$

$$\text{Slope of AP} = \frac{-1}{\text{Slope of (BC)}} = \frac{-3}{5}$$

Equation of line AP: $y - 2 = \frac{-3}{5}(x-6)$

$\Rightarrow 5y - 10 = -3x + 18$

$\Rightarrow 3x + 5y = 28$

Let AQ be the median $\Rightarrow$ Q is mid point of BC

$$\Rightarrow Q = \left(\frac{-5+1}{2}, \frac{-1+9}{2}\right) = (-2, 4)$$

$$\text{Slope of AQ} = \frac{4-2}{-2-6}$$

$$= \frac{2}{-8} = \frac{-1}{4}$$

$$\therefore \text{equation of line AQ: } y - 2 = \frac{-1}{4} \times (x-6)$$

$$\Rightarrow \quad y - 2 = \frac{-1}{4}(x-6)$$

$$4y - 8 = -x + 6$$

$$\Rightarrow \quad x + 4y = 14$$

$$\Rightarrow \quad x + 4y - 14 = 0$$

Question 10.

Find the equation of a straight line which -5 has slope $\frac{-5}{4}$ and passing through the point (-1, 2).

Solution:

$$m = \frac{-5}{4} \qquad \text{point} = (-1, 2)$$

$$\Rightarrow \quad y - 2 = \frac{-5}{4}(x - (-1))$$

$$\Rightarrow \quad y - 2 = \frac{-5}{4}(x+1)$$

$$\Rightarrow 4(y-2) = -5(x+1)$$

$$\Rightarrow \quad 4y - 8 = -5x - 5$$

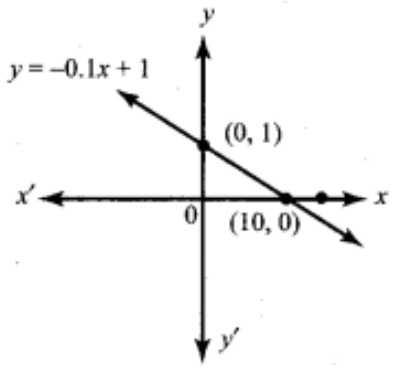
$$\Rightarrow \boxed{5x + 4y = 3} \Rightarrow 5x + 4y - 3 = 0$$

Question 11.

You are downloading a song. The percent y (in decimal form) of mega bytes remaining to get downloaded in x seconds is given by y = -0.1x + 1.

(i) graph the equation.

(ii) find the total MB of the song.
(iii) after how many seconds will 75% of the song gets downloaded?
(iv) after how many seconds the song will be downloaded completely?
Solution:
(i) y = -0.1x + 1
when x = 0 ⇒ y = 1
when y = 0 ⇒ y = 10



(ii) Total MB of song can be obtained when time = 0
∴ x = 0
⇒ y = 1 MB
(iii) time when 75% of song is downloaded
⇒ remaining % = 25% ⇒ y = 0.25
0.25 = -0. 1x + 1
⇒ 0.1x = 0.75
⇒ 7.5 Seconds

(iv) song will downloaded completely when , remaining % = 0 ⇒ y = 0
⇒ 0 = -0.1x + 1
⇒ x = 10
∴ 10 seconds

Question 12.
Find the equation of a line whose intercepts on the x and y axes are given below.
(i) 4, -6
(ii) $-5\frac{3}{4}$

Solution:

(i) $\frac{x}{a}+\frac{y}{b}=1$

$\Rightarrow \frac{x}{4}+\frac{y}{-6}=1$

$\Rightarrow \frac{3x-2y}{12}=1$

$\Rightarrow 3x-2y=12$

$\Rightarrow 3x-2y-12=0$

(ii) $\frac{x}{-5}+\frac{y}{\frac{3}{4}}=1$

$\Rightarrow \frac{-x}{5}+\frac{4y}{3} = 1$

$\Rightarrow \frac{-3x+20y}{15} = 1$

$\Rightarrow -3x+20y = 15$

$3x-20y+15 = 0$

Question 13.

Find the intercepts made by the following lines on the coordinate axes,

(i) 3x – 2y – 6 = 0

(ii) 4x + 3y + 12 = 0

Solution:

(i) The given equation is

3x – 2y – 6 = 0

3x – 2y = 6

Divided by 6

$\frac{3x}{6}-\frac{2y}{6}=\frac{6}{6}$

$\frac{x}{2}-\frac{y}{3}=1 \Rightarrow \frac{x}{2}+\frac{y}{-3}=1$

(Comparing with $\frac{x}{a}+\frac{y}{b}=1$)

∴ x intercept = 2; y intercept = -3

(ii) The given equation is

4x + 3y + 12 = 0

4x + 3y = -12
Divided by -12
$\frac{4x}{-12} + \frac{3y}{-12} = \frac{-12}{-12}$
$\frac{x}{-3} + \frac{y}{-4} = 1$
(Comparing with $\frac{x}{a} + \frac{y}{b} = 1$)
∴ x intercept = -3; y intercept = -4

Question 14.
Find the equation of a straight line
(i) passing through (1, -4) and has intercepts which are in the ratio 2 : 5
(ii) passing through (-8, 4) and making equal intercepts on the coordinate axes
Solution:
(i) ratio of intercept = 2 : 5

$$\therefore \text{Slope of line} = \frac{-5}{2} \Rightarrow m = \frac{-5}{2}$$

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow \quad y - (-4) = \frac{-5}{2}(x - 1)$$

$$\Rightarrow \quad 2(y + 4) = -5(x - 1)$$

$$\Rightarrow \quad 2y + 8 = -5x + 5$$

$$\Rightarrow \quad 5x + 2y + 3 = 0$$

(ii) $$\text{Slope of line} = \frac{y \text{ intercept}}{x \text{ intercept}} \times -1 = -1$$

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow \quad y - 4 = -1(x - (-8))$$

$$y - 4 = -x - 8$$

$$\Rightarrow \quad x + y + 4 = 0$$

Ex 5.4

Question 1.
Find the slope of the following straight lines
(i) 5y – 3 = 0
(ii) $7x - \frac{3}{17} = 0$
Solution:

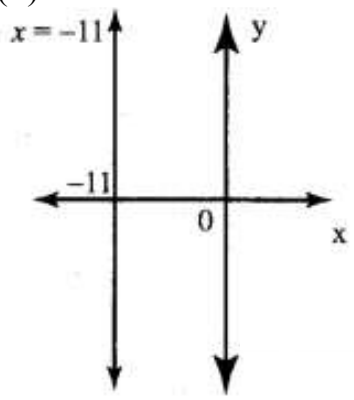
$$\text{(i)} \quad 5y - 3 = 0$$
$$5y = 3$$
$$y = \frac{3}{5}$$
$$\Rightarrow \quad y = mx + c$$
$$y = 0x + \frac{3}{5} \qquad \therefore \text{ Slope } m = 0$$

$$\text{(ii)} \quad 7x - \frac{3}{17} = 0 \Rightarrow 7x + 0y - \frac{3}{17} = 0$$

$$\text{Slope} = \frac{-\text{Co-efficient of } x}{\text{Co-efficient of } y}$$
$$= \frac{-7}{0} = \infty \text{ (undefined)}$$

Question 2.
Find the slope of the line which is
(i) parallel to y = 0.7x -11
(ii) perpendicular to the line x = -11
Solution:
(i) y = 0.7x – 11
line parallel to y = 0.7x – 11 is y = 0.7x + C

If the lines are parallel, slopes are equal
∴ The slope of the required line is 0.7.
(ii) m = tan θ = tan 90°= ∞ undefined.



Question 3.
Check whether the given lines are parallel or perpendicular

(i) $\frac{x}{3}+\frac{y}{4}+\frac{1}{7}=0$ **and** $\frac{2x}{3}+\frac{y}{2}+\frac{1}{10}=0$

(ii) $5x + 23y + 14 = 0$ **and** $23x - 5y + 9 = 0$

Solution:
(i) $\frac{x}{3}+\frac{y}{4}+\frac{1}{7}=0$

and $\frac{2x}{3} + \frac{y}{2} + \frac{1}{10} = 0$

$$\frac{y}{4} = -\frac{x}{3} - \frac{1}{7}$$

$$y = -\frac{4x}{3} - \frac{4}{7}$$

$$m_1 = -\frac{4}{3}$$

$$\frac{y}{2} = -\frac{2x}{3} - \frac{1}{10}$$

$$y = -\frac{4x}{3} - \frac{1}{10}$$

$$m_2 = -\frac{4}{3}$$

$$m_1 = m_2$$

$\therefore$ They are Parallel.

(ii) $5x + 23y + 14 = 0$

$23y = -5x - 14$

$y = -\frac{5x}{23} x - \frac{14}{23}$

$m_1 = -\frac{5}{23}$

$23x - 5y + 9 = 0$

$-5y = -23x - 9$

$y = \frac{-23}{-5} x - \frac{-9}{-5}$

$y = \frac{23}{5} x + \frac{9}{5}$

$m_2 = \frac{23}{5}$

$\therefore m_1 \times m_2 = \frac{-5}{23} \times \frac{23}{5} = -1$

$m_1 \times m_2 = -1$

$\therefore$ They are $\perp^r$.

Question 4.

If the straight lines 12y = -(p + 3)x + 12, 12x – 7y = 16 are perpendicular then find 'p'.

Solution:

$12y = -(p + 3)x + 12,$

$y = -\frac{p+3}{12} x + \frac{12}{12}$

$y = -\frac{p+3}{12} x + 1 \quad ...(1)$

$$\Rightarrow \quad m_1 = -\frac{p+3}{12}$$

$$12x - 7y = 16$$

$$-7y = -12x + 16$$

$$y = \frac{-12}{-7}x + \frac{16}{-7}$$

$$y = \frac{12}{7}x - \frac{16}{7} \qquad \ldots(2)$$

$$\Rightarrow \quad m_2 = \frac{12}{7}$$

$$m_1 \times m_2 = -1$$

$$-\frac{p+3}{12} \times \frac{12}{7} = -1$$

$$-\left(\frac{p+3}{7}\right) = -1$$

$$\therefore \quad \frac{p+3}{7} = 1$$

$$p + 3 = 7$$

$$p = 4$$

Question 5.
Find the equation of a straight line passing through the point P (-5, 2) and parallel to the line joining the points Q(3, -2) and R(-5, 4).

Solution:

$$\text{Slope of QR} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{4-(-2)}{-5-3} = \frac{4+2}{-8} = \frac{6}{-8}$$

$$m = \frac{-3}{4}$$

P is (–5, 2)

Require equation is

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{-3}{4}(x + 5)$$

$$4y - 8 = -3x - 15$$

$$3x + 4y + 7 = 0$$

Question 6.

Find the equation of a line passing through ; (6, -2) and perpendicular to the line joining the points (6, 7) and (2, -3).

Solution:

Slope of line joining (6, 7) and (2,-3) is

$$= \frac{-3-7}{2-6} = \frac{-10}{-4} = \frac{5}{2}$$

Slope of the $\perp^r$ line $= \frac{-2}{5}$

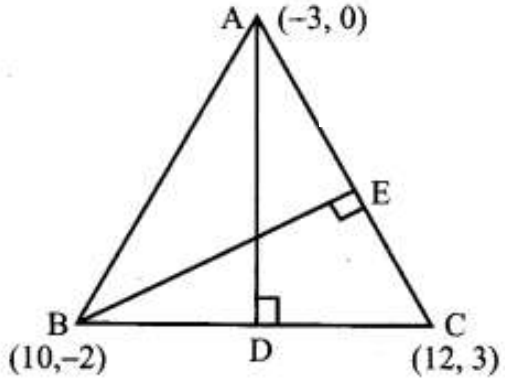
Required equation is

$$y + 2 = \frac{-2}{5}(x - 6)$$

$$5y + 10 = -2x + 12$$

$$2x + 5y - 2 = 0$$

Question 7.
A(-3, 0) B(10, – 2) and C(12, 3) are the vertices of ΔABC . Find the equation of the altitude through A and B.
Solution:
A(-3, 0), B(10, -2), C(12, 3)
Since AD ⊥ BC



$$\text{Slope AD} = \frac{-1}{\text{Slope of BC}}$$

$$\therefore \text{Slope of BC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-2)}{12-10} = \frac{5}{2}$$

$$\therefore \text{Slope of AD} = \frac{-1}{5/2} = \frac{-2}{5}$$

$$\therefore \text{Equation of AD} \Rightarrow y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{-2}{5}(x - (-3))$$

$$5y = -2x - 6$$

$$2x + 5y + 6 = 0 \qquad \ldots(1)$$

Since BE ⊥ AC

$$\text{Slope of BE} = \frac{-1}{\text{Slope of AC}}$$

$$\text{Slope of AC} = \frac{3-0}{12-(-3)} = \frac{\cancel{3}}{\cancel{15}_{5}} = \frac{1}{5}$$

$$\text{B}(10, -2), \text{ slope of BE} = \frac{-1}{1/5} = -5$$

$$\therefore \text{ Equation of BE} \Rightarrow y - (-2) = -5\,(x - 10)$$

$$y + 2 = -5x + 50$$

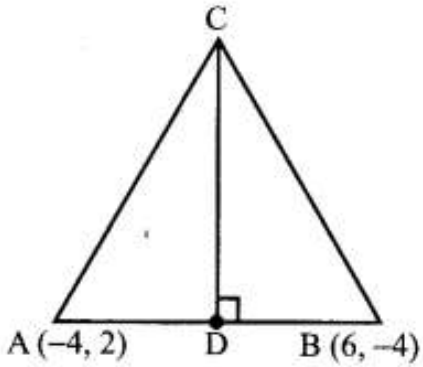
$$5x + y + 2 - 50 = 0$$

$$5x + y - 48 = 0 \qquad \ldots(2)$$

(1), (2) are the required equations of the altitudes through A and B.

Question 8.
Find the equation of the perpendicular bisector of the line joining the points A(-4, 2) and B(6, -4).
Solution:
Mid Point AB is

C is $\left(\frac{-4+6}{2}, \frac{2+(-4)}{2}\right) = \left(\frac{2}{2}, \frac{-2}{2}\right) = (1, -1)$



Slope of AB $= \frac{-4-2}{6-(-4)} = \frac{-6}{10} = \frac{-3}{5}$

$\therefore$ Slope of CD $= \frac{-1}{-3/5} = \frac{5}{3}$ [$\because$ CD $\perp$ AB]

$\therefore$ Equation of CD is

$$y-(-1) = \frac{5}{3}(x-1)$$

$$3(y+1) = 5x-5 \Rightarrow 3y+3 = 5x-5$$

$5x - 3y - 8 = 0$ is the required equation of the line.

Question 9.

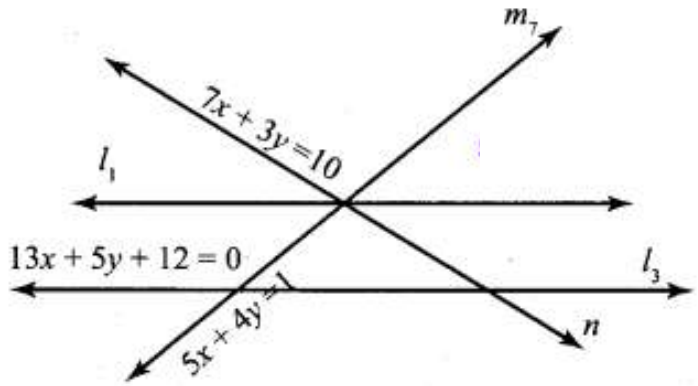
Find the equation of a straight line through the intersection of lines 7x + 3y = 10, 5x – 4y = 1 and parallel to the line 13x + 5y + 12 = O

Solution:

$l_1 \parallel l_2$

$\therefore$ Slope of $l_1 = \frac{-13}{5}$

l_1 passes through the intersecting point.



$\therefore$ By solving m and n, we get

$$7x + 3y = 10 \quad \ldots(n)$$
$$5x - 4y = 1 \quad \ldots(m)$$
$$(n) \times 5 \Rightarrow \quad 35x + 15y = 50$$
$$(m) \times 7 \Rightarrow \quad 35x - 28y = 7$$
$$(-) \quad (+) \quad (-)$$
$$43y = 43$$
$$\boxed{y = 1}$$

Substitute $y = 1$ in (m)

$$5x - 4 \times 1 = 1 \Rightarrow 5x = 1 + 4 = 5$$
$$\boxed{x = 1}$$

$\therefore$ The intersecting point is (1, 1).

$$m = \frac{-13}{5}$$

$$\therefore \text{ The equation} = y - 1 = \frac{-13}{5}(x - 1)$$

$$\Rightarrow \quad 5y - 5 = -13x + 13$$

$\boxed{13x + 5y - 18 = 0}$ is the required equation.

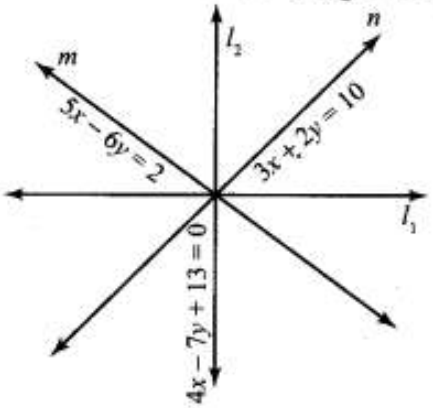
Question 10.

Find the equation of a straight line through the intersection of lines 5x – 6y = 2, 3x + 2y = 10 and perpendicular to the line 4x – 7y + 13 = 0

Solution:

$l_1 \perp^r l_2$

$\therefore$ Slope of l_1 $= \dfrac{-1}{\text{Slope of } l_2}$



$= \dfrac{-1}{\frac{4}{7}}$

$= \dfrac{-1}{\frac{4}{7}} = \dfrac{-7}{4}$

The intersecting point of m and n is got by solving the equations of m and n.

$5x - 6y = 2 \quad (m)$

$3x + 2y = 10 \quad (n)$

$(m) \times 3 \Rightarrow \quad 15x - 18y = 6$

$(n) \times 5 \Rightarrow \quad 15x + 10y = 50$

$(-) \quad (-) \quad (-)$

$-28y = -44 \Rightarrow 7y = 11$

$y = \dfrac{11}{7}$

Substitute $y = \dfrac{11}{7}$ in (m)

$5x - 6\left(\dfrac{11}{7}\right) = 2$

$5x - \dfrac{66}{7} = 2$

$5x = 2 + \dfrac{66}{7}$

$$5x = \frac{14+66}{7}$$

$$x = \frac{\cancel{80}^{16}}{\cancel{35}_{7}} = \frac{16}{7}$$

$$\text{Slope} = \frac{-7}{4}$$

$\therefore$ The intersecting point is $\left(\frac{16}{7}, \frac{11}{7}\right)$

$\therefore$ The required equation is

$$y - \frac{11}{7} = -\frac{7}{4}\left(\frac{x}{1} - \frac{16}{7}\right)$$

$$4y - 4\left(\frac{11}{7}\right) = -7\left(\frac{7x-16}{7}\right)$$

$$4y - \frac{44}{7} = -7x + 16$$

$$\frac{28y-44}{7} = -7x + 16$$

$$28y - 44 = -49x + 112$$

$\boxed{49x + 28y - 156 = 0}$ is the required equation of the line.

Question 11.
Find the equation of a straight line joining the point of intersection of 3x + y + 2 = 0 and x – 2y – 4 = 0 to the point of intersection of 7x – 3y = -12 and 2y = x + 3
Solution:

$$3x + y + 2 = 0 \quad \ldots(1)$$
$$x - 2y - 4 = 0 \quad \ldots(2)$$

Solving (1) and (2) we get the point of intersection of the lines (1) and (2).

$$3x + y = -2 \quad \ldots(1)$$
$$(2) \times 3 \Rightarrow \quad 3x - 6y = 12$$
$$(-) \quad (+) \quad (-)$$
$$7y = -14 \Rightarrow \boxed{y = -2}$$

Substitute $y = -2$ in (1), we get

$$3x + (-2) = -2$$
$$3x = -2 + 2 = 0$$
$$\boxed{x = 0}$$

The intersecting point is $(0, -2)$

$$7x - 3y = -12 \quad \ldots(3)$$
$$x - 2y = -3 \quad \ldots(4)$$

$(4) \times 7 \Rightarrow \quad 7x - 14y = -21$

$(3) \quad \Rightarrow \quad 7x - 3y = -12$

$\quad\quad\quad\quad (-) \quad (+) \quad\quad (+)$

$$-11y = -9$$

$$y = \frac{9}{11}$$

Substitute $y = \frac{9}{11}$ in (4), we get

$$x - 2 \times \frac{9}{11} = -3$$

$$x - \frac{18}{11} = -3$$

$$\Rightarrow x = -3 + \frac{18}{11} = \frac{-33+18}{11}$$

$$x = \frac{-15}{11}$$

$\therefore$ The intersecting point of (3) and (4) is $\left(\frac{-15}{11}, \frac{9}{11}\right)$

$\therefore$ The required line passes through (0, –2) and $\left(\frac{-15}{11}, \frac{9}{11}\right)$

$$\therefore \text{ Equation } = \frac{y-(-2)}{\frac{9}{11}+2} = \frac{x-0}{\frac{-15}{11}-0}$$

$$\Rightarrow \quad \frac{y+2}{\frac{31}{11}} = \frac{x}{\frac{-15}{11}}$$

$$\Rightarrow \quad \frac{31}{11}x = \frac{-15}{11}(y+2)$$

$31x + 15y + 30 = 0$ is the required equation of the line.

Question 12.

Find the equation of a straight line through the point of intersection of the lines 8x + 3+ = 18, 4x + 5+ = 9 and bisecting the line segment joining the points (5, -4) and (-7, 6).

Solution:

The intersecting point of the lines

$8x + 3y = 18$ ...(1)

$4x + 5y = 9$ ...(2)

solving (1) and (2)

(1) $\Rightarrow$ $8x + 3y = 18$

(2) $\times 2 \Rightarrow$ $8x + 10y = 18$

(–) (–) (+)

$-7y = 0$

$y = 0$

Substitute $y = 0$ in (1), we get

$$8x + 3(0) = 18$$

$$x = \frac{\cancel{18}^{9}}{\cancel{4}_{4}} = \frac{9}{4}$$

$\therefore$ The intersecting point is $\left(\frac{9}{4}, 0\right)$

The mid point of the line joining the two points (5, –4) and (–7, 6) is

$$\left(\frac{5+(-7)}{2}, \frac{-4+6}{2}\right) = \left(\frac{-2}{2}, \frac{2}{2}\right) = (-1, 1)$$

The required line is passing through the points

$\begin{pmatrix} \frac{9}{4}, & 0 \\ x_1 & y_1 \end{pmatrix}$ and $\begin{pmatrix} -1, & 1 \\ x_2 & y_2 \end{pmatrix}$

$$\frac{y-0}{1-0} = \frac{x-\frac{9}{4}}{-1-\frac{9}{4}}$$

$$\frac{y}{1} = \frac{\frac{4x-9}{4}}{\frac{-4-9}{4}}$$

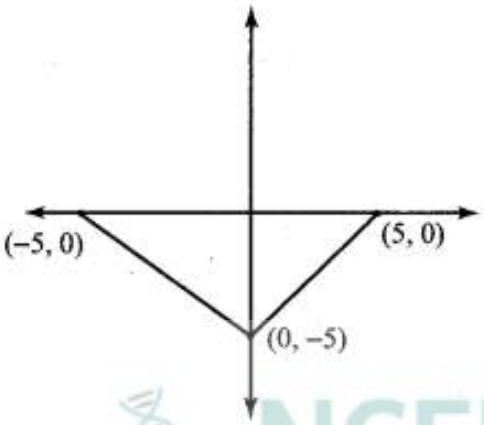
$$y = \frac{4x-9}{-13}$$

$$-13y = 4x - 9$$

$\therefore 4x + 13y - 9 = 0$ is the required equation.

Ex 5.5

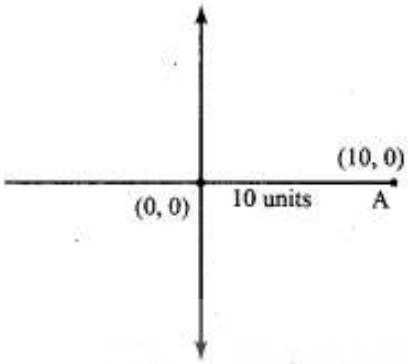
Question 1.
The area of triangle formed by the points (-5, 0), (0, -5) and (5, 0) is
(1) 0 sq. units
(2) 25 sq.units
(3) 5 sq. units
(4) none of these
Solution:
(2) 25 sq. units
Hint:



$$\Delta = \frac{1}{2}\begin{vmatrix} -5 & 0 & 5 & 5 \\ 0 & -5 & 0 & 0 \end{vmatrix}$$

$$= \frac{1}{2}(25-(-25)) = \frac{1}{2}(50) = 25$$

Question 2.
A man walks near a wall, such that the distance between him and the wall is 10 units. Consider the wall to be the Y axis. The path travelled by the man is
(1) x = 10
(2) y = 10
(3) x = 0
(4) y = 0
Solution:
(1) x = 10
Hint:

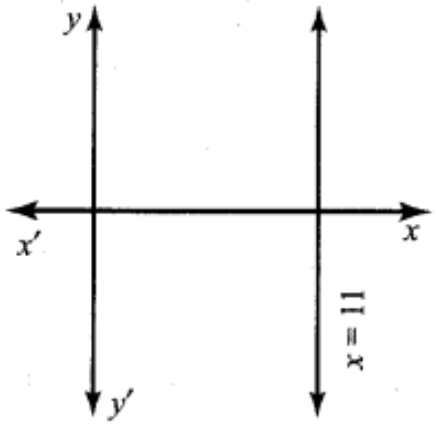


Distance $= \sqrt{10^2 + 0^2} = \sqrt{100} = 10$

$x = 10$ units

Question 3.
The straight line given by the equation x = 11 is
(1) parallel to X axis
(2) parallel to Y axis
(3) passing through the origin
(4) passing through the point (0,11)
Solution:
(2) Parallel to y axis

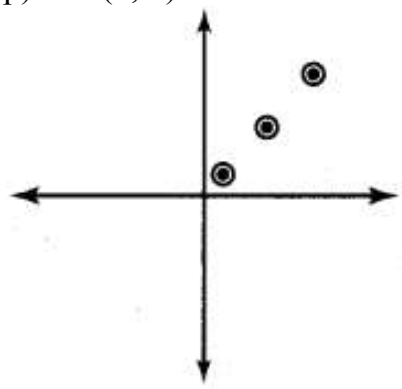
Hint:



$x = 11$ is parallel to y axis.

Question 4.
If (5, 7), (3, p) and (6, 6) are collinear, then the value of p is
(1) 3
(2) 6
(3) 9
(4) 12
Solution:
(3) 9

If (5, 7), (3, p) and (6, 6) are collinear $\Delta = 0$



$$\Rightarrow \quad \frac{1}{2}\begin{vmatrix} 5 & 3 & 6 & 5 \\ 7 & p & 6 & 7 \end{vmatrix} = 0$$

$$\begin{aligned} (5p + 18 + 42) - (21 + 6p + 30) &= 0 \\ 5p + 60 - (6p + 51) &= 0 \\ 5p - 6p &= -60 + 51 \\ -1p &= -9 \\ p &= +9 \end{aligned}$$

Question 5.
The point of intersection of 3x – y = 4 and x + y = 8 is
(1) (5, 3)
(2) (2, 4)
(3) (3, 5)
(4) (4, 4)
Solution:
(3) (3, 5)]
Hint:

$$\begin{aligned} 3x - y &= 4 \\ x + y &= 8 \\ \hline 4x &= 12 \\ x &= 3 \\ 3 + y &= 8 \\ y &= 5 \end{aligned}$$

∴ Point of intersection is (3, 5)

Question 6.
The slope of the line joining (12, 3), (4, a) is $\frac{1}{8}$. The value of 'a' is
(1) 1
(2) 4
(3) -5
(4) 2
Solution:
(4) 2
Hint:

$$m = \frac{a-3}{4-12} = \frac{1}{8}$$

$$\frac{a-3}{-8} = \frac{1}{8}$$
$$8a - 24 = -8$$
$$8a = -8 + 24 = 16$$
$$a = 2$$

Question 7.
The slope of the line which is perpendicular to a line joining the points (0, 0) and (-8, 8) is
(1) -1
(2) 1
(3) $\frac{1}{3}$
(4) -8
Solution:
(2) 1
Hint:
Slope of the line joining the points (0, 0) and (-8, 8) is

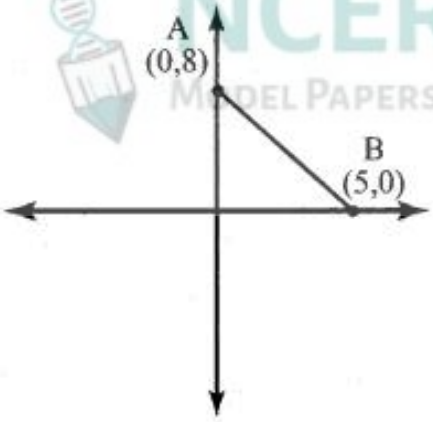
$$m = \frac{8-0}{-8-0} = \frac{8}{-8} = -1$$

Slope of the line $\perp^r$ to the given line is

$$m_2 = \frac{-1}{m} = \frac{-1}{-1} = 1$$

Question 8.
If slope of the line PQ is $\frac{1}{\sqrt{3}}$ then slope of the perpendicular bisector of PQ is
(1) $\sqrt{3}$
(2) $-\sqrt{3}(3)$
(3) $\frac{1}{\sqrt{3}}$
(4) 0
Solution:
(2) $-\sqrt{3}(3)$

Question 9.
If A is a point on the Y axis whose ordinate is 8 and B is a point on the X axis whose abscissa is 5 then the equation of the line AB is
(1) 8x + 5y = 40
(2) 8x – 5y = 40
(3) x = 8
(4) y = 5
Solution:
(1) 8x + 5y = 40
Hint:



$$\frac{y-8}{0-8} = \frac{x-0}{5-0}$$

$$\frac{y-8}{-8} = \frac{x}{5}$$

$$-8x = 5y - 40$$

$$8x + 5y - 40 = 0$$

Question 10.
The equation of a line passing through the origin and perpendicular to the line $7x - 3y + 4 = 0$ is
(1) $7x - 3y + 4 = 0$
(2) $3x - 7y + 4 = 0$
(3) $3x + 7y = 0$
(4) $7x - 3y = 0$
Solution:
(3) $3x + 7y = 0$
Hint:

Slope of $7x - 3y + 4 = 0$ is $\frac{-7}{-3} = \frac{7}{3}$

Slope of the line that is $\perp^r$ to $7x - 3y + 4 = 0$ is

$$m = \frac{-1}{\frac{7}{3}} = \frac{-3}{7}$$

Passing through origin i.e. (0, 0)

Required equation is $y - 0 = \frac{-3}{7}(x - 0)$

$3x + 7y = 0$

Question 11.
Consider four straight lines
(i) $l_1 : 3y = 4x + 5$
(ii) $l_2 : 4y = 3x - 1$
(iii) $l_3 : 4y = 3x = 7$
(iv) $l_4 : 4x + 3y = 2$
Which of the following statement is true ?
(1) l_1 and l_2 are perpendicular
(2) l_1 and l_4 are parallel
(3) l_2 and l_4 are perpendicular
(4) l_2 and l_3 are parallel
Solution:
(3) l_2 and l_4 are perpendicular

Hint:

$l_1: 4x - 3y + 5 = 0 \qquad m_1 = \frac{-4}{-3}$

$l_2: 3x - 4y - 1 = 0 \qquad m_2 = \frac{-3}{-4}$

$l_3: 3x + 4y - 7 = 0 \qquad m_3 = \frac{-3}{4}$

$l_4: 4x + 3y - 2 = 0 \qquad m_4 = \frac{-4}{3}$

Question 12.
A straight line has equation 8y = 4x + 21. Which of the following is true
(1) The slope is 0.5 and the y intercept is 2.6
(2) The slope is 5 and they intercept is 1.6
(3) The slope is 0.5 and the y intercept is 1.6
(4) The slope is 5 and they intercept is 2.6
Solution:
(1) The slope is 0.5 and the y intercept is 2.6
Hint:

$$8y = 4x + 21 \Rightarrow 4x - 8y + 21 = 0$$

$$m = \frac{-4}{-8} = \frac{1}{2} = 0$$

y intercept is $\frac{21}{8} = 2.6$

Question 13.
When proving that a quadrilateral is a trapezium, it is necessary to show ……
(1) Two sides are parallel.
(2) Two parallel and two non-parallel sides.
(3) Opposite sides are parallel.
(4) All sides are of equal length.
Answer:
(2) Two parallel and two non-parallel sides.

Question 14.
When proving that a quadrilateral is a parallelogram by using slopes you must find
(1) The slopes of two sides
(2) The slopes of two pair of opposite sides
(3) The lengths of all sides
(4) Both the lengths and slopes of two sides
Solution:
(2) The slopes of two pair of opposite sides

Question 15.
(2, 1) is the point of intersection of two lines.
(1) $x - y - 3 = 0; 3x - y - 7 = 0$
(2) $x + y = 3; 3x + y = 7$
(3) $3x + y = 3; x + y = 7$
(4) $x + 3y - 3 = 0; x - y - 7 = 0$
Answer:
(2) $x + y = 3; 3x + y = 7$
Hint:
Substitute the value of $x = 2$ and $y = 1$ in the given equation.
$(1) \Rightarrow x - y - 3 = 0$
$2 - 1 - 3 = 0$
$2 - 4 = 0$
$-2 \neq 0$
not true

$3x - y - 7 = 0$
$3(2) - 1 - 7 = 0$
$6 - 8 = 0$
$-2 \neq 0$
not true

$(2) \Rightarrow x + y = 3$
$2 + 1 = 3$
$3 = 3$
True

$3x + y = 7$
$3(2) + 1 = 7$
$6 + 1 = 7$
$7 = 7$
True
$\therefore$ (2, 1) is the point of intersection

(3) ⇒ 3x + y = 3
3(2) + 1 = 3
6 + 1 = 3
7 = 3
not true

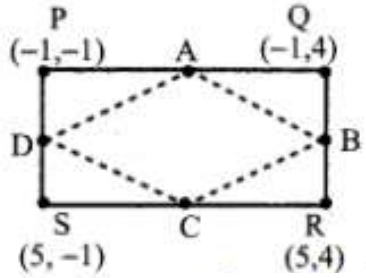
x + y = 7
2 + 1 = 7
3 = 7
not true

(4) ⇒ x + 3y – 3 = 0
2 + 3 – 3 = 0
5 – 3 = 0
2 ≠ 0
not true

x – y – 7 = 0
2 – 1 – 7 = 0
2 – 8 = 0
-6 ≠ 0
not true

Unit Exercise 5

Question 1.
PQRS is a rectangle formed by joining the points P(-1, -1), Q(-1, 4) ,R(5, 4) and S(5, -1). A, B, C and D are the mid-points of PQ, QR, RS and SP respectively. Is the quadrilateral ABCD a square, a rectangle or a rhombus? Justify your answer.
Solution:
A, B, C and D are mid points of PQ, QR, RS & SP respectively.



$$\therefore A = \left(\frac{-1+-1}{2}, \frac{-1+4}{2}\right) = \left(-1, \frac{3}{2}\right)$$

$$B = \left(\frac{-1+5}{2}, \frac{4+4}{2}\right) = (2, 4)$$

$$C = \left(\frac{5+5}{2}, \frac{4+(-1)}{2}\right) = \left(5, \frac{3}{2}\right)$$

$$D = \left(\frac{5+(-1)}{2}, \frac{-1+-1}{2}\right) = (2, -1)$$

$$\text{Slope of AC} = \frac{\frac{3}{2}-\frac{3}{2}}{-1-5} = 0$$

$$\text{Slope of BD} = \frac{4-(-1)}{2-2} = \infty$$

∴ AC is perpendicular to BD.

∴ ABCD can be a square or rhombus.

$$\text{Slope of AB} = \frac{4-\frac{3}{2}}{2-(-1)} = \frac{\frac{5}{2}}{3} = \frac{5}{6}$$

$$\text{Slope of BC} = \frac{\frac{3}{2}-4}{5-2} = \frac{\frac{-5}{2}}{3} = \frac{-5}{6}$$

∴ AB and BC are not perpendicular

⇒ ABCD is rhombus as diagonals are perpendicular and sides are not perpendicular.

Question 2.

The area of a triangle is 5 sq.units. Two of its vertices are (2, 1) and (3, -2). The third Vertex is (x, y) where y = x + 3 . Find the coordinates of the third vertex.

Solution:

Area of triangle formed by points (x_1, y_1),

(x_2, y_2), and $(x_3, y_3) = \frac{1}{2}\{(x_1y_2 + x_2y_3 + x_3y_1) -$

$(x_2\, y_1) + x_3\, y_2 + x_1\, y_3)\}$

$$\begin{array}{|cccc|} \hline 2 & 3 & x & 2 \\ 1 & -2 & x+3 & 1 \\ \hline \end{array}$$

A B C

$\Rightarrow$ (2, 1) (3, –2) $(x, x + 3)$

$$\therefore \text{Area} = \frac{1}{2}\{(-4 + 3x + 9 + x) - (3 - \cancel{2x} + \cancel{2x} + 6)\}$$

$$\Rightarrow \quad \text{Area} = 5 \text{ (given)}$$

$$\Rightarrow \quad 5 = \frac{1}{2}\{(4x + 5\} - (9)\}$$

$$\Rightarrow \quad 4x - 4 = 10$$

$$\Rightarrow \quad 4x = 14$$

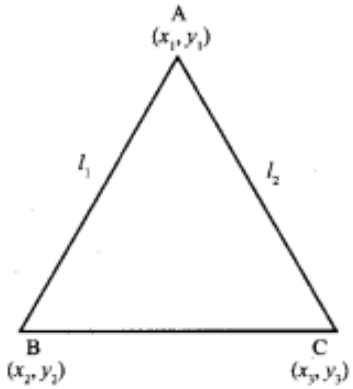
$$\Rightarrow \quad x = \frac{14}{4} = \frac{7}{2}$$

$$y = x + 3$$

$$\Rightarrow \quad y = \frac{7}{2} + 3 \Rightarrow y = \frac{13}{2}$$

$$(x, y) = \left(\frac{7}{2}, \frac{13}{2}\right)$$

Question 3.
Find the area of a triangle formed by the lines 3x + y – 2 = 0, 5x + 2y – 3 = 0 and 2x – y – 3 = 0
Solution:



$l_1: 3x + y - 2 = 0$

$l_2: 5x + 2y - 3 = 0$

$l_3: 2x - y - 3 = 0$

$\rightarrow$ Solving l_1 and $l_2, l_2 - 2l_1$

$\Rightarrow 5x + 2y - 3 - 6x - 2y + 4 = 0$

$\Rightarrow \quad -x + 1 = 0 \Rightarrow \quad x = 1,$

$\therefore 3(1) + y - 2 = 0 \Rightarrow \quad y = -1$ } B = (1, – 1)

$\rightarrow$ Solving l_2 and l_3,

$l_2 + 2l_3$

$\Rightarrow 5x + 2y - 3 + 4x - 2y - 6 = 0$

$\Rightarrow \quad 9x - 9 = 0 \Rightarrow \quad x = 1$

$\therefore 2(1) + y - 3 = 0 \Rightarrow \quad y = -1$ } C = (1, –1)

$\rightarrow$ Solving l_1 and l_3,

$l_1 + l_3$

$\Rightarrow 3x + y - 2 + 2x - y - 3 = 0$

$\Rightarrow \quad 5x - 5 = 0 \Rightarrow \quad x = 1,$

$\therefore 2(1) - y - 3 = 0 \Rightarrow \quad y = -1$ } A = (1, –1)

$\therefore$ l_1, l_2 and l_3 do not form a triangle as they intersect at the same point (1, –1).

$\therefore$ Area is 0 sq. units.

Question 4.

If vertices of a quadrilateral are at A(-5, 7), B(-4, k) , C(-1, -6) and D(4, 5) and its area is

72 sq.units. Find the value of k.
Area (quadrilateral ABCD)

$$= \frac{1}{2}\begin{vmatrix} -5 & -4 & -1 & 4 & -5 \\ 7 & k & -6 & 5 & 7 \end{vmatrix}$$

$$\Rightarrow \frac{1}{2}[(-5k+24-5+28)-(-28-k-24-25)]=72$$

$$\Rightarrow (47-5k)-(-77-k) = 144$$

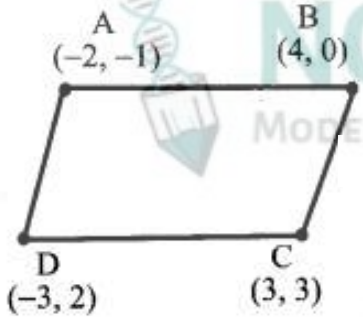
$$\Rightarrow 47-5k+77+k = 144$$

$$\Rightarrow 124-4k = 144$$

$$\Rightarrow -4k = 20$$

$$\boxed{k = -5}$$

Question 5.
Without using distance formula, show that the points (-2, -1) , (4, 0) , (3, 3) and (-3, 2) are vertices of a parallelogram.
Solution:



$$\text{Slope of AB} = \frac{0-(-1)}{4-(-2)} = \frac{+1}{6}$$

$$\text{Slope of BC} = \frac{3-0}{3-4} = -3$$

$$\text{Slope of CD} = \frac{2-3}{-3-3} = \frac{+1}{6}$$

$$\text{Slope of DA} = \frac{-1-2}{-2-(-3)} = -3$$

Slope of AB = Slope of CD
Slope of BC = Slope of DA
Hence ABCD forms a parallelogram.

Question 6.
Find the equations of the lines, whose sum and product of intercepts are 1 and -6 respectively.
Let the intercepts be x_1, y_1 respectively

$$x_1 + y_1 = 1 \Rightarrow y_1 = 1 - x_1 \quad \ldots(1)$$

$$x_1 \cdot y_1 = -6 \quad \ldots(2)$$

$$y = mx + c \text{ where } m = \frac{-y_1}{x_1}$$

Solving (1) and (2)

$$\Rightarrow x_1(1 - x_1) = -6$$
$$\Rightarrow x_1 - x_1^2 + 6 = 0$$
$$\Rightarrow x_1^2 - x_1 - 6 = 0$$
$$\Rightarrow x_1^2 - 3x_1 + 2x_1 - 6 = 0$$
$$\Rightarrow x_1(x_1 - 3) + 2(x_1 - 3) = 0$$
$$\Rightarrow (x_1 - 3)(x_1 + 2) = 0$$
$$\therefore x_1 = 3 \text{ (or) } x_1 = -2$$
$$\therefore y_1 = -2 \text{ (or) } y_1 = 3$$

l_1	l_2
$m = \frac{-y_1}{x_1} = \frac{2}{3}$	$m = \frac{-y_1}{x_1} = \frac{3}{2}$
$\Rightarrow y - 0 = \frac{2}{3}(x - 3)$	$\Rightarrow y - 0 = \frac{3}{2}(x - (-2))$
$\Rightarrow \boxed{2x - 3y = 6}$	$\Rightarrow \boxed{3x - 2y + 6 = 0}$
$\Rightarrow 2x - 3y - 6 = 0$	

Question 7.
The owner of a milk store finds that, he can sell 980 litres of milk each week at □ 14/litre and 1220 litres of milk each week at □ 16 litre. Assuming a linear relationship
between selling price and demand, how many litres could he sell weekly at □ 17/litre?

Solution:

x_1 y_1

980 L @ ₹ 14/L

x_2 y_2

1220 L @ ₹ 16/L

x L @ ₹ 17/L

$$\frac{17-16}{x-1220} = \frac{16-14}{1220-980}$$

$$\Rightarrow \quad \frac{1}{x-1220} = \frac{2}{240}$$

$$\Rightarrow \quad x - 1220 = 120$$

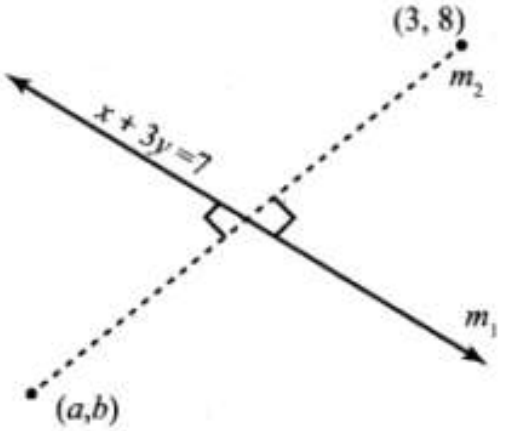
$$\Rightarrow \quad x = 1340$$

He can sell 1340 L @ ₹ 17/L.

Question 8.

Find the image of the point (3, 8) with respect to the line x + 3y = 7 assuming the line to be a plane mirror.

Solution:



$$m_2 = \frac{8-b}{3-a}$$

$$m_1 = \frac{-1}{3}$$

$$m_1 \times m_2 = -1$$

$$\frac{-1}{3}\times\frac{8-b}{3-a} = -1$$

$$8-b = (3-a)\times 3$$

$$8-b = 9-3a$$

$$3a-b = 1 \Rightarrow b = 3a-1 \quad ...(1)$$

Mid point of line joining (3, 8) and (a, b) lies on $x + 3y = 7$.

$$\text{mid point} = \left(\frac{a+3}{2}, \frac{b+8}{2}\right)$$

$$\therefore \frac{a+3}{2}+3\left(\frac{b+8}{2}\right) = 7 \quad ...(2)$$

$\therefore$ Solving (1) and (2)

$$\frac{a+3}{2}+\frac{3}{2}(3a-1+8) = 7$$

as $b = 3a - 1$ from (1).

$$a+3+9a+21 = 14$$

$$10a = -10$$

$$a = -1$$

$$\therefore b = 3(-1) - 1 = -4$$

$$(a, b) = (-1, -4)$$

Question 9.
Find the equation of a line passing through the point of intersection of the lines 4x + 7y – 3 = 0 and 2x – 3y + 1 = 0 that has equal intercepts on the axes.
Solution:
4x + 7y – 3 = 0
2x – 3y + 1 = 0

4x + 7y – 3 – 2(2x – 3y + 1) = 0

$$4x + 7y - 3 = 0$$
$$2x - 3y + 1 = 0$$

$4x + 7y - 3 - 2(2x - 3y + 1) = 0$

$\Rightarrow 4x + 7y - 3 - 4x + 6y - 2 = 0$

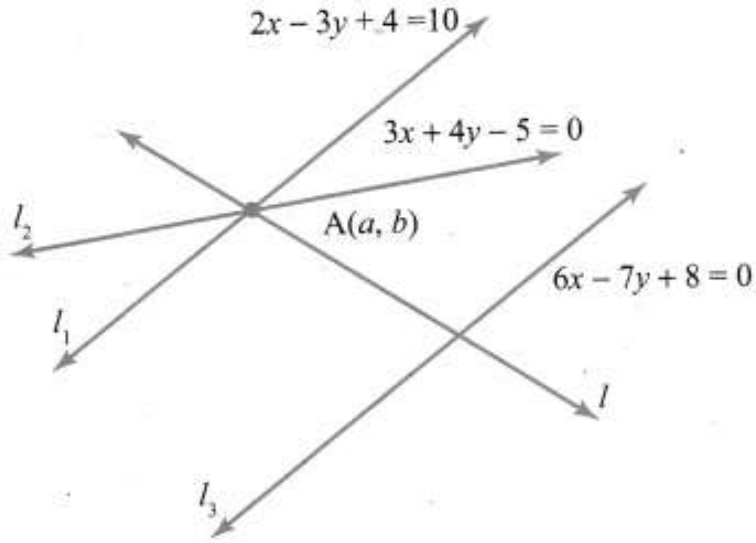
$$\Rightarrow \quad 13y = 5 \Rightarrow y = \frac{5}{13}$$

$$2x - \frac{15}{13} + 1 = 0$$
$$2x = \frac{2}{13} \Rightarrow x = \frac{1}{13}$$
$$(x, y) = \left(\frac{1}{13}, \frac{5}{13}\right),$$

point of intersection. Equal intercepts

$$\Rightarrow \quad \text{Slope} = -1$$
$$\therefore \quad y - \frac{5}{13} = -1\left(x - \frac{1}{13}\right)$$
$$\Rightarrow \quad x + y = \frac{5}{13} + \frac{1}{13}$$
$$\Rightarrow \quad 13x + 13y = 6$$
$$\Rightarrow \quad 13x + 13y - 6 = 0$$

Question 10.
A person standing at a junction (crossing) of two straight paths represented by the equations 2x – 3y + 4 = 0 and 3x + 4y – 5 = 0 seek to reach the path whose equation is 6x – 7y + 8 = 0 in the least time. Find the equation of the path that he should follow.
Solution:



l_1 and l_2 intersect at A(a, b) $3l_1 - 2l_2 = 0$

$$6x - 9y + 12 - 6x - 8y + 10 = 0$$

$$-17y = -22$$

$$\Rightarrow \quad y = \frac{22}{7} = b$$

$$2a - 3 \times \frac{22}{7} + 4 = 0$$

$$2a = \frac{66}{7} - \frac{28}{7}$$

$$\Rightarrow a = \frac{19}{7}$$

$$\text{Slope of } l_3 = \frac{-6}{-7} = \frac{6}{7}$$

$$\text{Slope of } l = \frac{-1}{\left(\frac{6}{7}\right)} = \frac{-7}{6}$$

Equation of $l : y - b = m(x - a)$

$$\Rightarrow \quad y - \frac{22}{7} = \frac{-7}{6}\left(x - \frac{19}{7}\right)$$

$$\Rightarrow \quad \frac{7y - 22}{\cancel{7}} = \frac{-7}{6}\left(\frac{7x - 19}{\cancel{7}}\right)$$

$$\Rightarrow \quad 6(7y - 22) = -7(7x - 19)$$

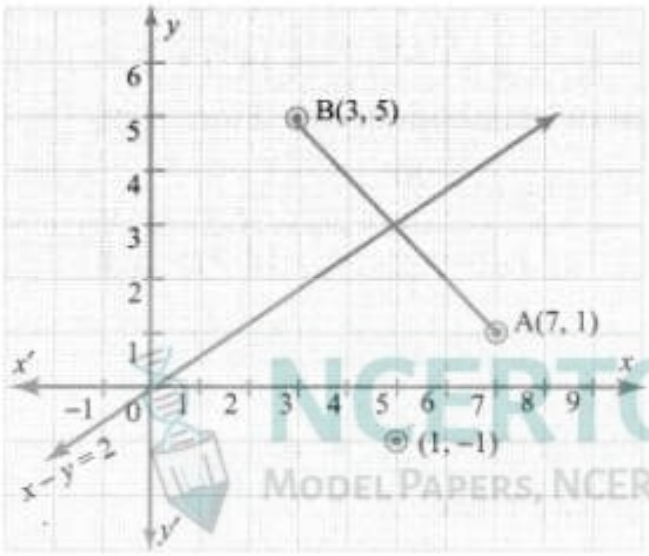
$$\Rightarrow \quad 42y - 132 = -49x + 133$$

$$\Rightarrow \quad 49x + 42y = 265$$

$$49x + 42y - 265 = 0$$

Additional Questions

Question 1.
Find a relation between x and y such that the point (x, y) is equidistant from the points (7, 1) and (3, 5).
Solution:
Let P(x, y) be equidistant from the points A(7, 1) and B(3, 5).
We are given that AP = BP. So, $AP^2 = BP^2$
$(x - 7)^2 + (y - 1)^2 = (x - 3)^2 + (y - 5)^2$
$x^2 - 14x + 49 + y^2 - 2y + 1 = x^2 - 6x + 9 + y^2 - 10y + 25$
$x - y = 2$
Which is the required relation



Question 2.
Show that the points (1, 7), (4, 2), (-1, -1) and (-4, 4) are the vertices of a square.
Solution:

Let A(1, 7), B(4, 2), C(-1, -1) and D(-4, 4) be the given points. To prove that ABCD is a square, we have to prove that all its sides are equal and both its diagonals are equal.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AB = \sqrt{(1-4)^2 + (7-4)^2} = \sqrt{9+25} = \sqrt{34}$$

$$BC = \sqrt{(4+1)^2 + (2+1)^2} = \sqrt{25+9} = \sqrt{34}$$

$$CD = \sqrt{(-1+4)^2 + (-1-4)^2} = \sqrt{9+25} = \sqrt{34}$$

$$DA = \sqrt{(1+4)^2 + (7-4)^2} = \sqrt{25+9} = \sqrt{34}$$

$$AC = \sqrt{(1+1)^2 + (7+1)^2} = \sqrt{4+64} = \sqrt{68}$$

$$BD = \sqrt{(4+4)^2 + (2-4)^2} = \sqrt{64+4} = \sqrt{68}$$

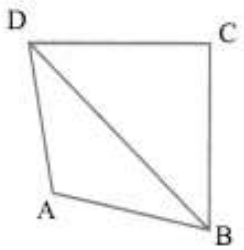
Since, AB = BC = CD = DA and AC = BD, all the four sides of the quadrilateral ABCD are equal and its diagonals AC and BD are also equal. Therefore, ABCD is a square.

Question 3.
If A (-5, 7), B (-4, -5), C (-1, -6) and D (4, 5) are the vertices of a quadrilateral, find the area of the quadrilateral ABCD.
Solution:
By joining B to D, you will get two triangles ABD and BCD.
Now, the area of ΔABD

$$= \frac{1}{2}[-5(-5-5) + (-4)(5-7) + 4(7+5)]$$

$$= \frac{1}{2}(50 + 8 + 48)$$

$$= \frac{106}{2} = 53 \text{ square units.}$$



Also, the area of ΔBCD

$$= \frac{1}{2}[-4(-6-5) - 1(5+5) + 4(-5+6)]$$

$$= \frac{1}{2}(44 - 10 + 4)$$

$$= 19 \text{ square units.}$$

So, the area of quadrilateral ABCD = 53 + 19 = 72 square units.

Question 4.
Find the coordinates of the points of trisection (i.e. points dividing in three equal parts) of the line segment joining the points A(2, -2) and B(-7, 4).
Solution:
Let P and Q be the points of trisection at AB. i.e., AP = PQ = QB

Therefore, P divides AB internally in the ratio 1 : 2. Therefore, the coordinates at P, by applying the section formula, are

$$\left[\frac{1(-7)+2(2)}{1+2}, \frac{1(7)+2(-2)}{1+2}\right] \text{ i.e., } (-1, 10)$$

Now, Q also divides AB internally in the ratio 2 : 1, so, the coordinates at Q are

$$\left[\frac{2(-7)+1(2)}{2+1}, \frac{2(4)+(-2)}{2+1}\right] \text{ i.e., } (-4, 2)$$

Therefore, the coordinates of the points at trisection of the line segment joining A and B are (-1, 0) and (-4, 2).

Question 5.
If the points A(6, 1), B(8, 2), C(9, 4) and D(P, 3) are the vertices of a parallelogram, taken in order. Find the value of P.
Solution:
We know that diagonals of a parallelogram bisect each other.
So, the coordinates at the mid-point of AC = coordinates of the mid-point of BD.

$$\text{i.e., } \left[\frac{6+9}{2}, \frac{1+4}{2}\right] = \left[\frac{8+P}{2}, \frac{2+3}{2}\right]$$

$$\left[\frac{15}{2}, \frac{5}{2}\right] = \left[\frac{8+P}{2}, \frac{5}{2}\right]$$

$$\frac{15}{2} = \frac{8+P}{2}$$

$$P = 7$$

Question 6.
Find the area of a triangle whose vertices are (1,-1), (-4, 6) and (-3, -5).
Solution:
The area of the triangle formed by the vertices A(1, -1), B(-4, 6) and C(-3, -5), by using the formula

$$\Delta = \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

sq.units

$$= \frac{1}{2}[1(6+5) + (-4)(-5+1) + (-3)(-1-6)]$$

$$= \frac{1}{2}[11 + 16 + 21] = 24 \text{ square units.}$$

So, the area of the triangle is 24 square units.

Question 7.
If A(-2, -1), B(a, 0), C(4, b) and D(1, 2) are the vertices of a parallelogram, find the values of a and b.
Solution:
We know that the diagonals of a parallelogram bisect each other. Therefore the co-ordinates of the

midpoint of AC are same as the co-ordinates of the mid-point of BD. i.e.

$$\left(\frac{-2+4}{2}, \frac{-1+b}{2}\right) = \left(\frac{a+1}{2}, \frac{0+2}{2}\right)$$

$$\Rightarrow \quad \left(1, \frac{b-1}{2}\right) = \left(\frac{a+1}{2}, 1\right)$$

$$\Rightarrow \quad \frac{a+1}{2} = 1 \Rightarrow a+1=2 \Rightarrow a=1$$

$$\Rightarrow \quad \frac{b-1}{2} = 1 \Rightarrow b-1=2 \Rightarrow b=3$$

Question 8.
Find the area of the quadrilateral whose vertices, taken in order, are (-3, 2), (5, 4), (7, -6) and (-5, -4).
Solution:
We have Area of the quadrilateral

$$= \frac{1}{2}\begin{vmatrix} -3 & 5 & 7 & -5 & -3 \\ +2 & 4 & -6 & -4 & 2 \end{vmatrix}$$

$$= \frac{1}{2}[(-12-30-28-10)-(+10+28+30+12)]$$

$$\frac{1}{2}[-80-(80)]$$

$$\frac{1}{2}[-160] = -80 = 80 \text{ square units.}$$

$$(\because \text{Area is always +ve}).$$

Question 9.
Find the area of the triangle formed by the points P(-1.5, 3), Q(6, -2) and R(-3, 4).
Solution:
The area of the triangle formed by the given points is equal to

$$= \frac{1}{2}[-1.5\,(-2-4) + 6(4-3) + (-3)(3+2)]$$

$$= \frac{1}{2}[9 + 6 - 15] = 0$$

Can we have a triangle of area 0 square units? What does this mean?
If the area of a triangle is 0 square units, then its vertices will be collinear.

Question 10.
Find the value of k if the pointsA(2, 3), B(4, k) and (6, -3) are collinear.
Since the given points are collinear, the area a the triangle formed by them must be 0, i.e.

$$= \frac{1}{2}[2(k+3) + 4(-3-3) + 6(3-k)] = 0$$

$$= \frac{1}{2}[-4k] = 0$$

$$k = 0$$

Area of ΔABC

$$= \frac{1}{2}[2(0+3) + 4(-3-3) + 6(3-0)] = 0$$