Algebra

Ex 3.1

Question 1.
Solve the following system of linear equations in three variables
(i) $x + y + z = 5$; $2x – y + z = 9$; $x – 2y + 3z = 16$
(ii) $\frac{1}{x} - \frac{2}{y} + 4 = 0$; $\frac{1}{y} - \frac{1}{z} + 1 = 0$; $\frac{2}{z} + \frac{3}{x} = 14$
(iii) $x + 20 = \frac{3y}{2} + 10 = 2z + 5 = 110 – (y + z)$
Solutions:
(i) $x + y + z = 5$ ………….. (1)
$2x – y + z = 9$ …………. (2)
$x – 2y + 3z = 16$ …………. (3)

$$\begin{array}{lrl} (1) + (2) \Rightarrow & x + \cancel{y} + z = 5 & \\ & 2x - \cancel{y} + z = 5 & \\ \hline & 3x + 2z = 14 & \ldots(4) \end{array}$$

$$\begin{array}{lrl} (2) \times 2 \Rightarrow & 4x - \cancel{2y} + 2z = 18 & \\ (3) \Rightarrow & \underset{(-)}{x} - \underset{(+)}{\cancel{2y}} + \underset{(-)}{3z} = \underset{(-)}{16} & \\ \hline & 3x \quad - z = 2 & \ldots(5) \end{array}$$

$$\begin{array}{lr} (4) - (5) \Rightarrow & 3x + 2z = 14 \\ \Rightarrow & \underset{(-)}{3x} - \underset{(+)}{z} = \underset{(-)}{2} \\ \hline & 3z = 12 \\ & z = 4 \end{array}$$

Substitute z = 4 in (4)
$3x + 2(4) = 14$
$3x + 8 = 14$
$3x = 6$
$x = 2$
Substitute x = 2, z = 4 in (1)

$2 + y + 4 = 5 \Rightarrow y = -1$
$x = 2, y = -1, z = 4$

(ii)
$$\frac{1}{x} - \frac{2}{y} + 4 = 0 \quad \ldots(1)$$
$$\frac{1}{y} - \frac{1}{z} + 1 = 0 \quad \ldots(2)$$
$$\frac{2}{z} + \frac{3}{x} = 14 \quad \ldots(3)$$
$$\text{Put } \frac{1}{x} = a$$

$\frac{1}{y} = b$
$\frac{1}{z} = c$ in (1), (2) & (3)
$a - 2b + 4 = 0 \Rightarrow a - 2b = -4$ …………. (1)
$b - c + 1 = 0 \Rightarrow b - c = -1$ ……….. (2)

2c + 3a = 14 ⇒ 2c + 3a = 14 …………. (3)

$$\begin{array}{lrl} (1) & \Rightarrow & a - \cancel{2b} = -4 \\ (2)\times 2 & \Rightarrow & \underline{-2c + \cancel{2b} = -2} \\ & & a - 2c = -6 \qquad \ldots(4) \\ & & \underline{3a + 2c = 14} \\ (4)+(3) & \Rightarrow & 4a \quad = 8 \\ & & a \quad = 2 \end{array}$$

Substitute $a = 2$ in (1), we get

$$\begin{aligned} 2 - 2b &= -4 \\ -2b &= -6 \\ b &= 3 \end{aligned}$$

Substitute $b = 3$ in (2), we get

$$\begin{aligned} 3 - c &= -1 \\ -c &= -4 \\ \Rightarrow c &= 4 \end{aligned}$$

$$a = \frac{1}{x} = 2 \Rightarrow x = \frac{1}{2}$$

$$b = \frac{1}{y} = 3 \Rightarrow y = \frac{1}{3}$$

$$c = \frac{1}{z} = 4 \Rightarrow z = \frac{1}{4}$$

$$x = \frac{1}{2}, y = \frac{1}{3}, z = \frac{1}{4}$$

(iii) x + 20 = $\frac{3y}{2}$ + 10 = 2z + 5 = 110 – (y + z)

x = $\frac{3y}{2}$ – 10 …………. (1)

2z + 5 = 110 – (y + z)

2z = 105 – y – z

y = 105 – 3z ………….. (2)

Substitute (2) in (1), x = $\frac{315}{2}$ – $\frac{9z}{2}$ – 10

= 2z + 5 – 20

∴ 315 – 9z – 20 = 4z – 30

13 z = 315 – 20 + 30
= 325
z = $\frac{325}{13}$ = 25
x + 20 = 2z + 5
x + 20 = 50 + 5
x = 35
Substitute z = 25 in (2)
y = 105 – 3z = 105 – 75 = 30
y = 30
x = 35, y = 30, z = 25
The system has unique solutions.

Question 2.
Discuss the nature of solutions of the following system of equations
(i) x + 2y – z = 6 ; -3x – 2y + 5z = -12 ; x – 2z = 3
(ii) 2y + z = 3 (-x + 1); -x + 3y -z = -4 3x + 2y + z = – $\frac{1}{2}$
(iii) $\frac{y+z}{4} = \frac{z+x}{3} = \frac{x+y}{2}$; x + y + z = 27
Solution:
(i) x + 2y – z = 6 …………. (1)
-3x – 2y + 5z = -12 ……… (2)
x – 2z = 3 …………… (3)

$$\begin{array}{lrcl}
 & x + \cancel{2y} - z &=& 6 \quad \ldots(1) \\
 & -3x - \cancel{2y} + 5z &=& -12 \quad \ldots(2) \\
\hline
(1) + (2) \Rightarrow & -2x \qquad + 4z &=& -6 \\
 & -x \qquad + 2z &=& -3 \\
 & x \qquad - \cancel{2z} &=& 3 \\
(3) & \underset{(-)}{x} \qquad \underset{(+)}{-\cancel{2z}} &=& \underset{(-)}{3} \\
\hline
 & & 0 = 0 &
\end{array}$$

We see that the system has an infinite number of solutions.
(ii) 2y + z = 3(-x + 1);
-x + 3y – z = -4;
3x + 2y + z = $-\frac{1}{2}$
2y + z + 3x = 3 ⇒ 3x + 2y + z = 3 ………….. (1)
-x + 3y – z = -4 …………. (2)
3x + 2y + z = $-\frac{1}{2}$ ………………. (3)

$$
\begin{array}{lrl}
 & 3x + 2y + \cancel{z} &= 3 \\
 & -x + 3y - \cancel{z} &= -4 \\
\hline
(1) + (2) \Rightarrow & 2x + 5y &= -1 \qquad \text{..... (4)} \\
(2) + (3) \Rightarrow & -x + 3y - \cancel{z} &= -4 \\
 & 3x + 2y + \cancel{z} &= -\frac{1}{2} \\
\hline
 & 2x + 5y &= -\frac{9}{2} \qquad (5) \\
 & \overset{(-)}{2x} \overset{(-)}{+} 5y &= \overset{(+)}{-1} \\
\hline
 & 0 &\neq \frac{-7}{2}
\end{array}
$$

This is a contradiction. This means the system is inconsistent and has no solutions.

(iii) $\frac{y+z}{4} = \frac{z+x}{3} = \frac{x+y}{2}$

$x + y + z = 27$

$$\frac{y+z}{4} = \frac{z+x}{3} \Rightarrow \begin{aligned} 3y + 3z &= 4z + 4x \quad \ldots(1) \\ 4x - 3y + z &= 0 \end{aligned}$$

$$\frac{z+x}{3} = \frac{x+y}{2} \Rightarrow \begin{aligned} 2z + 2x &= 3x + 3y \\ 1x + 3y - 2z &= 0 \quad (2) \\ x + y + z &= 27 \quad (3) \end{aligned}$$

$$\begin{array}{l} 4x \cancel{-3y} + z = 0 \\ x + \cancel{3y} - 2z = 0 \\ \hline 5x \quad - z = 0 \quad \ldots(4) \end{array}$$

$$\begin{array}{lll} (3) \times 3 & \Rightarrow & 3x \cancel{+3y} + 3z = 81 \\ & & (-)\ (-)\ \ (+)\ \ \ (-) \\ (2) & \Rightarrow & x + 3y - 2z = 0 \\ \hline & & 2x \quad \cancel{+5z} = 81 \quad \ldots(4) \\ (4) \times 5 & \Rightarrow & 25x \quad \cancel{-5z} = 0 \\ \hline & & 27x \quad = 81 \\ & & x \quad = 3 \end{array}$$

Sub. x = 3 in (4) ⇒ 5(3) – z = 0
15 – z = 0
-z = -15
z = 15
Sub, x = 3, z = 15 in (3)
x + y + z = 27
3 + y + 15 = 27
y = 27 – 18 = 9
x = 3, y = 9, z = 15
∴ The system has unique solutions.

Question 3.
Vani, her father and her grand father have an average age of 53. One-half of her grand father’s age plus one-third of her father’s age plus one fourth of Vani’s age is 65. Four years ago if Vani’s

grandfather was four times as old as Vani then how old are they all now?
Solution:
Let Vani's age be x
Let Vani's father's age be y
Let Vani's grand father's age be z.

$$\frac{x+y+z}{3} = 53 \Rightarrow x+y+z = 159 \quad ...(1)$$

$$\frac{x}{4}+\frac{y}{3}+\frac{z}{2} = 65 \Rightarrow 3x+4y+6z = 780 \quad ...(2)$$

$$(x-4)\,4 = z-4 \Rightarrow 4x - z = 12 \quad ...(3)$$

$$(1) \times 4 \Rightarrow 4x + 4y + 4z = 636$$

$$(3) \Rightarrow \overset{(-)}{4x} \quad \overset{(+)}{-}\ z = \overset{(-)}{12}$$

$$4y + 5z = 624 \quad ...(4)$$

$$(2) \times 4 \Rightarrow 12x + 16y + 24z = 3120$$

$$(3) \times 3 \Rightarrow \overset{(-)}{12x} \quad \overset{(+)}{-}\ 3z = \overset{(-)}{36}$$

$$16y + 27z = 3084 \quad ...(5)$$

$$(4) \times 4 \Rightarrow \overset{(-)}{16y} \overset{(-)}{+} 20z = \overset{(-)}{2496}$$

$$7z = 588$$

$$z = 84$$

Sub, z = 84 in (3), we get
4x – 84 = 12
4x = 96
x = 24
Sub, x = 24, z = 84 in (1) we get
24 + y + 84 = 159
y = 159 – 108
= 51
∴ Vani's age = 24 years
Her father's age =51 years
Her grand father's age = 84 years.

Question 4.
The sum of the digits of a three-digit number is 11. If the digits are reversed, the new number is 46 more than five times the former number. If the hundreds digit plus twice the tens digit is equal to

the units digit, then find the original three digit number?
Solution:
Let the number be $100x + 10y + z$.
Reversed number be $100z + 10y + x$.
$x + y + z = 11$ …………… (1)
$100z + 10y + x = 5(100x + 10y + z) + 46$
$100z + 10y + x = 500x + 50y + 5z + 46$
$499x + 40y – 95z -46$ ………….. (2)
$x + 2y = z$
$x + 2y – z = 0$ ……………. (3)

$$\begin{array}{lrl}
 & x+y+z = 11 & \ldots(1) \\
 & x+2y-z = 0 & \ldots(3) \\
\hline
(1)+(3) \Rightarrow & 2x+3y = 11 & \ldots(4)
\end{array}$$

$$\begin{array}{lll}
(2) \Rightarrow & 499x + 40y - 95z = -46 & \\
(3)\times 95 \Rightarrow & \overset{(-)}{95x} \overset{(-)}{+} 190y \overset{(+)}{-} 95z = -\ 0 & \\
\hline
 & 404x - 150y = -46 & \ldots(5)
\end{array}$$

$$\begin{array}{ll}
(4)\times 50 \Rightarrow & 100x + 150y = 550 \\
(5) \Rightarrow & 404x - 150y = -46 \\
\hline
 & 504x = 504 \\
 & x = 1
\end{array}$$

Sub. $x = 1$ in (4)

$$\begin{aligned}
2 \times 1 + 3y &= 11 \\
3y &= 9 \\
y &= 3
\end{aligned}$$

Sub. $x = 1, y = 3$ in (1)

$$\begin{aligned}
1 + 3 + z &= 11 \\
z &= 7
\end{aligned}$$

$\therefore$ The number is $xyz = 137$

Question 5.
There are 12 pieces of five, ten and twenty rupee currencies whose total value is ₹105. When first 2 sorts are interchanged in their numbers its value will be increased by ₹20. Find the number of

currencies in each sort.
Solution:
Let x, y and z be number of currency pieces of 5,10,20 rupees
x + y + z = 12 ………. (1)
5x + 10y + 20z = 105 ………… (2)
10x + 5y + 20z = 125 …………. (3)

$$
\begin{array}{llrrrll}
(1)\times 5 & \Rightarrow & 5x + & 5y + & 5z & = 60 & \\
 & & (-) & (-) & (-) & (-) & \\
(3) & \Rightarrow & 5x + & 10y + & 20z & = 105 & \\
\hline
 & & & -5y & -15z & = -45 & \ldots(4)
\end{array}
$$

$$
\begin{array}{llrrrll}
(2)\times 2 & \Rightarrow & 10x + & 20y + & 40z & = 210 & \\
 & & (-) & (-) & (-) & (-) & \\
(3) & \Rightarrow & 10x + & 5y + & 20z & = 125 & \\
\hline
 & & & 15y & +20z & = 85 & \ldots(5)
\end{array}
$$

$$
\begin{array}{llrrl}
(4)\times 3 & \Rightarrow & -\cancel{15y} & -45z & = -135 \\
 & & \cancel{15y} & +20z & = 85 \\
\hline
(5) & & & -25z & = -50 \\
 & & & z & = 2
\end{array}
$$

Sub, z = 2 in (5), we get
15y + 20 × 2 = 85
15y = 45
y = 3
Sub; y = 3, z = 2 in (1)
x + y + z = 12
x = 7
∴ The solutions are
the number of □ 5 are 7
the number of □ 10 are 3
the number of □ 20 are 2

Ex 3.2

Question 1.
Find the GCD of the given polynomials
(i) $x^4 + 3x^3 - x - 3, x^3 + x^2 - 5x + 3$
(ii) $x^4 - 1, x^3 - 11x^2 + x - 11$
(iii) $3x^4 + 6x^3 - 12x^4 - 24x, 4x^4 + 14x^3 + 8x^2 - 8x$
(iv) $3x^3 + 3x^2 + 3x + 3, 6x^3 + 12x^2 + 6x + 12$
Solution:
$x^4 + 3x^3 - x - 3, x^3 + x^2 - 5x + 3$
Let $f(x) = x^4 + 3x^3 - x - 3$
$g(x) = x^3 + x^2 - 5x + 3$

$$
\begin{array}{r|l}
 & x + 2 \\
\hline
x^3 + x^2 - 5 + 3 & \cancel{x^4} + 3x^3 + 0x^2 \quad x - 3 \\
 & \overset{(-)}{\cancel{x^4}} \overset{(-)}{+} x^3 \overset{(-)}{-} 5x^2 \overset{(-)}{+} 3x \\
\hline
 & \cancel{2x^3} + 5x^2 - 4x \quad - 3 \\
 & \overset{(-)}{\cancel{2x^3}} \overset{(-)}{+} 2x^2 \overset{(-)}{-} 10x \overset{(-)}{+} 6 \\
\hline
 & 3x^2 \quad + 6x - 9
\end{array}
$$

$= 3(x^2 + 2x - 3) \neq 0$

Note that 3 is not a divisor of g(x). Now dividing $g(x) = x^3 + x^2 - 5x + 3$ by the new remainder $x^2 + 2x - 3$ (leaving the constant factor 3) we get

$$\begin{array}{r|l} & x-1 \\ \hline x^2+2x-3 & x^3+x^2-5x+3 \\ & x^3+2x^2-3x \\ & (-)\ (-)\ (+) \\ \hline & \cancel{-x^2}\cancel{-2x}\cancel{+3} \\ & (+)\ (+)\ (-) \\ & \cancel{-x^2}\cancel{-2x}\cancel{+3} \\ \hline & 0 \end{array}$$

Here we get zero remainder

G.C.D of $(x^4 + 3x^3 - x - 3)$, $(x^3 + x^2 - 5x + 3)$ is $(x^2 + 2x - 3)$

(ii) $x^4 - 1$, $x^3 - 11x^2 + x - 11$

$$\begin{array}{r|l} & x+11 \\ \hline x^3-11x^2+x-11 & \cancel{x^4}+0x^3+0x^2+0x-1 \\ & \cancel{x^4}-11x^3+x^2-11x \\ & (-)\ (+)\ (-)\ (+) \\ \hline & \cancel{11x^3}-x^2+11x-1 \\ & \cancel{11x^3}-121x^2+11x-121 \\ & (-)\ (+)\ (-)\ (+) \\ \hline & 120x^2+120 \neq 0 \end{array}$$

$= 120(x^2 + 0x + 1)$

$$\begin{array}{r|l} & x \\ \hline x^2+0x+1 & \cancel{x^3}-11x^2\cancel{+x}-11 \\ & \cancel{x^3}+0x^2\cancel{+x} \\ & (-)\ (-)\ (-) \\ \hline & -11x^2-11 \neq 0 \\ \hline & -11(x^2+1) \neq 0 \end{array}$$

$$\begin{array}{r|l} & 1 \\ \hline x^2+1 & \cancel{x^2}+\cancel{0x}\cancel{+1} \\ & \cancel{x^2}+\cancel{0x}\cancel{+1} \\ & (-)\ (-)\ (-) \\ \hline & 0 \text{ Remainder} \end{array}$$

G.C.D. $= x^2 + 1$

(iii) $3x^4 + 6x^3 - 12x^2 - 24x$, $4x^4 + 14x^3 + 8x^2 - 8x$

$4x^4 + 14x^3 + 8x^2 - 8x = 2(2x^4 + 7x^3 + 4x^2 - 4x)$

Let us divide

$(2x^4 + 7x^3 + 4x^2 + 4x)$ by $x^4 + 2x^3 - 4x^2 - 8x$

$$
\begin{array}{r|l}
 & \quad 2 \\
\hline
x^4 + 2x^3 - 4x^2 - 8x & 2x^4 + 7x^3 + 4x^2 - 4x \\
 & 2x^4 + 4x^3 - 8x^2 - 16x \\
 & (-)\ \ (-)\ \ (+)\ \ (+) \\
\hline
 & \quad 3x^3 + 12x^2 + 12x \div 3
\end{array}
$$

$(x^3 + 4x^3 + 4x) \neq 0$

Now let us divide

$x^4 + 2x^3 - 4x^2 - 8x$ by $x^3 + 4x^2 + 4x$

$$
\begin{array}{r|l}
 & \quad x-2 \\
\hline
x^3 + 4x^2 + 4x & x^4 + 2x^3 - 4x^2 - 8x \\
 & x^4 + 4x^3 + 4x^2 \\
 & (-)\ \ (-)\ \ (-) \\
\hline
 & -2x^3 - 8x^2 - 8x \\
 & \overset{(+)}{-}2x^3 \overset{(+)}{-} 8x \overset{(+)}{-} 8x \\
\hline
 & \quad 0 \\
\hline
\end{array}
$$

$\therefore$ $x^3 + 4x^2 + 4x$ is the G.C.D of $3x^4 + 6x^3 - 12x^2 - 24x$, $4x^4 + 14x^3 + 8x^2 - 8x$

$\therefore$ Ans $x(x^2 + 4x + 4)$

(iv) $f(x) = 3x^3 + 3x^2 + 3x + 3 = 3(x^3 + x^2 + x + 1)$

$g(x) = 6x^3 + 12x^2 + 6x + 12$

$= 6(x^3 + 2x^2 + x + 2)$

$= 2 \times 3\,(x^3 + 2x^2 + x + 2)$

$f(x) \Rightarrow x^3 + x^2 + x + 1$

$$\frac{g(x)}{6} \Rightarrow x^3 + 2x^2 + x + 2$$

$$\begin{array}{r|l} & 1 \\ \hline x^3 + x^2 + x + 1 & \cancel{x^3} + 2x^2 + x + 2 \\ & \cancel{x^3} + x^2 + x + 1 \\ & (-)\ (-)\ (-)\ (-) \\ \hline & x^2 \qquad + 1 \neq 0 \end{array}$$

$$\begin{array}{r|l} & x \\ \hline x^2 + 1 & \cancel{x^3} + x^2 + \cancel{x} + 1 \\ & \cancel{x^3} + \quad + \cancel{x} \\ & (-) \qquad (-) \\ \hline & x^2 \qquad + 1 \neq 0 \end{array}$$

$$\begin{array}{r|l} & 1 \\ \hline x^2 + 1 & \cancel{x^2} + \cancel{1} \\ & \cancel{x^2} + \cancel{1} \\ & (-)\ (-) \\ \hline & 0 \text{ Remainder} \end{array}$$

G.C.D. $= 3(x^2 + 1)$

Question 2.
Find the LCM of the given expressions,
(i) $4x^2y$, $8x^3y^2$
(ii) $-9a^3b^2$, $12a^2b^2c$
(iii) $16m$, $-12m^2n^2$, $8n^2$
(iv) $p^2 – 3p + 2$, $p^2 – 4$
(v) $2x^2 – 5x – 3$, $4x^2 – 36$
(vi) $(2x^2 – 3xy)^2$, $(4x – 6y)^3$, $8x^3 – 27y^3$
Solution:
(i) $4x^2y$, $8x^3y^2$
$4x^2y = \underline{2} \times \underline{2}\ x^2y$
$8x^3y^2 = \underline{2} \times \underline{2} \times \underline{2}\ x^3y^2$
L.C.M. $= 2 \times 2 \times 2\ x^3y^2$
$= 8x^3\ y^2$

(ii) $-9a^3b^2 = \underline{-3} \times \underline{3}\ a^3b^2$
$12a^2b^2c = \underline{2} \times \underline{3} \times 2a^2b^2c$

L.C.M. = $-3 \times 3 \times 2 \times 2\ a^3b^2c$
$= -36a^3b^2c$

(iii) $16m, -12m^2n^2, 8n^2$
$16\ m = \underline{2} \times \underline{2} \times 2 \times 2 \times m$
$-12m^2n^2 = \underline{-2} \times \underline{2} \times 3 \times m^2n^2$
$8n^2 = \underline{2} \times \underline{2} \times 2 \times n^2$
L.C.M.= $-2 \times 2 \times 2 \times 2 \times 3\ m^2n^2$
$= -48\ m^2n^2$

(iv) $p^2 - 3p + 2, p^2 - 4$

$$p^2 - 3p + 2 = (p-2)(p-1)$$
$$p^2 - 4 = (p+2)(p-2)$$
$$\text{L.C.M.} = (p-2)(p+2)(p-1)$$

Hint:
2
−2 −1

(v) $2x^2 - 5x - 3, 4x^2 - 36$
$2x^2 - 5x - 3 = \underline{(x-3)}(2x+1)$
$4x^2 - 36 = 4(x+3)\underline{(x-3)}$
L.C.M. = $4(x+3)(x-3)(2x+1)$

(vi) $(2x^2 - 3xy)^2 = (x(2x-3y))^2$
$(4x - 6y)^3 = (2(2x-3y))^3$
$8x^3 - 27y^3 = (2x)^3 - (3y)^3$
$= (2x-3y)(4x^2 + 6xy + 9y^2)$
L.C.M. = $2^3 \times x^2 (2x-3y)^3 (4x^2 + 6xy + 9y^2)$

Ex 3.3

Question 1.
Find the LCM and GCD for the following and verify that f(x) × g(x) = LCM × GCD.
(i) $21x^2y, 35xy^2$
(ii) $(x^3 - 1)(x + 1), x^3 + 1$
(ii) $(x^3 - 1)(x + 1), (x^3 - 1)$
(iii) $(x^2y + xy^2), (x^2 + xy)$
Solution:
(i) $f(x) = 21x^2y = 3 \times 7x^2y$
$g(x) = 35xy^2 = 7 \times 5xy^2$
G.C.D. = 7xy
L.C.M. = $7 \times 3 \times 5 \times x^2y^2 = 105x^2 \times y^2$
L.C.M × G.C.D = f(x) × g(x)
$105x^2y^2 \times 7xy = 21x^2y \times 35xy^2$
$735x^3y^3 = 735x^3y^3$
Hence verified.

(ii) $(x^3 - 1)(x + 1) = (x - 1)(x^2 + x + 1)(x + 1)$
$x^3 + 1 = (x + 1)(x^2 - x + 1)$
G.C.D = $(x+ 1)$
L.C.M = $(x - 1)(x + 1)(x^2 + x + 1)(x^2 - x + 1)$
∴ L.C.M. × G.C.D = f(x) × g(x)
$(x - 1)(x + 1)(x^2 + x + 1)(x^2 - x + 1) = (x - 1)$
$(x^2 + x + 1) \times (x + 1)(x^2 - x + 1)$
$(x^3 - 1)(x + 1)(x^3 + 1) = (x^3 - 1)(x + 1)(x^3 + 1)$
∴ Hence verified.

(iii) $f(x) = x^2y + xy^2 = xy(x + y)$

$g(x) = x^2 + xy = x(x + y)$

L.C.M. $= x\,y\,(x + y)$

G.C.D. $= x\,(x + y)$

To verify:

L.C.M. × G.C.D. $= xy(x + y) \times (x + y)$

$= x^2y\,(x + y)^2$ ……….. (1)

$f(x) \times g\,(x) = (x^2y + xy^2)(x^2 + xy)$

$= x^2y\,(x + y)^2$ …………… (2)

∴ L.C.M. × G.C.D $= f(x) \times g\{x)$.

Hence verified.

Question 2.

Find the LCM of each pair of the following polynomials

(i) $a^2 + 4a - 12$, $a^2 - 5a + 6$ whose GCD is $a - 2$

(ii) $x^4 - 27a^3x$, $(x - 3a)^2$ whose GCD is $(x - 3a)$

Solution:

(i) $f(x) = a^2 + 4a - 12 = (a + 6)(a - 2)$

$g(x) = a^2 - 5a + 6 = (a-3)(a-2)$

G.C.D. $= (a-2)$,

L.C.M. $= (a-2)(a-3)(a+6)$

Hint:
−12
6 −2

$$\text{L.C.M.} = \frac{f(x) \times g(x)}{\text{G.C.D}}$$

$$= \frac{(a+b)(a-2) \times (a-3)(a-2)}{(a-2)}$$

$$= (a-2)\,(a-3)(a+6)$$

L.C.M. $= (a-3)(a^2 + 4a - 12)$

Hint:
6
−3 −2

(ii) $f(x) = x^4 - 27a^3x = x(x^3 - (3a)^3)$

$g(x) = (x - 3a)^2$

G.C.D $= (x - 3a)$

L.C.M. × G.C.D $= f(x) \times g(x)$

L. C.M $= \frac{x\left(x^3-(3a)^3\right) \times (x-3a)^2}{(x-3a)}$

L.C.M = $x(x^3 - (3a)^3) . (x - 3a)$
$= x(x - 3a)^2 (x^2 + 3ax + 9a^2)$

Question 3.
Find the GCD of each pair of the following polynomials
(i) $12(x^4 - x^3)$, $8(x^4 - 3x^3 + 2x^2)$ whose LCM is $24x^3 (x - 1)(x - 2)$
(ii) $(x^3 + y^3)$, $(x^4 + x^2y^2 + y^4)$ whose LCM is $(x^3 + y^3) (x^2 + xy + y^2)$
Solution:
(i) $f(x)= 12(x^4 - x^3)$
$g(x) = 8(x^4 - 3x^3 + 2x^2)$
L.C.M = $24x^3 (x - 1)(x - 2)$

$$\text{G.C.D.} = \frac{f(x)\times g(x)}{\text{L.C.M.}}$$
$$= \frac{12(x^4 - x^3)\times 8(x^4 - 3x^3 + 2x^2)}{24x^3(x-1)(x-2)}$$
$$= \frac{4x^3(x-1)x^2(x^2-3x+2)}{x^3(x-1)(x-2)}$$
$$= \frac{4x^2(x-2)(x-1)}{(x-2)}$$
$$= 4x^2(x-1)$$

(ii) $(x^3 + y^3)$, $(x^4 + x^2y^2 + y^4)$
L.C.M. = $(x^3 + y^3)(x^2 + xy + y^2)$
$$\text{G.C.D} = \frac{f(x)(g(x))}{\text{L.C.M.}}$$
$$= \frac{\cancel{(x^3+y^3)}(x^4+x^2y^2+y^4)}{\cancel{(x^3+y^3)}(x^2+xy+y^2)}$$
$$= \frac{(x^2-xy+y^2)(x^2+\cancel{xy}+y^2)}{(x^2+\cancel{xy}+y^2)} = (x^2 - xy + y^2)$$

Question 4.
Given the LCM and GCD of the two polynomials p(x) and q(x) find the unknown polynomial in the following table

S. No	LCM	GCD	$p(x)$	$q(x)$
(i)	$a^3 - 10a^2 + 11a + 70$	$a - 7$	$a^2 - 12a + 35$	
(ii)	$(x^2 + y^2)(x^4 + x^2y^2 + y^4)$	$(x^2 - y^2)$		$(x^4 - y^4)(x^2 + y^2 - xy)$

Solution:

(i) L.C.M $= a^3 - 10a^2 + 11a + 70$

G.C.D $= a - 7$

$p(x) = a^2 - 12a + 35$

$$q(x) = \frac{\text{L.C.M.} \times \text{G.C.D}}{p(x)}$$

Hint:

$$\begin{array}{c|cccc} 7 & 1, & -10, & 11, & 70 \\ & 0, & 7, & -21, & -70 \\ \hline & 1, & -3, & -10, & \underline{0} \end{array}$$

$$= \frac{(a^3 - 10a^2 + 11a + 70)(a-7)}{(a^2 - 12a + 35)}$$

Hint: 35 → −5, −7

$$= \frac{(a^2 - 3a - 10)(a-7)\cancel{(a-7)}}{(a-5)\cancel{(a-7)}}$$

$$= \frac{(a+2)\cancel{(a-5)}(a-7)}{\cancel{(a-5)}}$$

Hint: −10 → +2, −5

$\therefore q(x) = (a+2)(a-7)$

(ii) L.C.M $= (x^2 + y^2)(x^4 + x^2y^2 + y^4)$

G.C.D. $= (x^2 - y^2)$

$q(x) = (x^4 - y^4)(x^2 + y^2 - xy)$

$$p(x) = \frac{\text{L.C.M} \times \text{G.C.D}}{q(x)}$$

$$= \frac{(x^2 + y^2)(x^4 + x^2y^2 + y^4)(x^2 - y^2)}{(x^4 - y^4)(x^2 + y^2 - xy)}$$

$$= \frac{\cancel{(x^2 + y^2)}(x^2 + xy + y^2)\cancel{(x^2 - xy + y^2)}\cancel{(x^2 - y^2)}}{\cancel{(x^2 + y^2)}\cancel{(x^2 - y^2)}\cancel{(x^2 + y^2 - xy)}}$$

$= (x^2 + xy + y^2)$

Ex 3.4

Question 1.
Reduce each of the following rational expressions to its lowest form.

(i) $\frac{x^2-1}{x^2+x}$ (ii) $\frac{x^2-11x+18}{x^2-4x+4}$

(iii) $\frac{9x^2+81x}{x^3+8x^2-9x}$

(iv) $\frac{p^2-3p-40}{2p^3-24p^2+64p}$

Solution:

(i) $\frac{x^2-1}{x^2+x} = \frac{(x+1)(x-1)}{x(x+1)} = \frac{x-1}{x}$

(ii) $\frac{x^2-11x+18}{x^2-4x+4} = \frac{(x-2)(x-9)}{(x-2)(x-2)} = \frac{x-9}{x-2}$

(iii) $\frac{9x^2+81x}{x^3+8x^2-9x} = \frac{9x(x+9)}{x(x^2+8x-9)} = \frac{9(x+9)}{(x+9)(x-1)}$

$= \frac{9}{x-1}$

Hint:

18: -2, -9

4: -2, -2

(iv) $\frac{p^2-3p-40}{2p^3-24p^2+64p} = \frac{(p-8)(p+5)}{2p(p^2-12p+32)}$

$= \frac{\cancel{(p-8)}(p+5)}{2p(p-4)\cancel{(p-8)}}$

$= \frac{p+5}{2p(p-4)}$

Question 2.
Find the excluded values, if any of the following expressions.

(i) $\frac{y}{y^2-25}$ **(ii)** $\frac{t}{t^2-5t+6}$

(iii) $\frac{x^2+6x+8}{x^2+x-2}$ **(iv)** $\frac{x^3-27}{x^3+x^2-6x}$

Solution:
(i) $\frac{y}{y^2-25} = \frac{y}{(y+5)(y-5)}$ is undefined when (y + 5) (y – 5) = 0 that is y = -5, 5
∴ The excluded values are -5, 5

(ii) $\frac{t}{t^2-5t+6}$ is undefined when $t^2 - 5t + 6 = 0$ i.e.
(t – 3) (t – 2) = 0 ⇒ t = 3, 2

∴ The excluded values are 3, 2

(iii) $\frac{x^2+6x+8}{x^2+x-2}$ is undefined when $x^2 + x - 2 = 0$ i.e.
$(x + 2)(x + 1) = 0$
∴ The excluded values are 2, 1

(iv) $\frac{x^3-27}{x^3+x^2-6x}$ is undefined when $x^3 + x^2 - 6x = 0$, i.e
$x(x^2 + x - 6) = 0$
$x(x + 3)(x - 2) = 0$
∴ The excluded values are -3, 2

Ex 3.5

Question 1.
Simplify

(i) $\dfrac{4x^2y}{2z^2} \times \dfrac{6xz^3}{20y^4}$

(ii) $\dfrac{p^2-10p+21}{p-7} \times \dfrac{p^2+p-12}{(p-3)^2}$

(iii) $\dfrac{5t^3}{4t-8} \times \dfrac{6t-12}{10t}$

Solution:

(i) $\dfrac{4x^2y}{2z^2} \times \dfrac{6xz^3}{20y^4} = \dfrac{3x^3z}{5y^3}$

(ii) $\dfrac{p^2-10p+21}{p-7} \times \dfrac{p^2+p-12}{(p-3)^2}$

$= \dfrac{\cancel{(p-7)}\,\cancel{(p-3)}}{\cancel{(p-7)}} \times \dfrac{(p+4)\,\cancel{(p-3)}}{\cancel{(p-3)}\,\cancel{(p-3)}} = p+4$

(iii) $\dfrac{5t^3}{4t-8} \times \dfrac{6t-12}{10t} = \dfrac{\cancel{5}t^3 \times \cancel{6}^3\,\cancel{(t-2)}}{4\,\cancel{(t-2)} \times \cancel{10}_2\,t} = \dfrac{3t^2}{4}$

Question 2.
Simplify

(i) $\dfrac{x+4}{3x+4y} \times \dfrac{9x^2-16y^2}{2x^2+3x-20}$

(ii) $\dfrac{x^3-y^3}{3x^2+9xy+6y^2} \times \dfrac{x^2+2xy+y^2}{x^2-y^2}$

Solution:

(i) $\dfrac{x+4}{3x+4y} \times \dfrac{9x^2-16y^2}{2x^2+3x-20}$

$$= \frac{\cancel{(x+4)}\,((3x)^2-(4y)^2)}{(3x+4y)\,\cancel{(x+4)}\,(2x-5)}$$

$$= \frac{(3x+4y)(3x-4y)}{(3x+4y)(2x-5)} = \frac{3x-4y}{(2x-5)}$$

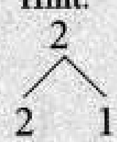
(ii) $\dfrac{x^3-y^3}{3x^2+9xy+6y^2} \times \dfrac{x^2+2xy+y^2}{x^2-y^2}$

$$= \frac{\cancel{(x-y)}\,(x^2+xy+y^2)(x^2+2xy+y^2)}{3(x^2+3xy+2y^2)(x+y)\,\cancel{(x-y)}}$$

$$= \frac{(x^2+xy+y^2)(x+y)^2}{3(x+2y)(x+y)(x+y)}$$

$$= \frac{(x^2+xy+y^2)}{3(x+2y)}$$

Hint:



Question 3.
Simplify

(i) $\dfrac{2a^2+5a+3}{2a^2+7a+6} \div \dfrac{a^2+6a+5}{-5a^2-35a-50}$

(ii) $\dfrac{b^2+3b-28}{b^2+4b+4} \div \dfrac{b^2-49}{b^2-5b-14}$

(iii) $\dfrac{x+2}{4y} \div \dfrac{x^2-x-6}{12y^2}$

(iv) $\dfrac{12t^2-22t+8}{3t} \div \dfrac{3t^2+2t-8}{2t^2+4t}$

Solution:

(i) $\dfrac{2a^2+5a+3}{2a^2+7a+6} \div \dfrac{a^2+6a+5}{-5a^2-35a-50}$

Hint:
6
$\frac{3}{2}$ $\frac{2}{2}$

$= \dfrac{2a^2+5a+3}{2a^2+7a+6} \times \dfrac{-5a^2-35a-50}{a^2+6a+5}$

$= \dfrac{(2a+3)\cancel{(a+1)}}{2a^2+7a+6} \times \dfrac{-5(a^2+7a+10)}{(a+5)\cancel{(a+1)}}$

$= \dfrac{(2a+3)(-5)\cancel{(a+5)}(a+2)}{(2a^2+7a+6)\cancel{(a+5)}}$

Hint:
+10
5 +2

$= \dfrac{(2a+3)(-5)\cancel{(a+2)}}{(2a+3)\cancel{(a+2)}} \qquad = -5$

(ii) $\dfrac{b^2+3b-28}{b^2+4b+4} \div \dfrac{b^2-49}{b^2-5b-14}$

Hint:
−28
7 −4
4
2 2
−14
−7 2

$= \dfrac{b^2+3b-28}{b^2+4b+4} \times \dfrac{b^2-5b-14}{b^2-49}$

$= \dfrac{\cancel{(b+7)}(b-4)}{(b+2)\cancel{(b+2)}} \times \dfrac{\cancel{(b-7)}\cancel{(b+2)}}{\cancel{(b+7)}\cancel{(b-7)}}$

$= \dfrac{b-4}{b+2}$

(iii) $\dfrac{x+2}{4y} \div \dfrac{x^2-x-6}{12y^2} = \dfrac{x+2}{\cancel{4y}} \times \dfrac{{}^{3}\cancel{12y^2}}{x^2-x-6}$

$$= \frac{\cancel{(x+2)}}{1} \times \frac{3y}{(x-3)\cancel{(x+2)}}$$

$$= \frac{3y}{x-3}$$

(iv) $\dfrac{12t^2-22t+8}{3t} \div \dfrac{3t^2+2t-8}{2t^2+4t}$

$$= \frac{12t^2-22t+8}{3t} \times \frac{2t^2+4t}{3t^2+2t-8}$$

$$= \frac{2(6t^2-11t+4)}{3\cancel{t}} \times \frac{2\cancel{t}(t+2)}{3t^2+2t-8}$$

Hint:

24: $\dfrac{-\cancel{8}^{4}}{\cancel{6}_{3}}$, $\dfrac{-\cancel{3}^{1}}{\cancel{6}_{2}}$

-24: $\dfrac{\cancel{6}^{2}}{\cancel{3}}$, $\dfrac{-4}{3}$

$$= \frac{2\cancel{(3t-4)}(2t-1)}{3} \times \frac{2\cancel{(t+2)}}{\cancel{(t+2)}\cancel{(3t-4)}}$$

$$= \frac{4(2t-1)}{3}$$

Question 4.
If x = $\frac{a^2+3a-4}{3a^2-3}$ and y = $\frac{a^2+2a-8}{2a^2-2a-4}$ find the value of x^2y^2.

Solution:

$$\text{(i)}\quad x = \frac{a^2+3a-4}{3a^2-3},\; y = \frac{a^2+2a-8}{2a^2-2a-4}$$

$$x^2y^{-2} = \left(\frac{a^2+3a-4}{3a^2-3}\right)\left(\frac{a^2+3a-4}{3a^2-3}\right)\times\left(\frac{2a^2-2a-4}{a^2+2a-8}\right)^2$$

$$= \frac{\cancel{(a+4)}\,\cancel{(a-1)}}{3\cancel{(a+1)}\,\cancel{(a-1)}}\times\frac{\cancel{(a+4)}\,\cancel{(a-1)}}{3\cancel{(a+1)}\,\cancel{(a-1)}}$$

$$\times\frac{2\cancel{(a-2)}\,\cancel{(a+1)}\,2\cancel{(a-2)}\,\cancel{(a+1)}}{\cancel{(a+4)}\,\cancel{(a-2)}\,\cancel{(a+4)}\,\cancel{(a-2)}}$$

$$= \frac{4}{9}$$

Question 5.

If a polynomial $p(x) = x^2 - 5x - 14$ is divided by another polynomial $q(x)$ we get $\frac{x-7}{x+2}$ find $q(x)$.

Solution:

$p(x) = x^2 - 5x - 14$

$$(x^2-5x-14)\div q(x) = \frac{x-7}{x+2}$$

$$(x^2-5x-14)\times\frac{1}{q(x)} = \frac{x-7}{x+2}$$

$$\frac{1}{q(x)} = \frac{x-7}{x+2}\times\frac{1}{x^2-5x-14}$$

$$\frac{1}{q(x)} = \frac{\cancel{x-7}}{x+2}\times\frac{1}{\cancel{(x-7)}(x+2)} = \frac{1}{(x+2)^2}$$

$$\therefore q(x) = (x+2)^2 = x^2+4x+4$$

Ex 3.6

Question 1. Simplify

(i) $\dfrac{x(x+1)}{x-2}+\dfrac{x(1-x)}{x-2}$

(ii) $\dfrac{x+2}{x+3}+\dfrac{x-1}{x-2}$ (iii) $\dfrac{x^3}{x-y}+\dfrac{y^3}{y-x}$

Solution:

(i) $\dfrac{x(x+1)}{x-2}+\dfrac{x(1-x)}{x-2}=\dfrac{x(x+1)+x(1-x)}{(x-2)}=\dfrac{2x}{x-2}$

(ii) $\dfrac{x+2}{x+3}+\dfrac{x-1}{x-2}=\dfrac{(x-2)(x+2)+(x+3)(x-1)}{(x+3)(x-2)}$

$$=\frac{x^2-4+x^2+2x-3}{(x+3)(x-2)}$$

$$=\frac{2x^2+2x-7}{(x+3)(x-2)}$$

(iii) $\dfrac{x^3}{x-y}+\dfrac{y^3}{y-x}=\dfrac{x^3}{x-y}-\dfrac{y^3}{x-y}=\dfrac{x^3-y^3}{(x-y)}$

$$=\frac{(x-y)(x^2+xy+y^2)}{x-y}$$

$$=x^2+xy+y^2$$

Question 2.
Simplify

(i) $\dfrac{(2x+1)(x-2)}{x-4}-\dfrac{(2x^2-5x+2)}{x-4}$

(ii) $\dfrac{4x}{x^2-1}-\dfrac{x+1}{x-1}$

Solution:

(i) $\frac{(2x+1)(x-2)}{x-4}-\frac{(2x^2-5x+2)}{x-4}$

$= \frac{2x^2-3x-2-2x^2+5x-2}{x-4} = \frac{2x-4}{x-4}$

$= \frac{2(x-2)}{x-4}$

(ii) $\frac{4x}{x^2-1}-\frac{x+1}{x-1} = \frac{4x}{(x+1)(x-1)}-\frac{x+1}{(x-1)}$

$= \frac{4x-(x+1)(x+1)}{(x+1)(x-1)} = \frac{4x-(x^2+2x+1)}{(x+1)(x-1)}$

$= \frac{4x-x^2-2x-1}{(x+1)(x-1)}$

$= \frac{-x^2+2x-1}{(x+1)(x-1)} = \frac{-(x^2-2x+1)}{(x+1)(x-1)}$

$= \frac{-(x-1)\cancel{(x-1)}}{(x+1)\cancel{(x-1)}}$

$= \frac{-(x-1)}{(x+1)} = \frac{1-x}{1+x}$

Question 3.
Subtract $\frac{1}{x^2+2}$ from $\frac{2x^3+x^2+3}{(x^2+2)^2}$

Solution:

$$\frac{2x^3+x^2+3}{(x^2+2)^2}-\frac{1}{x^2+2}$$

$$=\frac{2x^3+x^2+3-(x^2+2)}{(x^2+2)^2}$$

$$=\frac{2x^3+\cancel{x^2}+3-\cancel{x^2}-2}{(x^2+2)^2}=\frac{2x^3+1}{(x^2+2)^2}$$

Question 4.
Which rational expression should be subtracted from $\frac{x^2+6x+8}{x^3+8}$ to get $\frac{3}{x^2-2x+4}$
Solution:

$$\frac{x^2+6x+8}{x^3+8}-q(x)=\frac{3}{x^2-2x+4}$$

$$q(x)=\frac{x^2+6x+8}{x^3+8}-\frac{3}{x^2-2x+4}$$

$$=\frac{(x+4)(x+2)}{(x+2)\left(x^2-2x+4\right)}-\frac{3}{x^2-2x+4}$$

$$=\frac{(x+4)(x+2)}{(x+2)\left(x^2-2x+4\right)}-\frac{3}{x^2-2x+4}$$

$$=\frac{x+4-3}{x^2-2x+4}$$

$$=\frac{x+1}{x^2-2x+4}$$

Question 5.

If $A=\dfrac{2x+1}{2x-1}$, $B=\dfrac{2x-1}{2x+1}$ **find** $\dfrac{1}{A-B}-\dfrac{2B}{A^2-B^2}$

Solution:

$$A=\frac{2x+1}{2x-1},\ B=\frac{2x-1}{2x+1}$$

$$=\frac{1}{A-B}-\frac{2B}{A^2-B^2}=\frac{A+B-2B}{(A+B)(A-B)}$$

$$=\frac{(A-B)}{(A+B)(A-B)}=\frac{1}{A+B}$$

$$=\frac{1}{\dfrac{2x+1}{2x-1}+\dfrac{2x-1}{2x+1}}=\frac{1}{\dfrac{(2x+1)^2+(2x-1)^2}{(2x-1)(2x+1)}}$$

$$=\frac{1}{\dfrac{4x^2+4x+1+4x^2-4x+1}{(2x-1)(2x+1)}}=\frac{(2x-1)(2x+1)}{8x^2+2}$$

$$=\frac{4x^2-1}{2(4x^2+1)}$$

Question 6. If A = $\frac{x}{x+1}$, B = $\frac{1}{x+1}$, prove that

$$\frac{(A+B)^2+(A-B)^2}{A \div B}=\frac{2(x^2+1)}{x(x+1)^2}.$$

Solution:

$$A = \frac{x}{x+1}, B = \frac{1}{x+1}$$

$$= \frac{(A+B)^2 + (A-B)^2}{A \div B}$$

$$= \frac{A^2 + 2AB + B^2 + A^2 - 2AB + B^2}{(A \div B)}$$

$$= \frac{2A^2 + 2B^2}{A \div B} = \frac{2(A^2 + B^2)}{(A \div B)}$$

$$= \frac{2\left(\left(\frac{x}{x+1}\right)^2 + \left(\frac{1}{x+1}\right)^2\right)}{\frac{x}{x+1} \div \frac{1}{x+1}}$$

$$= \frac{2\left(\frac{x^2}{(x+1)^2} + \frac{1}{(x+1)^2}\right)}{\frac{x\cancel{(x+1)}}{\cancel{(x+1)}}} = 2\left(\frac{(x^2+1)}{x(x+1)^2}\right)$$

Question 7.
Pari needs 4 hours to complete a work. His friend Yuvan needs 6 hours to complete the same work. How long will be take to complete if they work together?
Answer:
Let the work done by Pari and Yuvan together be x
Work done by part = $\frac{1}{4}$
Work done by Yuvan = $\frac{1}{6}$
By the given condition
$\frac{1}{4} + \frac{1}{6} = \frac{1}{x} \Rightarrow \frac{3+2}{12} = \frac{1}{x}$
$\frac{5}{12} = \frac{1}{x}$
$5x = 12 \Rightarrow x = \frac{12}{5}$
$x = 2\frac{2}{5}$ hours (or) 2 hours 24 minutes

Question 8.
Iniya bought 50 kg of fruits consisting of apples and bananas. She paid twice as much per kg for the apple as she did for the banana. If Iniya bought Rs. 1800 worth of apples and Rs. 600 worth bananas, then how many kgs of each fruit did she buy?
Answer:
Let the quantity of apples and bananas purchased be 'x' and 'y'
By the given condition
x + y = 50 ………(1)
Cost of one kg of apple = $\frac{1800}{x}$
Cost of one kg of banana = $\frac{600}{y}$
By the given condition
One kg of apple = $2\,\frac{(600)}{y}$
Total cost of fruits purchased = 1800 + 600
$x \times 2\,\frac{(600)}{y} + y\,\frac{(600)}{y} = 2400$
$\frac{1200x}{y} = 2400 - 600$
$\frac{1200x}{y} = 1800$
1200 x = 1800 × y
$x = \frac{1800x}{1200} = \frac{3y}{2}$
Substitute the value of x in (1)
$\frac{3y}{2} + y = 50$
$\frac{5y}{2} = 50$
$5y = 100 \Rightarrow y = \frac{100}{5} = 20$
$x = \frac{3y}{2} = \frac{3\times20}{2}$
= 30
The quantity of apples = 30 kg
The quantity of bananas = 20 kg

Ex 3.7

Question 1.
Find the square root of the following rational expressions.

(i) $$\frac{400x^4y^{12}z^{16}}{100x^8y^4z^4}$$ **(ii)** $$\frac{7x^2+2\sqrt{14}x+2}{x^2-\frac{1}{2}x+\frac{1}{16}}$$

(iii) $$\frac{121(a+b)^8(x+y)^8(b-c)^8}{81(b-c)^4(a-b)^{12}(b-c)^4}$$

Solution:

(i) $$\sqrt{\frac{400x^4y^{12}z^{16}}{100x^8y^4z^4}} = \frac{20x^2y^6z^8}{10x^4y^2z^2}$$

$$= \frac{2|y^4z^6|}{|x^2|}$$

(ii) $$\sqrt{\frac{7x^2+2\sqrt{14}x+2}{x^2-\frac{1}{2}x+\frac{1}{16}}} = \sqrt{\frac{(\sqrt{7}x+\sqrt{2})(\sqrt{7}x+\sqrt{2})}{\left(x-\frac{1}{4}\right)\left(x-\frac{1}{4}\right)}}$$

$$= \frac{\sqrt{7}x+\sqrt{2}}{x-\frac{1}{4}} = \frac{\sqrt{7}x+\sqrt{2}}{\frac{4x-1}{4}}$$

$$= 4\left|\frac{(\sqrt{7}x+\sqrt{2})}{4x-1}\right|$$

Hint:

$$14 \to \sqrt{14} \quad \sqrt{14}$$

$$= \frac{\sqrt{2\times7}}{\sqrt{7}\times\sqrt{7}} \quad \frac{\sqrt{2\times7}}{\sqrt{7}\times\sqrt{7}}$$

$$= (\sqrt{7}x+\sqrt{2})(\sqrt{7}x+\sqrt{2})$$

(iii) $$\sqrt{\frac{121(a+b)^8(x+y)^8(b-c)^8}{81(b-c)^4(a-b)^{12}(b-c)^4}}$$

$$= \frac{11(a+b)^4(x+y)^4(b-c)^4}{9\cancel{(b-c)^2}(a-b)^6\cancel{(b-c)^2}}$$

$$= \frac{11}{9}\left|\frac{(a+b)^4(x+y)^4}{(a-b)^6}\right|$$

Question 2.
Find the square root of the following
(i) $4x^2 + 20x + 25$
(ii) $9x^2 - 24xy + 30xz - 40yz + 25z^2 + 16y^2$
(iii) $1 + \frac{1}{x^6} + \frac{2}{x^3}$
(iv) $(4x^2 - 9x + 2)(7x^2 - 13x - 2)(28x^2 - 3x - 1)$
(v) $(2x^2 + \frac{17}{6}x + 1)(\frac{3}{2}x^2 + 4x + 2)(\frac{4}{3}x^2 + \frac{11}{3}x + 2)$
Solution:

(i) $\sqrt{4x^2+20x+25} = \sqrt{(2x+5)^2} = |2x+5|$

(ii) $\sqrt{9x^2-24xy+30xz-40yz+25z^2+16y^2}$

$= \sqrt{(3x)^2+(-4y)^2+(5x)^2+(-24xy)+(-40yz)+(30xz)}$

$= \sqrt{(3x-4y+5z)^2}$ $\quad (\because (a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca)$

$= |3x-4y+5z|$

(iii) $\sqrt{1+\frac{1}{x^6}+\frac{2}{x^3}} = \sqrt{1^2+2\cdot 1\cdot \frac{1}{x^3}+\left(\frac{1}{x^3}\right)^2}$

$= \sqrt{\left(1+\frac{1}{x^3}\right)^2} = \left|1+\frac{1}{x^3}\right|$

(iv) $\sqrt{(4x^2-9x+2)(7x^2-13x-2)(28x^2-3x-1)}$

$= \sqrt{(x-2)(4x-1)(x-2)(7x+1)(4x-1)(7x+1)}$

$= \sqrt{(x-2)^2(4x-1)^2(7x+1)^2}$

$= |(x-2)(4x-1)(7x+1)|$

(v) $\sqrt{\left(2x^2+\frac{17}{6}x+1\right)\left(\frac{3}{2}x^2+4x+2\right)\left(\frac{4}{3}x^2+\frac{11}{3}x+2\right)}$

$= \sqrt{\frac{(12x^2+17x+6)}{6}\left(\frac{3x^2+8x+4}{2}\right)\left(\frac{4x^2+11x+6}{3}\right)}$

$= \frac{1}{6}\sqrt{(4x+3)(3x+2)(x+2)(3x+2)(4x+3)(x+2)}$

$= \frac{1}{6}\,|(4x+3)(3x+2)(x+2)|$

Ex 3.8

Question 1.
Find the square root of the following polynomials by division method
(i) $x^4 - 12x^3 + 42x^2 - 36x + 9$
(ii) $37x^2 - 28x^3 + 4x^4 + 42x + 9$
(iii) $16x^4 + 8x^2 + 1$
(iv) $121x^4 - 198x^3 - 183x^2 + 216x + 144$
Solution:
The long division method in finding the square root of a polynomial is useful when the degrees of a polynomial is higher.

(i)

$$
\begin{array}{r|l}
 & x^2 - 6x + 3 \\
\hline
x^2 & \cancel{x^4} - 12x^3 + 42x^2 - 36x + 9 \\
 & \cancel{x^4} \\
\hline
2x^2 - 6x & \cancel{-12x^3} + 42x^2 \\
 & \cancel{-12x^3} + 36x^2 \\
 & \quad (+) \qquad (-) \\
\hline
2x^2 - 12x + 3 & \qquad \cancel{6x^2} - \cancel{36x} + \cancel{9} \\
 & \qquad (-) \quad (+) \quad (-) \\
 & \qquad \cancel{6x^2} - \cancel{36x} + \cancel{9}
\end{array}
$$

$\therefore \sqrt{x^4 - 12x^3 + 42x^2 - 36x + 9} = |x^2 - 6x + 3|$

(ii) $\sqrt{37x^2 - 28x^3 + 4x^4 + 42x + 9} = ?$

$$
\begin{array}{r|l}
 & 2x^2 - 7x - 3 \\
\hline
x^2 & \cancel{4x^4} - 28x^3 + 37x^2 + 42x + 9 \\
 & \underset{(-)}{\cancel{4x^4}} \\
\hline
4x^2 - 7x & \quad -\cancel{28x^3} + 37x^2 \\
 & \quad \underset{(+)}{-}\cancel{28x^3} \underset{(-)}{+} 49x^2 \\
\hline
4x^2 - 14x - 3 & \qquad\quad -\cancel{12x^2} + \cancel{42x} + 9 \\
 & \qquad\quad \underset{(+)}{-}\cancel{12x^2} \underset{(-)}{+} \cancel{42x} \underset{(-)}{+} \cancel{9} \\
\hline
 & \qquad\qquad 0
\end{array}
$$

$\therefore \sqrt{37x^2 - 28x^3 + 4x^4 + 42x + 9} = |2x^2 - 7x - 3|$

(iii) $\sqrt{16x^4 + 8x^2 + 1}$

$$
\begin{array}{r|l}
 & 4x^2 + 0x + 1 \\ \hline
4x^2 & \cancel{16x^4} + 0x^3 + 8x^2 + 0x + 1 \\
 & \overset{(-)}{\cancel{16x^4}} \\ \hline
8x^2 + 0 & 0x^3 + 8x^2 \\
 & 0x^3 + 0x^2 \\ \hline
8x^2 + 0x + 1 & \cancel{8x^2} + \cancel{0x} + \cancel{1} \\
 & \overset{(-)}{\cancel{8x^2}} \overset{(-)}{+ \cancel{0x}} \overset{(-)}{+ \cancel{1}} \\ \hline
 & 0
\end{array}
$$

$\therefore \sqrt{16x^4 + 8x^2 + 1} = |4x^2 + 1|$

(iv) $\sqrt{121x^4 - 198x^3 - 183x^2 + 108x + 144}$

$$
\begin{array}{r|l}
 & 11x^2 - 9x - 12 \\ \hline
11x^2 & \cancel{121x^4} - 198x^3 - 183x^2 + 108x + 144 \\
 & \overset{(-)}{\cancel{121x^4}} \\ \hline
22x^2 - 9x & -\cancel{198x^3} - 183x^2 \\
 & \overset{(+)}{-\cancel{198x^3}} \overset{(-)}{+} 81x^2 \\ \hline
22x^2 - 9x - 12 & -\cancel{264x^2} + \cancel{108x} + \cancel{144} \\
 & \overset{(-)}{-\cancel{264x^2}} \overset{(-)}{+ \cancel{108x}} \overset{(-)}{+ \cancel{144}} \\ \hline
 & 0
\end{array}
$$

$\therefore \sqrt{121x^4 - 198x^3 - 183x^2 + 108x + 144}$
$= |11x^2 - 9x - 12|$

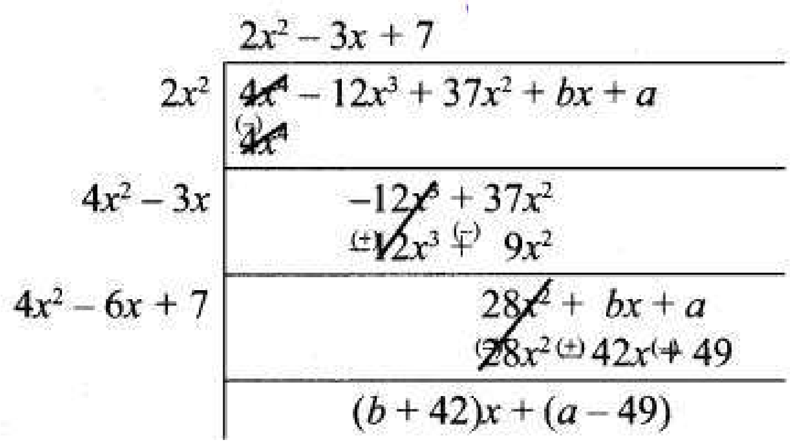
Question 2.
Find the square root of the expression $\frac{x^2}{y^2} - 10\frac{x}{y} + 27 - 10\frac{y}{x} + \frac{y^2}{x^2}$

Solution:

$$
\begin{array}{r|l}
 & \dfrac{x}{y} - 5 + \dfrac{y}{x} \\
\hline
\dfrac{x}{y} & \cancel{\dfrac{x^2}{y^2}} - 10\dfrac{x}{y} + 27 - 10\dfrac{y}{x} + \dfrac{y^2}{x^2} \\
 & {\scriptstyle (-)} \\
 & \cancel{\dfrac{x^2}{y^2}} \\
\hline
2\dfrac{x}{y} - 5 & \cancel{-10\dfrac{x}{y}} + 27 \\
 & {\scriptstyle (+)} \quad {\scriptstyle (-)} \\
 & \cancel{-10\dfrac{x}{y}} + 25 \\
\hline
2\dfrac{x}{y} - 10 + \dfrac{y}{x} & \cancel{2} \cancel{-10\dfrac{y}{x}} + \cancel{\dfrac{y^2}{x^2}} \\
 & {\scriptstyle (-)\,(+)} \quad {\scriptstyle (-)} \\
 & \cancel{2} \cancel{-10\dfrac{y}{x}} + \cancel{\dfrac{y^2}{x^2}} \\
\hline
 & \qquad 0
\end{array}
$$

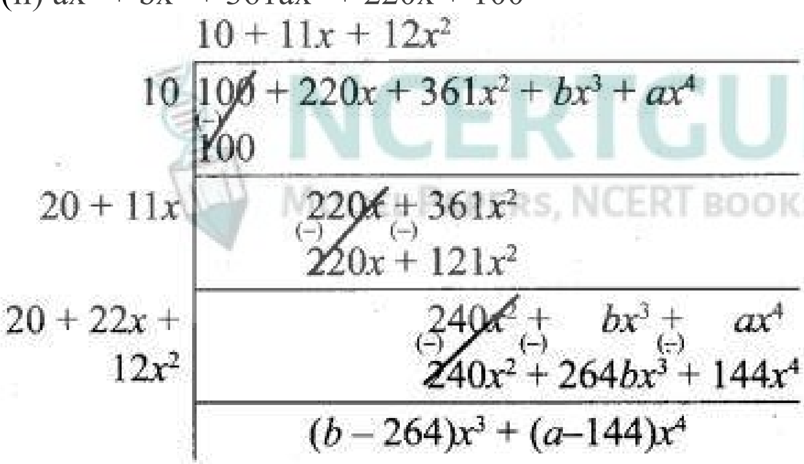
$$\therefore \sqrt{\frac{x^2}{y^2} - 10\frac{x}{y} + 27 - 10\frac{y}{x} + \frac{y^2}{x^2}} = \left|\frac{x}{y} - 5 + \frac{y}{x}\right|$$

Question 3.
Find the values of a and b if the following polynomials are perfect squares
(i) $4x^4 - 12x^3 + 37x^2 + bx + a$
(ii) $ax^4 + bx^3 + 361ax^2 + 220x + 100$
Solution:
(i)



Since it is a perfect square.
Remainder = 0
⇒ b + 42 = 0, a – 49 = 0
b = -42, a = 49

(ii) $ax^4 + bx^3 + 361ax^2 + 220x + 100$



Since remainder is 0
a = 144
b = 264

Question 4.
Find the values of m and n if the following expressions are perfect squares
(i) $\frac{1}{x^4} - \frac{6}{x^3} + \frac{13}{x^2} + \frac{m}{x} + n$
(ii) $x^4 - 8x^3 + mx^2 + nx + 16$
Solution:

(i)

$$
\begin{array}{r|l}
 & \frac{1}{x^2}-\frac{3}{x}+2 \\
\hline
\frac{1}{x^2} & \cancel{\frac{1}{x^4}}-\frac{6}{x^3}+\frac{13}{x^2}+\frac{m}{x}+n \\
 & \overset{(-)}{\cancel{\frac{1}{x^4}}} \\
\hline
\frac{2}{x^2}-\frac{3}{x} & -\cancel{\frac{6}{x^3}}+\frac{13}{x^2} \\
 & \overset{(+)}{-\cancel{\frac{6}{x^3}}}\overset{(-)}{+}\frac{9}{x^2} \\
\hline
\frac{2}{x^2}-\frac{6}{x}+2 & \cancel{\frac{4}{x^2}}+\frac{m}{x}+n \\
 & \overset{(-)}{\cancel{\frac{4}{x^2}}}\overset{(+)}{-}\frac{12}{x}\overset{(-)}{+}n \\
\hline
 &
\end{array}
$$

Since remainder is 0, $(x-12)\frac{1}{x}+(n-4)$

$\therefore m=-12, n=4$

(ii)

$$
\begin{array}{r|l}
 & x^2 - 4x + 4 \\
\hline
x^2 & \cancel{x^4} - 8x^3 + mx^2 + nx + 16 \\
 & \underset{(-)}{\cancel{x^4}} \\
\hline
2x^2 - 4x & \cancel{-8x^3} + mx^2 \\
 & \underset{(+)}{\cancel{-8x^3}} \underset{(-)}{+} 16x^2 \\
\hline
2x^2 - 8x + 4 & (m - 16)x^2 + nx + \cancel{16} \\
 & \underset{(-)}{8x^2} \underset{(+)}{-} 32x \underset{(-)}{+} \cancel{16} \\
\hline
 & (x - 24)x^2 + (x + 32) = 0
\end{array}
$$

Since remainder is 0,
m = 24, n = -32

Ex 3.9

Question 1.
Determine the quadratic equations, whose sum and product of roots are
(i) -9, 20
(ii) $\frac{5}{3}$, 4
(iii) $\frac{-3}{2}$, -1
(iv) $-(2 - a)^2$, $(a + 5)^2$
Solution:
If the roots are given, general form of the quadratic equation is x^2 – (sum of the roots) x + product of the roots = 0.
(i) Sum of the roots = -9
Product of the roots = 20
The equation = $x^2 - (-9x) + 20 = 0$
$\Rightarrow x^2 + 9x + 20 = 0$

(ii) Sum of the roots = $\frac{5}{3}$
Product of the roots = 4
Required equation = x^2 – (sum of the roots)x + product of the roots
= 0
$\Rightarrow x^2 - \frac{5}{3}x + 4 = 0$
$\Rightarrow 3x^2 - 5x + 12 = 0$

(iii) Sum of the roots = $(\frac{-3}{2})$
$(\alpha + \beta) = \frac{-3}{2}$
Product of the roots $(\alpha\beta) = (-1)$
Required equation = $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$x^2 - (\frac{-3}{2})x - 1 = 0$
$2x^2 + 3x - 2 = 0$

(iv) $\alpha + \beta = -(2 - a)^2$
$\alpha\beta = (a + 5)^2$
Required equation = $x^2 - (\alpha + \beta)x - \alpha\beta = 0$
$\Rightarrow x^2 - (-(2 - a)^2)x + (a + 5)^2 = 0$
$\Rightarrow x^2 + (2 - a)^2x + (a + 5)^2 = 0$

Question 2.
Find the sum and product of the roots for each of the following quadratic equations
(i) $x^2 + 3x - 28 = 0$
(ii) $x^2 + 3x = 0$
(iii) $3 + \frac{1}{a} = \frac{10}{a^2}$
(iv) $3y^2 - y - 4 = 0$

(i) $x^2 + 3x - 28 = 0$
Answer:
Sum of the roots $(\alpha + \beta) = -3$
Product of the roots $(\alpha\ \beta) = -28$

(ii) $x^2 + 3x = 0$
Answer:
Sum of the roots $(\alpha + \beta) = -3$
Product of the roots $(\alpha\ \beta) = 0$

(iii) $3 + \frac{1}{a} = \frac{10}{a^2}$
$3a^2 + a = 10$
$3a^2 + a - 10 = 0$ comparing this with $x^2 - (\alpha + \beta)$

x + αβ = 0

$$a^2 - \left(-\frac{1}{3}\right)a + \left(\frac{-10}{3}\right) = 0$$

$$\alpha + \beta = \frac{-1}{3}$$

$$\alpha\beta = \frac{-10}{3}$$

(iv) $3y^2 - y - 4 = 0 \div 3$

$$y^2 - \frac{y}{3} - \frac{4}{3} = 0$$

$$y^2 - \left(\frac{1}{3}\right)y + \left(\frac{-4}{3}\right) = 0$$

$$\therefore \alpha + \beta = \frac{1}{3}$$

$$\alpha\beta = \frac{-4}{3}$$

Ex 3.10

Question 1.
Solve the following quadratic equations by factorization method
(i) $4x^2 - 7x - 2 = 0$
(ii) $3(p^2 - 6) = p(p + 5)$
(iii) $\sqrt{a(a-7)} = 3\sqrt{2}$
(iv) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$
(v) $2x^2 - x + \frac{1}{8} = 0$
Solution:

(i)
$$\begin{aligned} 4x^2 - 7x - 2 &= 0 \\ 4x^2 - 8x + 1x - 2 &= 0 \\ 4x(x-2) + 1(x-2) &= 0 \\ (x-2)(4x+1) &= 0 \Rightarrow (x-2) = 0 \\ \Rightarrow \quad x &= 2 \text{ or } 4x + 1 = 0 \\ \Rightarrow \quad x &= -\frac{1}{4} \end{aligned}$$

Hint:
-8
$-8 \quad 1$

(ii) $3(p^2 - 6) = p(p + 5)$
$3p^2 - 18 = p^2 + 5p \Rightarrow 39^2 - 5p - 18 = 0$
$\Rightarrow 2p^2 - 5p - 18 = 0$
$\Rightarrow (2p - 9)(p + 2) = 0 \Rightarrow p = \frac{9}{2}, -2$

(iii) $\sqrt{a(a-7)} = 3\sqrt{2}$
Squaring on both sides
$a(a - 7) = 9 \times 2$

$a^2 - 7a - 18 = 0$
$a^2 - 9a + 2a - 18 = 0$
$a(a - 9) + 2(a - 9) = 0$
$(a - 9)(a + 2) = 0$
$\Rightarrow a = 9, a = -2$

(iv) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$$\sqrt{2}x^2 + 2x + 5x + 5\sqrt{2} = 0$$
$$\sqrt{2}x(x + \sqrt{2}) + 5(x + \sqrt{2}) = 0$$
$$(x + \sqrt{2})(\sqrt{2}x + 5) = 0$$
$$\Rightarrow \quad x = -\sqrt{2}$$
$$x = \frac{-5}{\sqrt{2}}$$

(v) $2x^2 - x + \frac{1}{8} = 0$
$16x^2 - 8x + 1 = 0$
$16x^2 - 4x - 4x + 1 = 0$
$4x(4x - 1) - 1(4x - 1) = 0$
$(4x - 1)(4x - 1) = 0$
$\Rightarrow x = \frac{1}{4}, \frac{1}{4}$

Question 2.
The number of volleyball games that must be scheduled in a league with n teams is given by G(n) = $\frac{n^2-n}{2}$ where each team plays with every other team exactly once. A league schedules 15 games. How many teams are in the league?
Answer:
Number of games = 15
G(n) = $\frac{n^2-n}{2}$
$\frac{n^2-n}{2} = 15$
$n^2 - n = 30 \Rightarrow n^2 - n - 30 = 0$
$\Rightarrow n^2 - 6n - 5n - 30 = 0$
$(n - 6)(n + 5) = 0$
$n - 6 = 0$ or $n + 5 = 0$
[Note: – 5 is neglected because number of team is not negative]

n = 6 or n = -5

∴ Number of teams = 6

Ex 3.11

Question 1.
Solve the following quadratic equations by completing the square method
(i) $9x^2 - 12x + 4 = 0$
(ii) $\frac{5x+7}{x-1} = 3x + 2$
Solution:

(i) $9x^2 - 12x + 4 = 0 \div 9$

$$\frac{9x^2}{9} - \frac{12}{9}x + \frac{4}{9} = 0$$

$$x^2 - \frac{4}{3}x = \frac{-4}{9}$$

Hint:

$$\left(\frac{-b}{2a}\right)^2 = \left(\frac{-2}{3}\right)^2$$

$$\left[\frac{4}{2(3)}\right]^2 = \left(\frac{2}{3}\right)^2$$

$$a = 1, b = -\frac{4}{3}$$

$$x^2 - \frac{4}{3}x + \left(\frac{+2}{3}\right)^2 = \frac{-4}{9} + \left(\frac{+2}{3}\right)^2$$

[Adding $\left(\frac{2}{3}\right)^2$ both sides]

$$\left(x-\frac{2}{3}\right)^2 = \frac{-4}{9}+\frac{4}{9}=0$$

$$x-\frac{2}{3} = 0,\ x-\frac{2}{3}=0$$

$$x = \frac{2}{3},\frac{2}{3}$$

(ii) $\frac{5x+7}{x-1}=3x+2$

$$5x+7 = (x-1)(3x+2)$$

$$5x+7 = 3x^2-3x+2x-2$$

$$3x^2-x-5x-2-7= 0$$

$$3x^2-6x-9 = 0 \div 3$$

$$x^2-2x-3 = 0$$

$$x^2-2x = 3$$

$$\boxed{a=1, b=-2 \quad \left(\frac{-b}{2a}\right)^2=\left(\frac{2}{2}\right)^2=1^2}$$

$$x^2-2x+1 = 3+1$$

[Adding 1 both sides]

$$(x-1)^2 = 4$$

$$(x-1) = \sqrt{4}=\pm 2$$

$$x-1=2 \Rightarrow x = 3$$

$$x-1=-2 \Rightarrow x = -1$$

Question 2.
Solve the following quadratic equations by formula method
(i) $2x^2 - 5x + 2 = 0$
(ii) $\sqrt{2}f^2 - 6f + 3\sqrt{2}$
(iii) $3y^2 - 20y - 3 = 0$
(iv) $36y^2 - 12ay + (a^2 - b^2) = 0$
Solution:
(i) $2x^2 - 5x + 2 = 0$
The formula for finding roots of a quadratic equation $ax^2 + bx + c = 0$ is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\underset{a}{2x^2} - \underset{b}{5x} + \underset{c}{2} = 0$$

$$\therefore x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 2 \times 2}}{2 \times 2}$$

$$= \frac{5 \pm \sqrt{25 - 16}}{4}$$

$$= \frac{5 \pm \sqrt{9}}{4} = \frac{5 \pm 3}{4} = \frac{8}{4}, \frac{2}{4}$$

$$= 2, \frac{1}{2}$$

$\therefore$ Solutions is 2, $\frac{1}{2}$

(ii) $\underset{a}{\sqrt{2}f^2} - \underset{b}{6f} + \underset{c}{3\sqrt{2}} = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\text{Here } f = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \times \sqrt{2} \times 3\sqrt{2}}}{2 \times \sqrt{2}}$$

$$= \frac{6 \pm \sqrt{36 - 24}}{2\sqrt{2}}$$

$$= \frac{6 \pm \sqrt{12}}{2\sqrt{2}} = \frac{6 \pm 2\sqrt{3}}{2\sqrt{2}} = \frac{\cancel{2}\left(3 \pm \sqrt{3}\right)}{\cancel{2}\sqrt{2}}$$

$$\Rightarrow \frac{3 + \sqrt{3}}{\sqrt{2}}, \frac{3 - \sqrt{3}}{2}$$

(iii) $\underset{a}{3y^2} - \underset{b}{20y} - \underset{c}{23} = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\text{Here } y = \frac{-(-20) \pm \sqrt{(-20)^2 - 4 \times 3 \times -23}}{2 \times 3}$$

$$= \frac{20 \pm \sqrt{400 + 276}}{6}$$

$$= \frac{20 \pm \sqrt{676}}{6} = \frac{20 \pm 26}{6}$$

$$= \frac{46}{6} \text{ or } \frac{-6}{6}$$

$$y \Rightarrow \frac{23}{3} \text{ or } -1$$

(iv) $\underset{a}{36y^2} - \underset{b}{12ay} + \underset{c}{(a^2 - b^2)} = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\text{Here } y = \frac{-(-12a) \pm \sqrt{(-12a)^2 - 4 \times 36 \times (a^2 - b^2)}}{2 \times 36}$$

$$= \frac{12a \pm \sqrt{144a^2 - 144a^2 + 144b^2}}{72}$$

$$= \frac{12a \pm 12b}{72} \Rightarrow \frac{a \pm b}{6}$$

$$= \frac{a+b}{6}, \frac{a-b}{6}$$

Question 3.
A ball rolls down a slope and travels a distance d = t^2 – 0.75t feet in t seconds. Find the time when the distance travelled by the ball is 11.25 feet.
Solution:
Distance d = t^2 – 0.75 t,
Given that d = 11.25 = t^2 – 0.75 t.

$t^2 - 0.75t - 11.25 = 0$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{(+0.75) \pm \sqrt{(-0.75)^2 - 4 \times 1 \times -11.25}}{2 \times 1}$$

$$= \frac{+0.75 \pm \sqrt{0.5625 + 45}}{2}$$

$$= \frac{+0.75 \pm \sqrt{45.5625}}{2}$$

$$= \frac{+0.75 \pm 6.75}{2}$$

$$= \frac{7.50}{2} \text{ or } \frac{-6}{2}$$

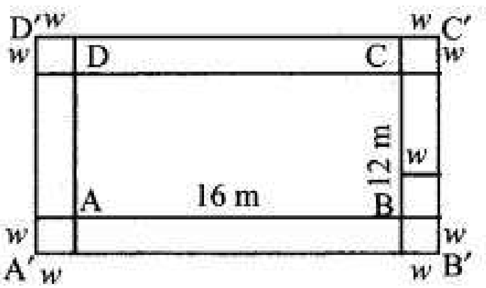
= 3.75 or –3 It is not possible.

$\therefore t = 3.75$ s.

Ex 3.12

Question 1.
If the difference between a number and its reciprocal is $\frac{24}{5}$, find the number.
Solution:
Let a number be x.
Its reciprocal is $\frac{1}{x}$
$x-\frac{1}{x}=\frac{24}{5}$
$\frac{x^2-1}{x}=\frac{24}{5}$
$5x^2 - 5 - 24x = 0 \Rightarrow 5x^2 - 24x - 5 = 0$
$5x^2 - 25x + x - 5 = 0$
$5x(x - 5) + 1(x - 5) = 0$
$(5x + 1)(x - 5) = 0$
$x = \frac{-1}{5}, 5$
∴ The number is $\frac{-1}{5}$ or 5.

Question 2.
A garden measuring 12m by 16m is to have a pedestrian pathway that is ‘w’ meters wide installed all the way around so that it increases the total area to 285 m^2. What is the width of the pathway?
Solution:
Area of ABCD = 16×12^{2}
= 192 m^2
Area of A’B’C’D’ (12 + 2w)(16 + 2w)
$192 + 32w + 24w + 4w^2 = 285$



$4w^2 + 56w - 93 = 0$

$4w^2 + 62w - 6w - 93 = 0$

$2w(2w + 31) - 3(2w + 31) = 0$

$(2w - 3)(2w + 31) = 0$

$w = 1.5$ or $\frac{-31}{2} = 15.5$

$w = -15.5$ cannot possible 3

$\therefore w = \frac{3}{2} = 1.5$ m

(w cannot be (-ve))

The width of the pathway = 1.5 m.

Question 3.

A bus covers a distance of 90 km at a uniform speed. Had the speed been 15 km/hour more it would have taken 30 minutes less for the journey. Find the original speed of the bus.

Solution:

Let x km/hr be the constant speed of the bus.

The time taken to cover 90 km = $\frac{90}{x}$ hrs.

When the speed is increased bus 15 km/hr.

$= \frac{90}{x+15}$

It is given that the time to cover 90 km is reduced by $\frac{1}{2}$ hrs.

$$\Rightarrow \frac{90}{x} - \frac{90}{x+15} = \frac{1}{2}$$

$$\frac{90(x+15) - 90x}{x(x+15)} = \frac{1}{2} \Rightarrow \frac{\cancel{90x} + 1350 - \cancel{90x}}{x^2 + 15x} = \frac{1}{2}$$

$$x^2 + 15x = 2700$$
$$x^2 + 15x - 2700 = 0$$
$$= \frac{-15 \pm \sqrt{225 + 10800}}{2}$$
$$= \frac{-15 \pm \sqrt{11025}}{2}$$
$$= \frac{-15 \pm 105}{2} \Rightarrow \frac{-15 + 105}{2} = \frac{-15 - 105}{2}$$
$$\Rightarrow \frac{90}{2}, \frac{-120}{2}$$

as the roots are real and equal

$$= 45, -60$$

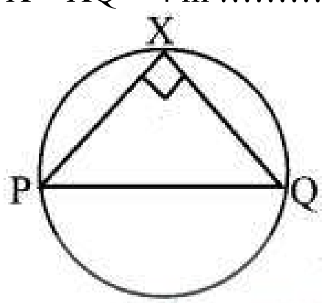
The speed of the bus cannot be -ve value.
∴ The original speed of the bus is 45 km/hr.

Question 4.
A girl is twice as old as her sister. Five years hence, the product of their ages (in years) will be 375. Find their present ages.
Solution:
Let the age of the girl be = 2y years
Her sister's age is = y years
(2y + 5)(y + 5) = 375
$2y^2 + 5y + 10y + 25 - 375 = 0$
$2y^2 + 15y - 350 = 0$

$$y = \frac{-15 \pm \sqrt{15^2 - 4 \times 2 \times -350}}{2 \times 2}$$
$$= \frac{-15 \pm \sqrt{3025}}{4}$$
$$= \frac{-15 \pm 55}{4} \text{ or } \frac{-15 - 55}{4}$$
$$= \frac{+40}{4} \text{ or } \frac{-70}{4}$$

y = 10, y cannot be (-ve).
∴ Girls age is 2y = 20 years.
Her sister's age = y = 10 years.

Question 5.
A pole has to be erected at a point on the T boundary of a circular ground of diameter j 20 m in such a way that the difference of its i distances from two diametrically opposite j fixed gates P and Q on the boundary is 4 m. Is i it possible to do so? If answer is yes at what j distance from the two gates should the pole j be erected?
Solution:
PQ = 20 m
PX – XQ = 4 m …………… (1)



Squaring both sides,
$PX^2 + XQ^2 - 2PX . QX = 16$ (∵ ∠Q × p = 90°)
$PQ^2 - 2P \times QX = 16$
400 – 16 = 2PX × QX
384 = 2PX – QX
PX . QX = 192
∴ $(PX + QX)^2 = PX^2 + QX^2 + 2PX . QX$
= 400 + 2 × 192
$= 784 = 28^2$
∴ PX + QX = 28
From (1) & (2) 2PX = 32 ⇒ PX = 16 m QX = 12 m
∴Yes, the distance from the two gates to the pole PX and QX is 12 m, 16m.

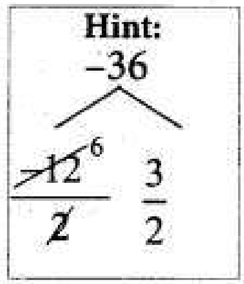
Question 6.
From a group of black bees $2x^2$, square root of half of the group went to a tree. Again eight- ninth of the bees went to the same tree. The remaining two got caught up in a fragrant lotus. How many bees were there in total?
Solution:
Total no. of bees = $2x^2$

No. of bees that went to a tree = $\sqrt{\frac{1}{2} \times 2x^2} = \sqrt{x^2} = x$

Second batch of bees that went to tree = $\frac{8}{9} \times 2x^2$

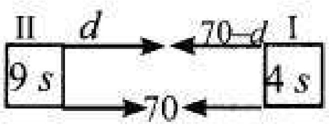
After this, only 2 are left.

$\therefore 2x^2 - x - \frac{16}{9}x^2 = 2$



$18x^2 - 9x - 16x^2 = 2 \times 9$
$2x^2 - 9x - 18 = 0$
$(x - 6)(2x + 3) = 0$
$x = 6, x = \frac{-3}{2}$ (it is not possible)
No. of bees in total = $2x^2$
$= 2 \times 6^2 = 72$

Question 7.
Music is been played in two opposite galleries with certain group of people. In the first gallery a group of 4 singers were singing and in the second gallery 9 singers were singing. The two galleries are separated by the distance of 70 m. Where should a person stand for hearing the same intensity of the singers voice? (Hint: The ratio of the sound intensity is equal to the square of the ratio of their corresponding distances).
Solution:
Let the person stand at a distance ‘d’ from 2nd gallery having 9 singers.



Given that ratio of sound intensity is equal to the square of the ratio of their corresponding distance.

$\therefore \frac{9}{4} = \frac{d^2}{(70-d)^2}$

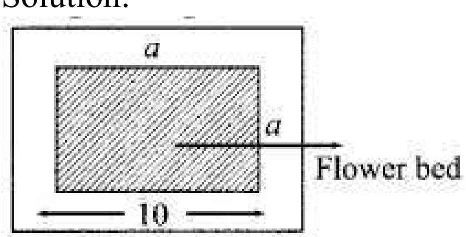
$4d^2 = 9(70 - d)^2$
$4d^2 = 9(70^2 - 140d + d^2)$
$4d^2 = 9 \times 70^2 - 9 \times 140d + 9d^2$
$\therefore 5d^2 - 9 \times 140d + 9 \times 70^2 = 0$

$5d^2 = 1260d + 44100 = 0$
$d^2 - 252d + 8820 = 0$

$$d = \frac{252 \pm \sqrt{252^2 - 4 \times 1 \times 8820}}{2 \times 1}$$
$$= \frac{252 \pm \sqrt{63504 - 35280}}{2}$$
$$= \frac{252 \pm \sqrt{98784}}{2}$$
$$= \frac{252 \pm 168}{2}$$
$$= \frac{252 + 168}{2} \text{ or } \frac{252 - 168}{2}$$

$= \frac{420}{2}$ or $\frac{84}{2}$
= 120 or 42
∴ The person stand at a distance 28m from the first and 42 m from second gallery.

Question 8.
There is a square field whose side is 10 m. A square flower bed is prepared in its centre leaving a gravel path all round the flower bed. The total cost of laying the flower bed and gravelling the path at □ 3 and □ 4 per square metre respectively is □364. Find the width of the gravel path.
Solution:

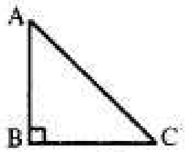


Area of the flower bed = a^2
Area of the gravel path = $100 - a^2$
Area of total garden =100
given cost of flower bed + gravelling = □ 364
$3a^2 + 4(100 - a^2) =$ □ 364
$3a^2 + 400 - 4a^2 = 364$
∴ $a^2 = 400 - 364$

= 36 ⇒ a = 6
width of gravel path $= \frac{10-6}{2} = \frac{4}{2}$ = 2 cm

Question 9.
Two women together took 100 eggs to a market, one had more than the other. Both sold them for the same sum of money. The first then said to the second: "If I had your eggs, I would have earned □ 15", to which the second replied: "If I had your eggs, I would have earned □ 6 $\frac{2}{3}$". How many eggs did each had in the beginning? Answer:
Number of eggs for the first women be 'x'
Let the selling price of each women be 'y'
Selling price of one egg for the first women = $\frac{y}{100-x}$
By the given condition
$(100 – x) \frac{y}{x}$ = 15 (for first women)
$y = \frac{15}{100-x}$ ……(1)
$x \times \frac{y}{(100-x)} = \frac{20}{3}$ [For second women]
$y = \frac{20(100-x)}{3x}$ ……..(2)
From (1) and (2) We get
$\frac{15}{100-x} = \frac{20(100-x)}{3x}$
$45x^2 = 20(100 – x)^2$
$(100 – x)^2 = \frac{45x^2}{20} = \frac{9}{4} x^2$
$\therefore 100 – x = \sqrt{\frac{9}{4}x^2}$
$100 – x = \frac{3x}{2}$
3x = 2(100 – x)
3x = 200 – 2x
3x + 2x = 200 ⇒ 5x = 200
$x = \frac{200}{5}$ ⇒ x = 40
Number of eggs with the first women = 40
Number of eggs with the second women = (100 – 40) = 60

Question 10.
The hypotenuse of a right-angled triangle is 25 cm and its perimeter 56 cm. Find the length of the smallest side.
Solution:



AB + BC + CA = 56 cm
AC = 25 cm
AB + BC = 56 – 25 = 31
$AB^2 + BC^2 = AC^2$
$(AB + BC)^2 - 2AB \cdot BC = AC2$ [$\because a^2 + b^2 = (a + b)^2 - 2ab$]
$31^2 - 2AB \cdot BC = 25^2$
$-2AB \cdot BC = 625 - 961$

$$-2AB \cdot BC = -336$$

$$AB \cdot BC = 168$$

Hint:
$\because \alpha + \beta = 31$
$\alpha\beta = 168$

$\therefore$ Quadratic equation is

$$x^2 - 31x + 168 = 0$$

$$x = \frac{-(-31) \pm \sqrt{(-31)^2 - 4\times1\times168}}{2\times1}$$

$$= \frac{31 \pm \sqrt{961 - 672}}{2}$$

$$= \frac{31 \pm \sqrt{289}}{2}$$

$$= \frac{31 \pm 17}{2} \Rightarrow \frac{31+17}{2} \text{ or } \frac{31-17}{2}$$

$$= \frac{48}{2}, \frac{14}{2} \Rightarrow 24, 7$$

$\therefore$ The length of the smallest side is 7 cm.

Ex 3.13

Question 1.
Determine the nature of the roots for the following quadratic equations
(i) $15.x^2 + 11.x + 2 = 0$
(ii) $x^2 - x - 1 = 0$
(iii) $\sqrt{2}t^2 - 3t + 3\sqrt{2} = 0$
(iv) $9y^2 - 6\sqrt{2}y + 2 = 0$
(v) $9a^2b^2x^2 - 24abcdx + 16c^2d^2 = 0$ $a \neq 0, b \neq 0$
Solution:
(i) $15x^2 + 11x + 2 = 0$ comparing with $ax^2 + bx + c = 0$.
Here a = 15, 6 = 11, c = 2.
$\Delta = b^2 - 4ac$
$= 11^2 -4 \times 15 \times 2$
$= 121 - 120$
$= 1 > 1.$
∴ The roots are real and unequal.

(ii) $x^2 - x - 1 = 0$,
Here a = 1, b = -1, c = -1 .
$\Delta = b^2 - 4ac$
$= (-1)^2 - 4 \times 1 \times -1$
$= 1 + 4 = 5 > 0.$
∴ The roots are real and unequal.

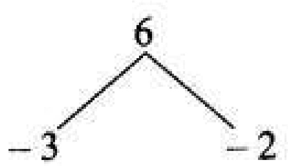
(iii) $\sqrt{2}t^2 - 3t + 3\sqrt{2} = 0$
Here a = $\sqrt{2}$, b = -3, c = $3\sqrt{2}$
$\Delta = b^2 - 4ac$
$= (-3)^2 - 4 \times \sqrt{2} \times 3\sqrt{2}$

$= 9 - 24 = -15 < 0$.
∴ The roots are not real.

(iv) $9y^2 - 6\sqrt{2}y + 2 = 0$
$a = 9$, $b = 6\sqrt{2}$, $c = 2$
$\Delta = b^2 - 4ac$
$= (6\sqrt{2})^2 - 4 \times 9 \times 2$
$= 36 \times 2 - 72$
$= 72 - 72 = 0$
∴ The roots are real and equal.

(v) $9a^2b^2x^2 - 24abcdx + 16c^2d^2 = 0$
$\Delta = b^2 - 4ac$
$= (-24abcd)^2 - 4 \times 9a^2b^2 \times 16c^2d^2$
$= 576a^2b^2c^2d^2 - 576a^2b^2c^2d^2$
$= 0$
∴ The roots are real and equal.

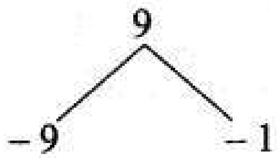
Question 2.
Find the value(s) of 'A' for which the roots of the following equations are real and equal.
(i) $(5k - 6)x^2 + 2kx + 1 = 0$
Answer:
Here $a = 5k - 6$; $b = 2k$ and $c = 1$
Since the equation has real and equal roots $\Delta = 0$.



∴ $b^2 - 4ac = 0$
$(2k)^2 - 4(5k - 6)(1) = 0$
$4k^2 - 20k + 24 = 0$
$(\div 4) \Rightarrow k^2 - 5k + 6 = 0$
$(k - 3)(k - 2) = 0$
$k - 3 = 0$ or $k - 2 = 0$
$k = 3$ or $k = 2$
The value of $k = 3$ or 2

(ii) $kx^2 + (6k + 2)x + 16 = 0$
Answer:

Here a = k, b = 6k + 2; c = 16
Since the equation has real and equal roots



$\Delta = 0$

$b^2 - 4ac = 0$

$(6k + 2)^2 - 4(k)(16) = 0$

$36k^2 + 4 + 24k - 4(k)(16) = 0$

$36k^2 - 40k + 4 = 0$

$(\div \text{ by } 4) \Rightarrow 9k^2 - 10k + 1 = 0$

$9k^2 - 9k - k + 1 = 0$

$9k(k - 1) - 1(k - 1) = 0$

$9k (k - 1) -1 (k - 1) = 0$

$(k - 1)(9k - 1) = 0$

$k - 1$ or $9k - 1 = 0$

$k = 1$ or $k = \frac{1}{9}$

The value of $k = 1$ or $\frac{1}{9}$

Question 3.
If the roots of $(a - b)x^2 + (b - c)x + (c - a) = 0$ are real and equal, then prove that b, a, c are in arithmetic progression.
Solution:

$(a - b)x^2 + (b - c)x + (c - a) = 0$

$A = (a - b), B = (b - c), C = (c - a)$

$\Delta = b^2 - 4ac = 0$

$\Rightarrow (b - c)^2 - 4(a - b)(c - a)$

$\Rightarrow b^2 - 2bc + c^2 -4 (ac - bc - a^2 + ab)$

$\Rightarrow b^2 - 2bc + c^2 - 4ac + 4bc + 4a^2 - 4ab = 0$

$\Rightarrow 4a^2 + b^2 + c^2 + 2bc - 4ac - 4ab = 0$

$\Rightarrow - (-2a + b + c)^2 = 0$ $[\because (a + b + c) = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca)]$

$\Rightarrow 2a + b + c = 0$

$\Rightarrow 2a = b + c$

$\therefore$ a, b, c are in A.P.

Question 4.
If a, b are real then show that the roots of the equation

$(a – b)x^2 – 6(a + b)x – 9(a – b) = 0$ are real and unequal.
Answer:
$(a – b)x^2 – 6(a + b)x – 9(a – b) = 0$
Here $a = a – b$; $b = – 6 (a + b)$; $c = – 9 (a – b)$
$\Delta = b^2 – 4ac$
$= [- 6(a + b)]^2 – 4(a – b)[-9(a – b)]$
$= 36(a + b)^2 + 36(a – b)(a – b)$
$= 36 (a + b)^2 + 36 (a – b)^2$
$= 36 [(a + b)^2 + (a – b)^2]$
The value is always greater than 0
$\Delta = 36 [(a + b)^2 + (a – b)^2] > 0$
∴ The roots are real and unequal.

Question 5.
If the roots of the equation $(c^2 – ab)x^2 – 2(a^2 – bc)x + b^2 – ac = 0$ are real and equal prove that either $a = 0$ (or) $a^3 + b^3 + c^3 = 3abc$.
Solution:
$(c^2 – ab)x^2 – 2(a^2 – bc)x + (b^2 – ac) – 0$
$\Delta = B^2 – 4AC = 0$ (since the roots are real and equal)
$\Rightarrow 4(a^{2'} – bc)^2 – 4 (c^2 – ab)(b^2 – ac) = 0$
$\Rightarrow 4(a^4 – 2a^2bc + b^2c^2) – 4(c^2b^2 – ab^3 – ac^3 + a^2bc) = 0$
$\Rightarrow 4a^4 + 4b^2c^2 – 8a^2bc – 4c^2b^2 + 4ab^3 + 4ac^3 – 4a^2bc = 0$
$\Rightarrow 4a^4 + 4ab^3 + 4ac^3 – 4a^2bc – 8a^2bc = 0$
$\Rightarrow 4a [a^3 + b^3 + c^3] = 0$ or $a = 0$
$\Rightarrow a = 0$ or $[a^3 + b^3 + c^3 – 3abc] = 0$
$\Rightarrow a^3 + b^3 + c^3 – 3abc = 0$
$\Rightarrow a^3 + b^3 + c^3 = 3abc$ or $a = 0$
Hence proved.

Ex 3.14

Question 1.
Write each of the following expression in terms of $\alpha + \beta$ and $\alpha\beta$.

(i) $\frac{\alpha}{3\beta}+\frac{\beta}{3\alpha}$ **(ii)** $\frac{1}{\alpha^2\beta}+\frac{1}{\beta^2\alpha}$

(iii) $(3\alpha-1)(3\beta-1)$ **(iv)** $\frac{\alpha+3}{\beta}+\frac{\beta+3}{\alpha}$

(i)
$$\frac{\alpha}{3\beta}+\frac{\beta}{3\alpha} = \frac{\alpha^2+\beta^2}{3\alpha\beta}$$
$$= \frac{(\alpha+\beta)^2-2\alpha\beta}{3\alpha\beta}$$

(ii)
$$\frac{1}{\alpha^2\beta}+\frac{1}{\beta^2\alpha} = \frac{\beta+\alpha}{\alpha^2\beta^2}$$
$$= \frac{\alpha+\beta}{(\alpha\beta)^2}$$

(iii)
$$(3\alpha-1)(3\beta-1) = 9\alpha\beta-3\beta-3\alpha+1$$
$$= 9\alpha\beta-3(\alpha+\beta)+1$$

(iv)
$$\frac{\alpha+3}{\beta}+\frac{\beta+3}{\alpha} = \frac{\alpha^2+3\alpha+\beta^2+3\beta}{\alpha\beta}$$
$$= \frac{\alpha^2+\beta^2+3(\alpha+\beta)}{\alpha\beta}$$
$$= \frac{(\alpha+\beta)^2-2\alpha\beta+3(\alpha+\beta)}{\alpha\beta}$$

Question 2.
The roots of the equation $2x^2 - 7x + 5 = 0$ are α and β. Without solving the root find

(i) $\frac{1}{\alpha}+\frac{1}{\beta}$ **(ii)** $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$

(iii) $\frac{\alpha+2}{\beta+2}+\frac{\beta+2}{\alpha+2}$

Solution:
$2x^2 - 7x + 5 = x^2 - \frac{7}{2}x + \frac{5}{2} = 0$
$\alpha + \beta = \frac{7}{2}$
$\alpha\beta = \frac{5}{2}$

(i) $\frac{1}{\alpha}+\frac{1}{\beta} = \frac{\beta+\alpha}{\alpha\beta} = \frac{\frac{7}{2}}{\frac{5}{2}} = \frac{7}{5}$

(ii) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta}$

$= \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$

$= \frac{(\alpha+\beta)^2}{\alpha\beta}-2$

$= \frac{\left(\frac{7}{2}\right)^2}{\frac{5}{2}}-2 = \frac{49}{4}\times\frac{2}{5}-2$

$= \frac{49}{10}-2 = \frac{49-20}{10} = \frac{29}{10}$

(iii) $\frac{\alpha+2}{\beta+2}+\frac{\beta+2}{\alpha+2}$

$$= \frac{(\alpha+2)^2+(\beta+2)^2}{\alpha\beta+2\alpha+2\beta+4}$$

$$= \frac{\alpha^2+4\alpha+4+\beta^2+4\beta+4}{\alpha\beta+2(\alpha+\beta)+4}$$

$$= \frac{(\alpha+\beta)^2-2\alpha\beta+4(\alpha+\beta)+8}{\alpha\beta+2(\alpha+\beta)+4}$$

$$= \frac{\frac{49}{4}-\frac{10}{2}+\frac{28}{2}+\frac{16}{2}}{\frac{5}{2}+\frac{14}{2}+\frac{8}{2}}$$

$$= \frac{49-20+56+32}{5+14+8}\times\frac{1}{2}$$

$$= \frac{117}{54}$$

Question 3.

The roots of the equation $x^2 + 6x - 4 = 0$ are α, β. Find the quadratic equation whose roots are

(i) α^2 and β^2

(ii) $\frac{2}{\alpha}$ and $\frac{2}{\beta}$

(iii) $\alpha^2\beta$ and $\beta^2\alpha$

Solution:

If the roots are given, the quadratic equation is x^2 – (sum of the roots) x + product the roots = 0.

For the given equation.

$x^2 + 6x - 4 = 0$

$\alpha + \beta = -6$

$\alpha\beta = -4$

(i) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (-6)^2 - 2(-4) = 36 + 8 = 44$

$\alpha^2\beta^2 = (\alpha\beta)^2 = (-4)^2 = 16$

∴ The required equation is $x^2 - 44x - 16 = 0$.

(ii) $$\frac{2}{\alpha}+\frac{2}{\beta} = \frac{2\beta+2\alpha}{\alpha\beta} = \frac{2(\alpha+\beta)}{\alpha\beta} = \frac{2(-6)}{-4}$$

$$= \frac{-12}{-4} = 3$$

$$\frac{2}{\alpha}+\frac{2}{\beta} = \frac{4}{\alpha\beta} = \frac{4}{-4} = -1$$

∴ The required equation is $x^2 - 3x - 1 = 0$.

(iii) $\alpha^2\beta + \beta^2\alpha = \alpha\beta(\alpha + \beta)$
$= -4(-6) = 24$
$\alpha^2\beta \times \beta^2\alpha = \alpha^3\beta^3 = (\alpha\beta)^3 = (-4)^3 = -64$
∴ The required equation $= x^2 - 24x - 64 - 0$.

Question 4.
If α, β are the roots of $7x^2 + ax + 2 = 0$ and if $\beta - \alpha = \frac{-13}{7}$ Find the values of a.

Solution:

$7x^2 + ax + 2 = 0$

$$x = \frac{-a \pm \sqrt{a^2 - 56}}{2 \times 7}$$

$$\alpha = \frac{-a + \sqrt{a^2 - 56}}{14}, \beta = \frac{-a - \sqrt{a^2 - 56}}{14}$$

$$\beta - \alpha = \frac{-a - \sqrt{a^2 - 56} + a - \sqrt{a^2 - 56}}{14}$$

$$= \frac{-2\sqrt{a^2 - 56}}{14} = \frac{-13}{7} \text{ (given)}$$

$$\Rightarrow \frac{-\sqrt{a^2 - 56}}{7} = \frac{-13}{7}$$

$$\Rightarrow -\sqrt{a^2 - 56} = -13$$

Squaring on both sides.

$$a^2 - 56 = 169$$

$$a^2 = 225$$

$$a = \pm 15$$

Question 5.

If one root of the equation $2y^2 - ay + 64 = 0$ is twice the other then find the values of a.

Solution:

Let one of the root $\alpha = 2\beta$

$\alpha + \beta = 2\beta + \beta = 3\beta$

Given

$$2y^2 - ay + 64 = 0$$

$$y^2 - \frac{a}{2}y + 32 = 0$$

$$\Rightarrow y^2 - \left(\frac{a}{2}\right)y + 32 = 0$$

Sum of the roots $\alpha + \beta = \frac{a}{2}$

i.e. $3\beta = \frac{a}{2} \Rightarrow \beta = \frac{a}{6}$

$$\alpha\beta = \alpha \times \frac{a}{6}$$

$$\Rightarrow, \quad 2\beta \times \beta = 2\left(\frac{a}{6}\right)\left(\frac{a}{6}\right)$$

$$(2\beta\beta) = 2\beta^2 = 32$$

$$2\left(\frac{a^2}{36}\right) = 36$$

$a^2 = 576$
$a = 24, -24$

Question 6.
If one root of the equation $3x^2 + kx + 81 = 0$ (having real roots) is the square of the other then find k.
Solution:
$3x^2 + kx + 81 = 0$

Let the roots be α and α^2

$$\alpha + \alpha^2 = \frac{-k}{3} \qquad ...(1)$$

$$\alpha \ \alpha^2 = \frac{81}{3}$$

$$\Rightarrow \qquad \alpha^3 = 27$$

$$\Rightarrow \qquad \alpha = 3 \qquad ...(2)$$

Sub (2) in (1) we get

$$3 + 3^2 = \frac{-k}{3}$$

$$\Rightarrow \qquad (3 + 9) = \frac{-k}{3}$$

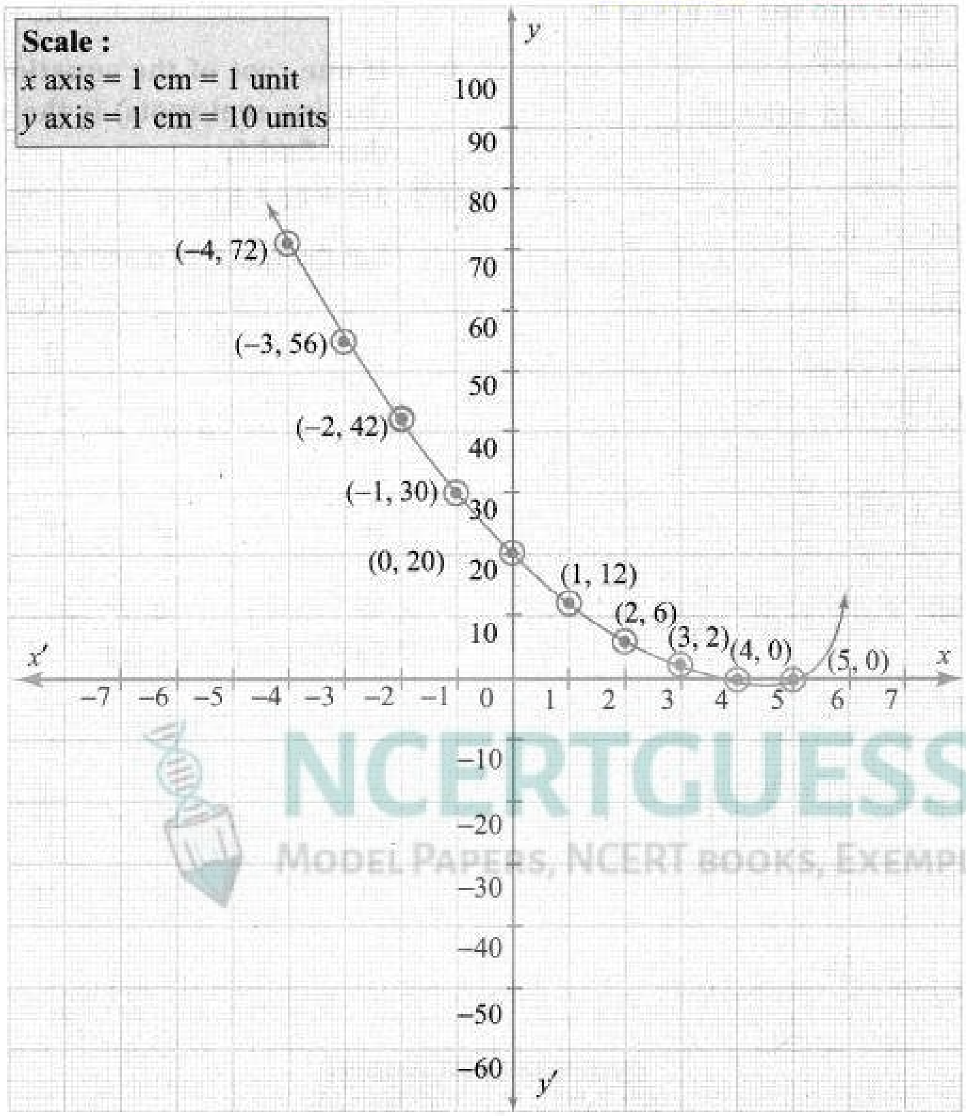
$$\Rightarrow \qquad k = -36.$$

Ex3.15

Question 1.
Graph the following quadratic equations and state their nature of solutions,
(i) $x^2 - 9x + 20 = 0$
Solution:

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$-9x$	+36	27	18	9	0	–9	–18	–27	–36
20	20	20	20	20	20	20	20	20	20
	72	56	42	30	20	12	6	2	0

Step 1:
Points to be plotted : (-4, 72), (-3, 56), (-2, 42), (-1, 30), (0, 20), (1, 12), (2, 6), (3, 2), (4, 0)
Step 2:
The point of intersection of the curve with x axis is (4, 0)
Step 3:



The roots are real & unequal

∴ Solution {4, 5}

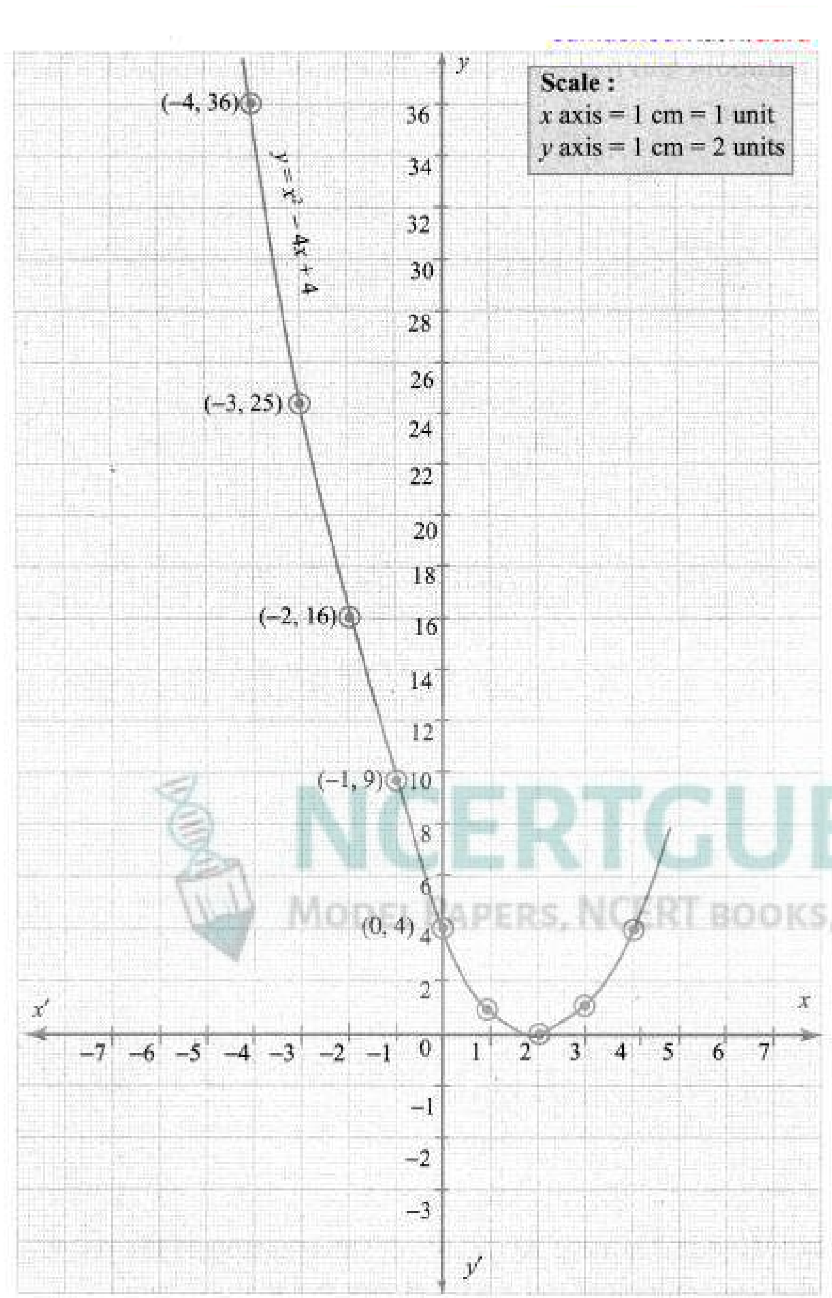
(ii) $x^2 - 4x + 4 = 0$

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$-4x$	16	12	8	4	0	-4	-8	-12	-16
4	4	4	4	4	4	4	4	4	4
$y = x^2 - 4x + 4$	36	25	16	9	4	1	0	1	4

Step 1: Points to be plotted : (-4, 36), (-3, 25), (-2, 16), (-1, 9), (0, 4), (1, 1), (2, 0), (3, 1), (4, 4)

Step 2: The point of intersection of the curve with x axis is (2, 0)

Step 3:

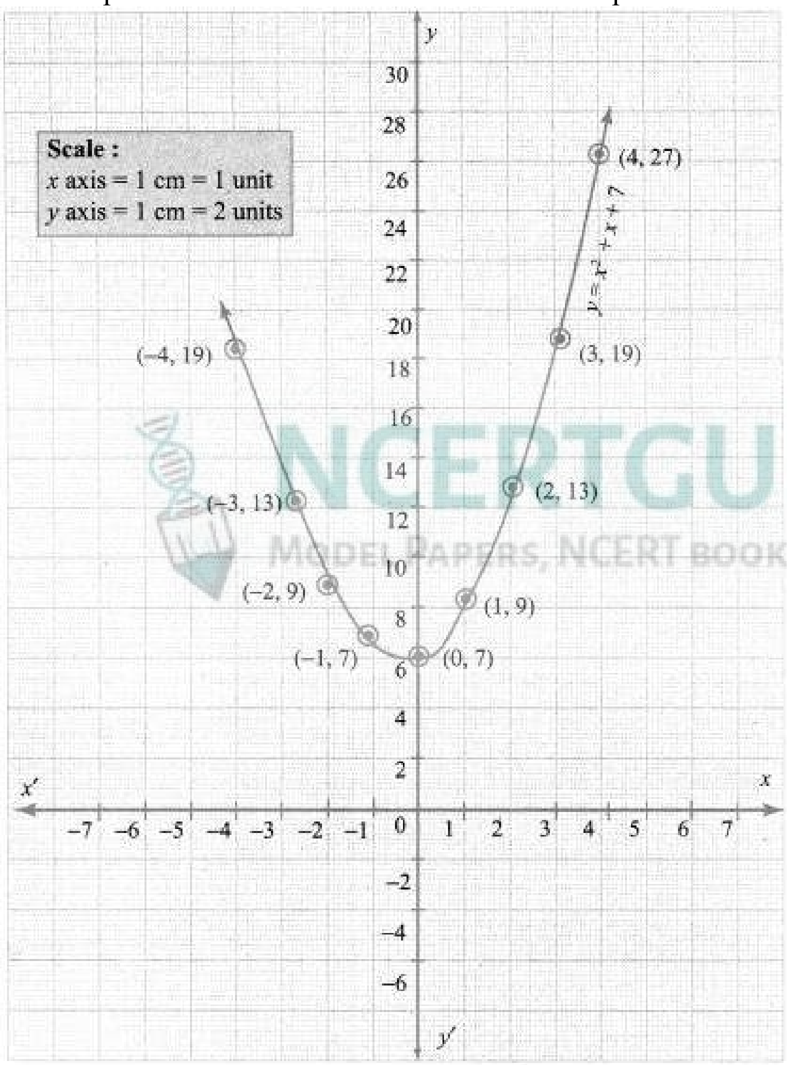


Since there is only one point of intersection with x axis, the quadratic equation $x^2 - 4x + 4 = 0$ has real and equal roots.
∴ Solution{2, 2}

(iii) $x^2 + x + 7 = 0$
Let $y = x^2 + x + 7$
Step 1:

x	−4	−3	−2	−1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
7	7	7	7	7	7	7	7	7	7
$y = x^2 + x + 7$	19	13	9	7	7	9	13	19	27

Step 2:
Points to be plotted: (-4, 19), (-3, 13), (-2, 9), (-1, 7), (0, 7), (1, 9), (2, 13), (3, 19), (4, 27)
Step 3:
Draw the parabola and mark the co-ordinates of the parabola which intersect with the x-axis.



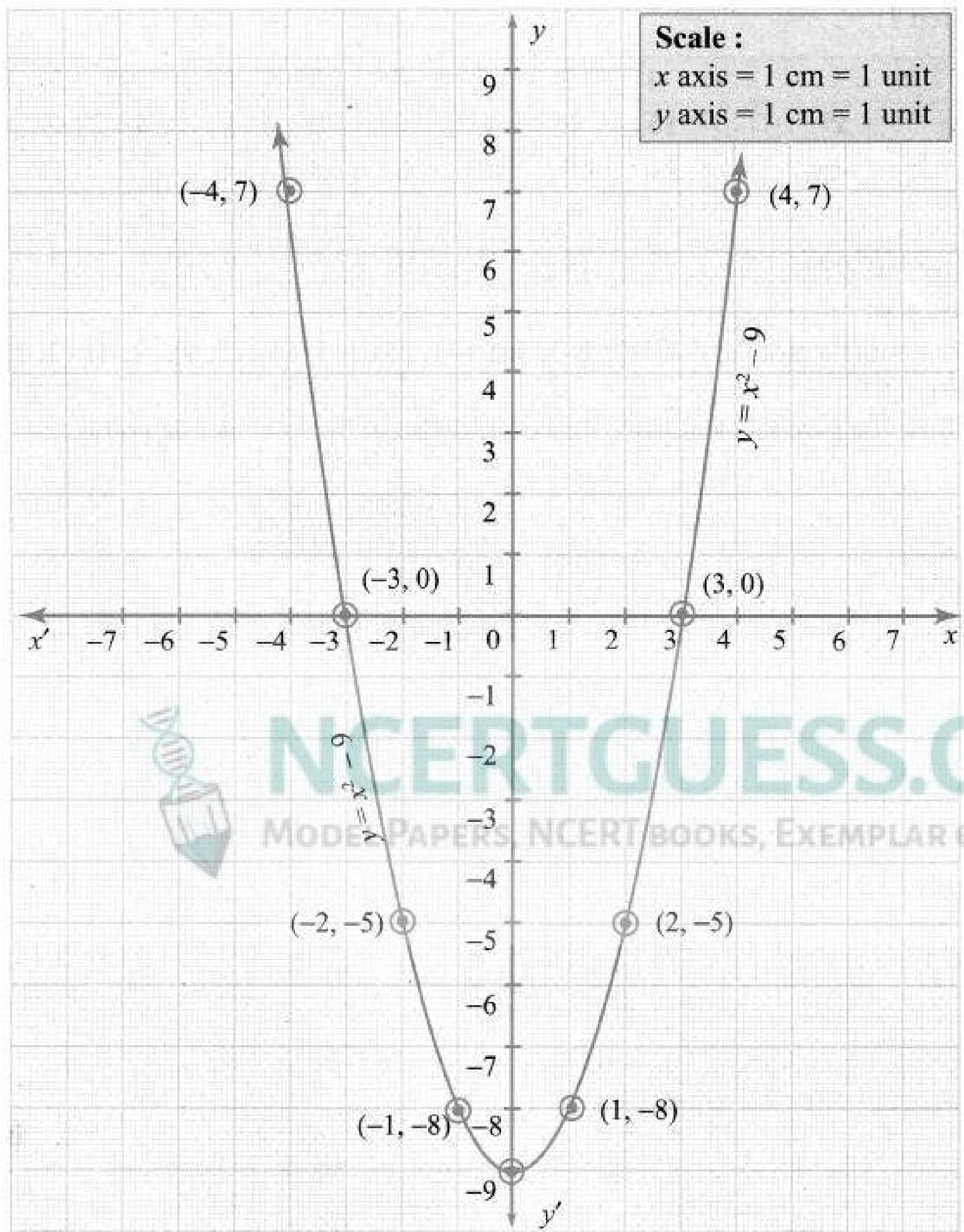
Step 4:
The roots of the equation are the points of intersection of the parabola with the x axis. Here the parabola does not intersect the x axis at any point.
So, we conclude that there is no real roots for the given quadratic equation,

(iv) $x^2 - 9 = 0$
Let $y = x^2 - 9$

Step 1:

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
–9	–9	–9	–9	–9	–9	–9	–9	–9	–9
$y = x^2 - 9$	7	0	–5	–8	–9	–8	–5	0	7

Step 2:
The points to be plotted: (-4, 7), (-3, 0), (-2, -5), (-1, -8), (0, -9), (1,-8), (2, -5), (3, 0), (4, 7)
Step 3:
Draw the parabola and mark the co-ordinates of the parabola which intersect the x-axis.

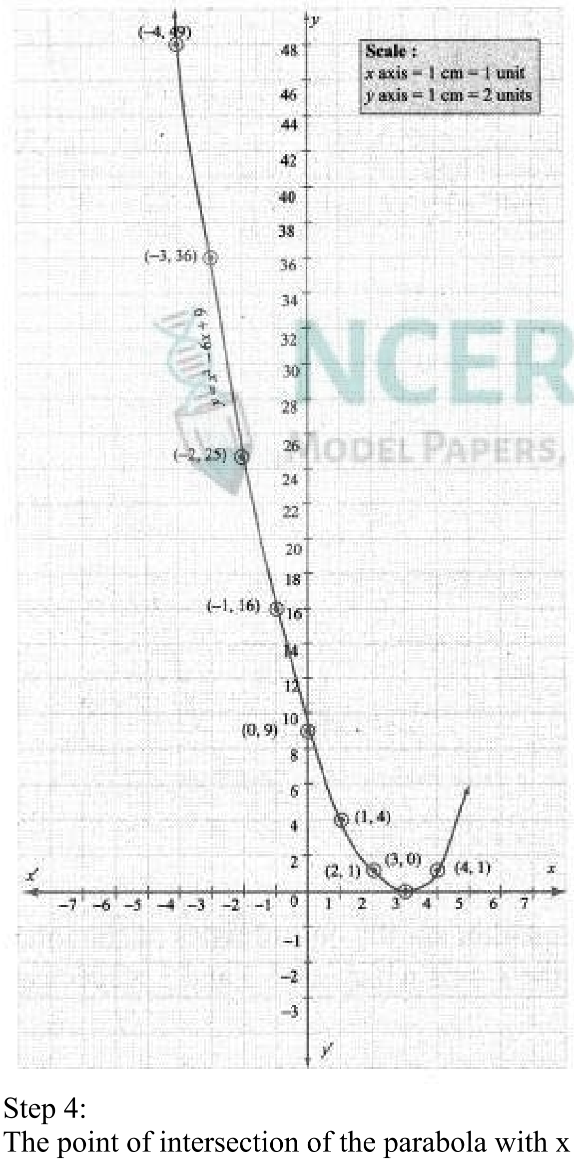


Step 4:
The roots of the equation are the co-ordinates of the intersecting points (-3, 0) and (3, 0) of the parabola with the x-axis which are -3 and 3 respectively.
Step 5:
Since there are two points of intersection with the x axis, the quadratic equation has real and unequal roots.
∴ Solution{-3, 3}

(v) $x^2 - 6x + 9 = 0$
Let $y = x^2 - 6x + 9$
Step 1:

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$-6x$	24	18	12	6	0	–6	–12	–18	–24
9	9	9	9	9	9	9	9	9	9
$y = x^2 - 6x + 9$	49	36	25	16	9	4	1	0	1

Step 2:
Points to be plotted: (-4, 49), (-3, 36), (-2, 25), (-1, 16), (0, 9), (1, 4), (2, 1), (3, 0), (4, 1)
Step 3:
Draw the parabola and mark the co-ordinates of the intersecting points.



Step 4:
The point of intersection of the parabola with x axis is (3, 0)
Since there is only one point of intersection with the x-axis, the quadratic equation has real and equal roots. .
∴ Solution (3, 3)

(vi) $(2x - 3)(x + 2) = 0$

$2x^2 - 3x + 4x - 6 = 0$

$2x^2 + 1x - 6 = 0$

Let $y = 2x^2 + x - 6 = 0$

Step 1:

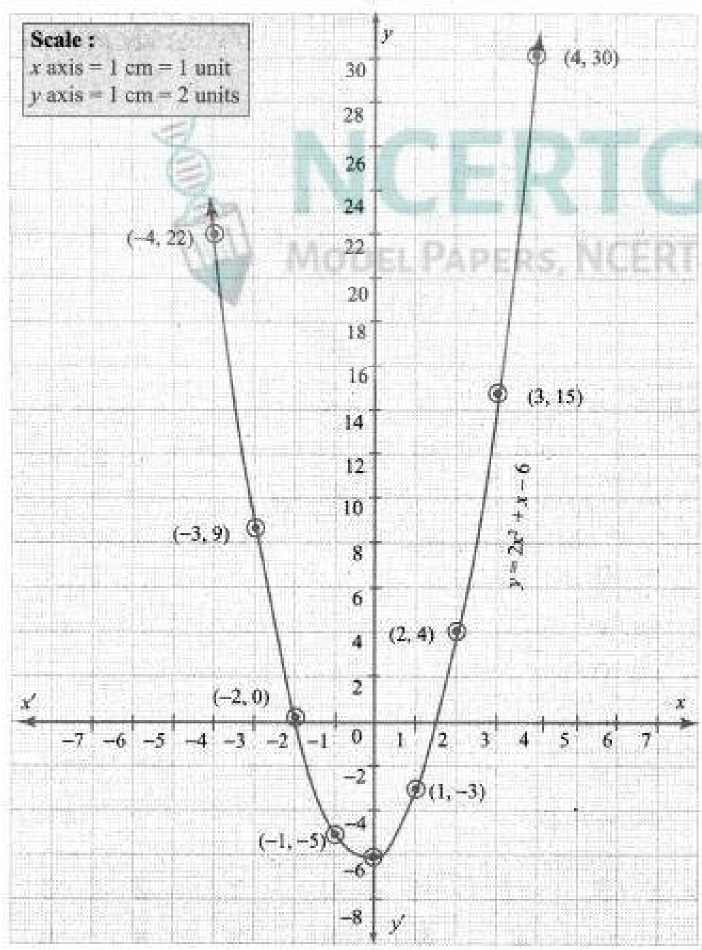
x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$2x^2$	32	18	8	2	0	2	8	18	32
x	-4	-3	-2	-1	0	1	2	3	4
-6	-6	-6	-6	-6	-6	-6	-6	-6	-6
$y = 2x^2 + x - 6$	22	9	0	-5	-6	-3	4	15	30

Step 2:

The points to be plotted: (-4, 22), (-3, 9), (-2, 0), (-1, -5), (0, -6), (1, -3), (2, 4), (3, 15), (4, 30)

Step 3:

Draw the parabola and mark the co-ordinates of the intersecting point of the parabola with the x-axis.



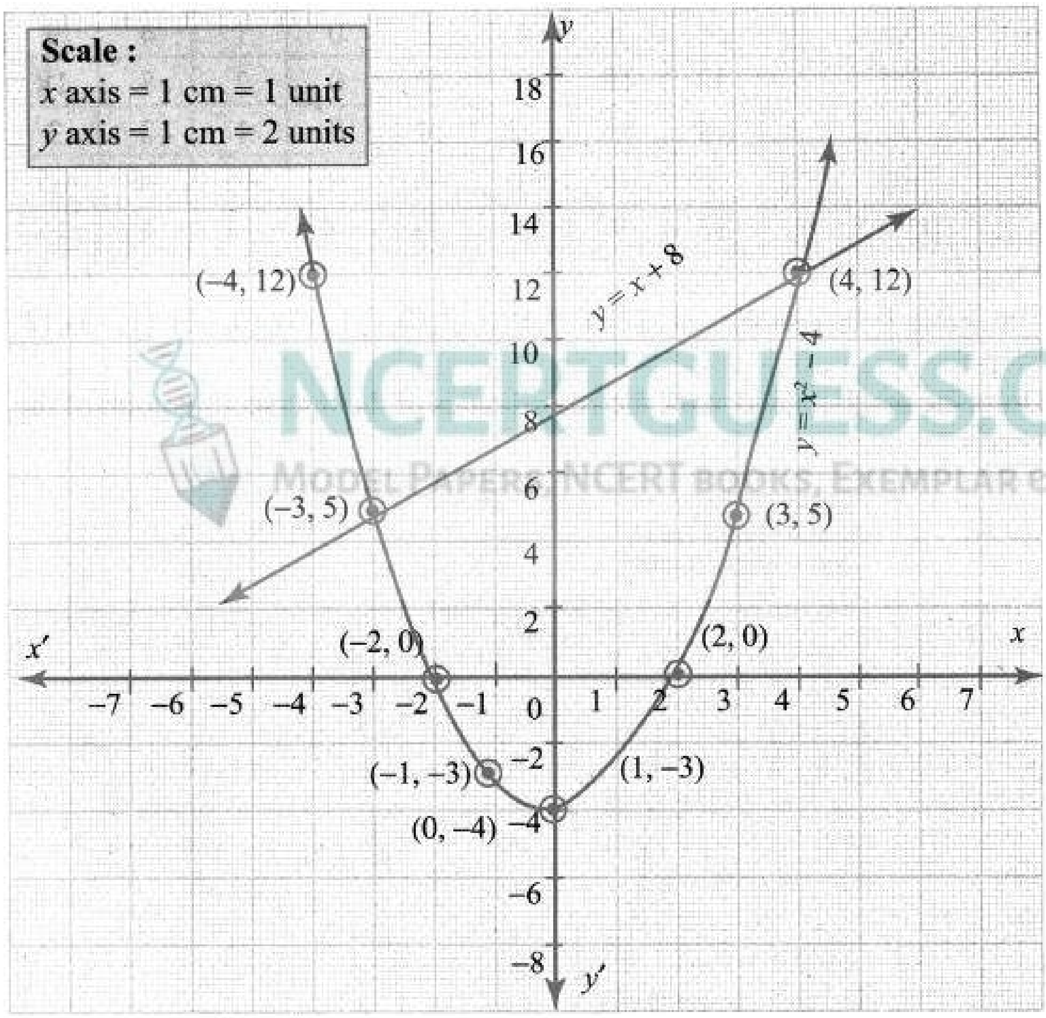
Step 4:

The points of intersection of the parabola with the x-axis are (-2, 0) and (1.5, 0).

Since the parabola intersects the x-axis at two points, the, equation has real and unequal roots.
∴ Solution {-2, 1.5}

Question 2.
Draw the graph of $y = x^2 - 4$ and hence solve $x^2 - x - 12 = 0$
Solution:

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
– 4	–4	–4	–4	–4	–4	–4	–4	–4	–4
$x^2 - 4$	12	5	0	–3	–4	–3	0	5	12



$$\begin{aligned}
\textbf{To solve } x^2 - x - 12 &= 0 \\
\cancel{x^2} - 4 &= y \\
\cancel{x^2} - x - 12 &= 0 \\
(-)\ (+)\ (+) \quad & \quad (-) \\
\hline
x + 8 &= y \\
y &= x + 8
\end{aligned}$$

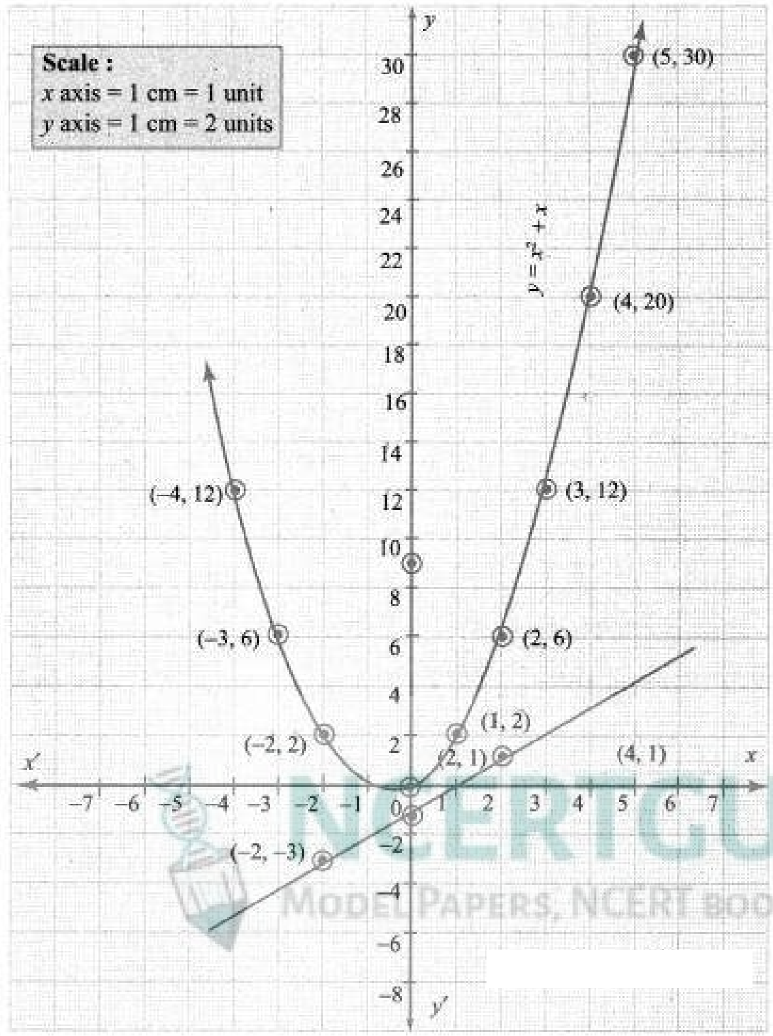
x	−4	−3	−2	−1	0	1	2	3	4
8	8	8	8	8	8	8	8	8	8
x − 8	4	5	6	7	8	9	10	11	12

Point of intersection (-3, 5), (4, 12) solution of $x^2 - x - 12 = 0$ is -3, 4

Question 3.
Draw the graph of $y = x^2 + x$ and hence solve $x^2 + 1 = 0$.
Solution:

x	−4	−3	−2	−1	0	1	2	3	4	5
x^2	16	9	4	1	0	1	4	9	16	25
$+x$	−4	−3	−2	−1	0	1	2	3	4	5
$y = x^2 + x$	12	6	2	0	0	2	6	12	20	30

Draw the parabola by the plotting the points (-4, 12), (-3, 6), (-2, 2), (-1, 0), (0, 0), (1, 2), (2, 6), (3, 12), (4, 20), (5, 30)



To solve: $x^2 + 1 = 0$, subtract $x^2 + 1 = 0$ from $y = x^2 + x$.

$x^2 + 1 = 0$ from $y = x^2 + x$

$x^2 + 1 = 0$ from $y = x^2 + x$

i.e.

$$\begin{aligned} y &= \cancel{x^2} + x \\ 0 &= \cancel{x^2} + 1 \\ (-) & \quad (-) \quad (-) \\ \hline y &= x - 1 \end{aligned}$$

This is a straight line.

Draw the line $y = x - 1$.

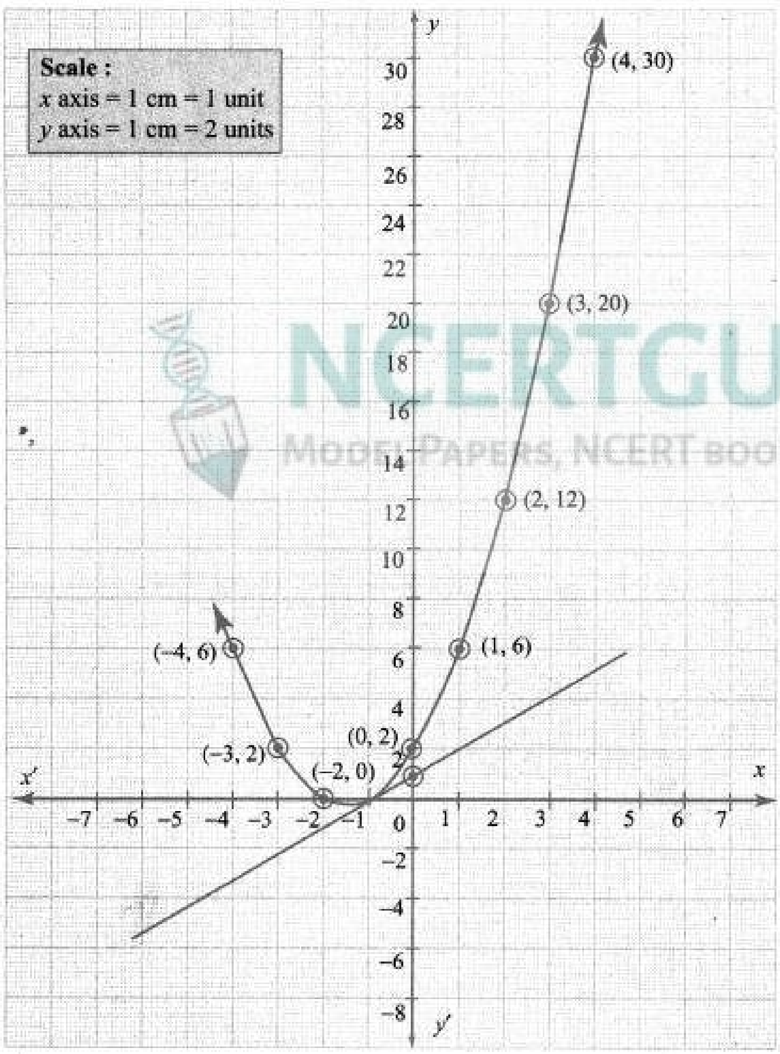
x	–2	0	2
–1	–1	–1	–1
y	–3	–1	1

Plotting the points (-2, -3), (0, -1), (2, 1) we get a straight line. This line does not intersect the parabola. Therefore there is no real roots for the equation $x^2 + 1 = 0$.

Question 4.
Draw the graph of $y = x^2 + 3x + 2$ and use it to solve $x^2 + 2x + 1 = 0$.
Solution:

x	−4	−3	−2	−1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$3x$	−12	−9	−6	−3	0	3	6	9	12
2	2	2	2	2	2	2	2	2	2
$y = x^2 + 3x + 2$	6	2	0	0	2	6	12	20	30

Draw the parabola by plotting the point (-4, 6), (-3, 2), (-2, 0), (-1, 0), (0, 2), (1, 6), (2, 12), (3, 20), (4, 30).



To solve $x^2 + 2x + 1 = 0$, subtract $x^2 + 2x + 1 = 0$ from $y = x^2 + 3x + 2$

$$
\begin{array}{rl}
y = & x^2 + 3x + 2 \\
0 = & x^2 + 2x + 1 \\
(-) & (-)\ (-)\quad (-) \\
\hline
y = & x + 1
\end{array}
$$

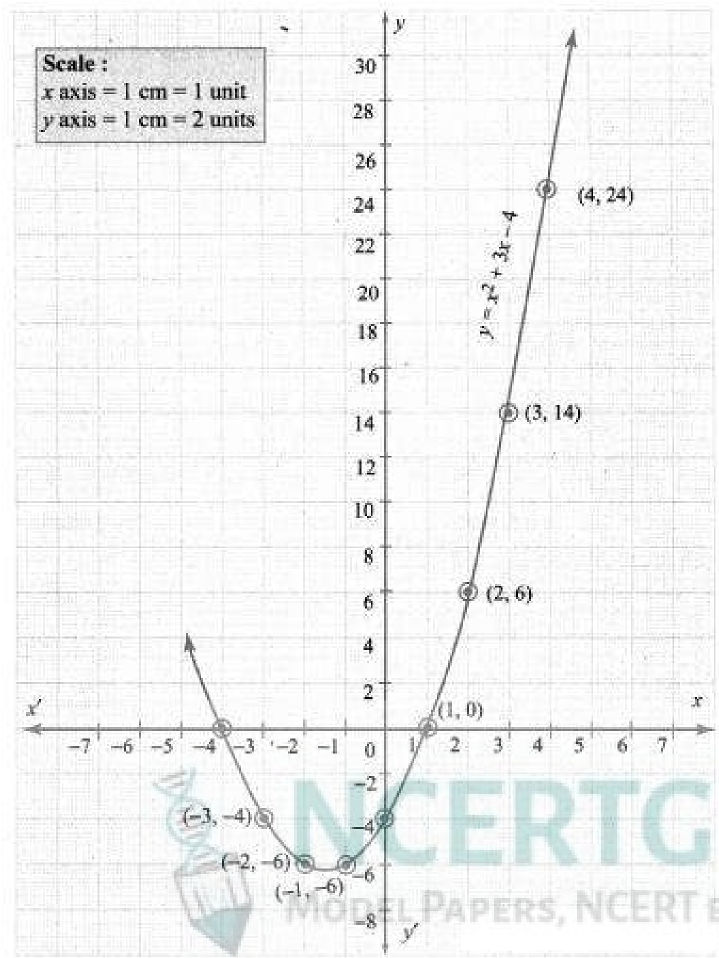
x	-2	0	2
1	1	1	1
$y = x + 1$	-1	1	3

Draw the straight line by plotting the points (-2, -1), (0, 1), (2, 3)
The straight line touches the parabola at the point (-1,0)
Therefore the x coordinate -1 is the only solution of the given equation

Question 5.
Draw the graph of $y = x^2 + 3x – 4$ and hence use it to solve $x^2 + 3x – 4 = 0$. $y = x^2 + 3x – 4$
Solution:

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$3x$	-12	-9	-6	-3	0	3	6	9	12
-4	-4	-4	-4	-4	-4	-4	-4	-4	-4
$y = x^2 + 3x - 4$	0	-4	-6	-6	-4	0	6	14	24

Draw the parabola using the points (-4, 0), (-3, -4), (-2, -6), (-1, -6), (0, -4), (1, 0), (2, 6), (3, 14), (4, 24).



To solve: $x^2 + 3x - 4 = 0$ subtract $x^2 + 3x - 4 = 0$ from $y = x^2 + 3x - 4$,

$$\begin{array}{rcl} y & = & x^2 + 3x - 4 \\ 0 & = & x^2 + 3x - 4 \\ & & (-)\ (-)\quad (+) \\ \hline y & = & 0 \end{array}$$

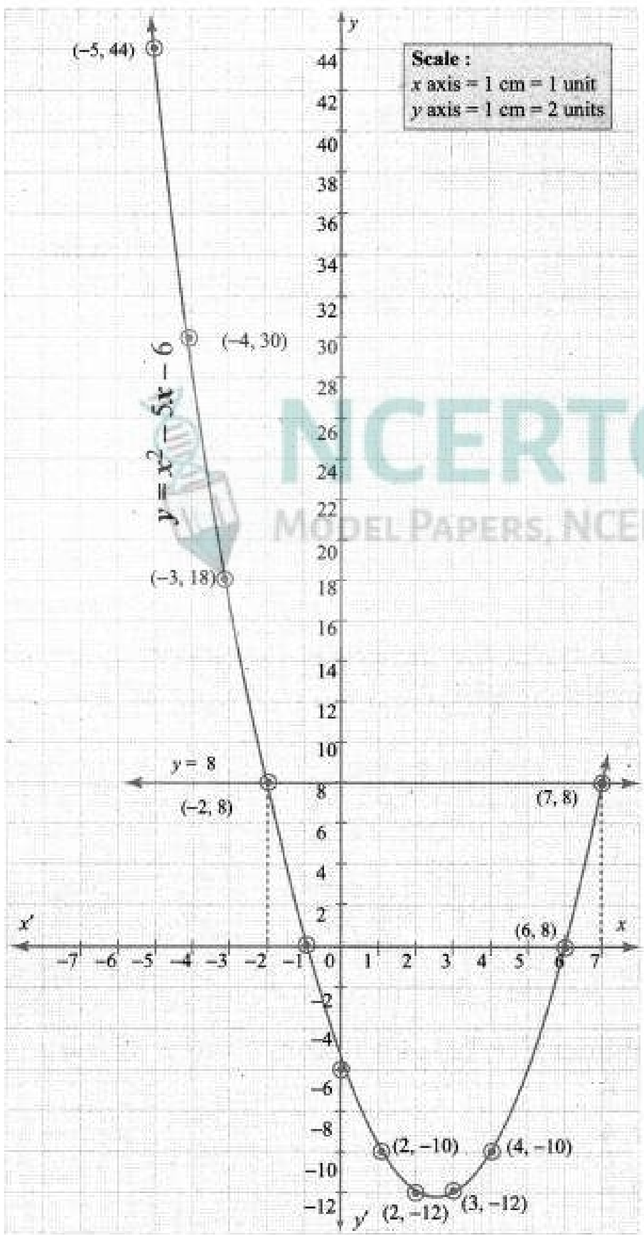
is the equation of the x axis.

The points of intersection of the parabola with the x axis are the points (-4, 0) and (1, 0), whose x – co-ordinates (-4, 1) is the solution, set for the equation $x^2 + 3x - 4 = 0$.

Question 6.
Draw the graph of $y = x^2 - 5x - 6$ and hence solve $x^2 - 5x - 14 = 0$.
Solution:

x	–5	–4	–3	–2	–1	0	1	2	3	4
x^2	25	16	9	4	1	0	1	4	9	16
$-5x$	25	20	15	10	5	0	–5	–10	–15	–20
-6	–6	–6	–6	–6	–6	–6	–6	–6	–6	–6
$y = x^2 + 5x - 6$	44	30	18	8	0	–6	–10	–12	–12	–10

Draw the parabola using the points (-5, 44), (-4, 30), (-3, 18), (-2, 8), (-1, 0), (0, -6), (1, -10), (2, -12), (3, -12), (4, -10)



To solve the equation $x^2 - 5x - 14 = 0$, subtract $x^2 - 5x - 14 = 0$ from $y = x^2 - 5x - 6$.

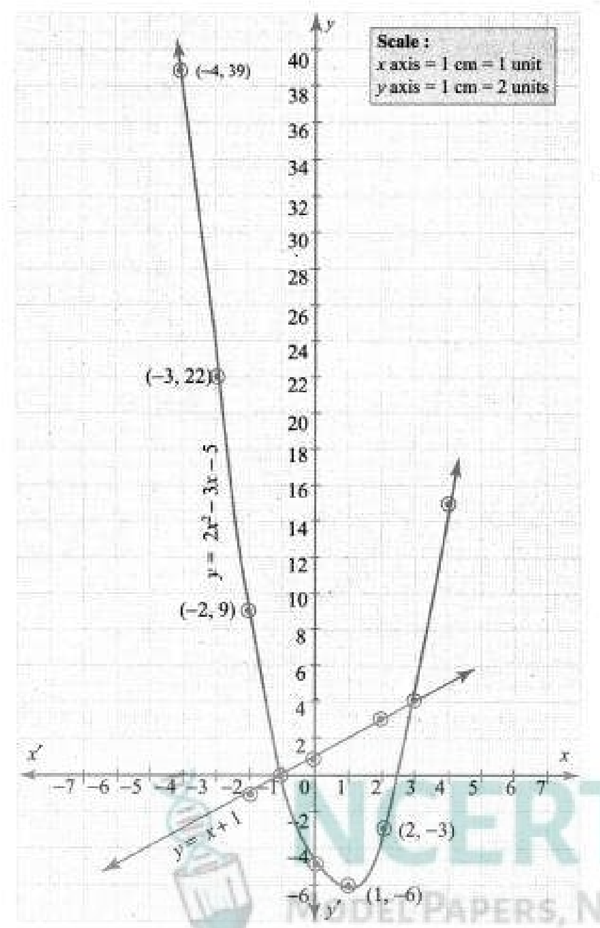
$$
\begin{array}{rcl}
y & = & x^2 - 5x - 6 \\
0 & = & x^2 - 5x - 14 \\
 & & (-)\ (+)\quad\ (+) \\
\hline
y & = & 8 \\
\hline
\end{array}
$$
is a straight line parallel to x axis.

The co-ordinates of the points of intersection of the line and the parabola forms the solution set for the equation x2 – 5x – 14 = 0.
∴ Solution {-2, 7}

Question 7.
Draw the graph of $y = 2x^2 – 3x – 5$ and hence solve $2x^2 – 4x – 6 = 0$. $y = 2x^2 – 3x – 5$
Solution:

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$2x^2$	32	18	8	2	0	2	8	18	32
$-3x$	12	9	6	3	0	–3	–6	–9	–12
-5	–5	–5	–5	–5	–5	–5	–5	–5	–5
$y = 2x^2 - 3x - 5$	39	22	9	0	–5	–6	–3	4	15

Draw the parabola using the points (-4, 39), (-3, 22), (-2, 9), (-1, 0), (0, -5), (1, -6), (2, -3), (3, 4), (4, 15).



To solve $2x^2 - 4x - 6 = 0$, subtract it from $y = 2x^2 - 3x - 5$

$$\begin{array}{rcl} y &=& \cancel{2x^2} - 3x - 5 \\ 0 &=& \cancel{2x^2} - 4x - 6 \\ & & (-)\ (+)\qquad (+) \\ \hline y &=& \qquad x + 1 \text{ is a straight line} \end{array}$$

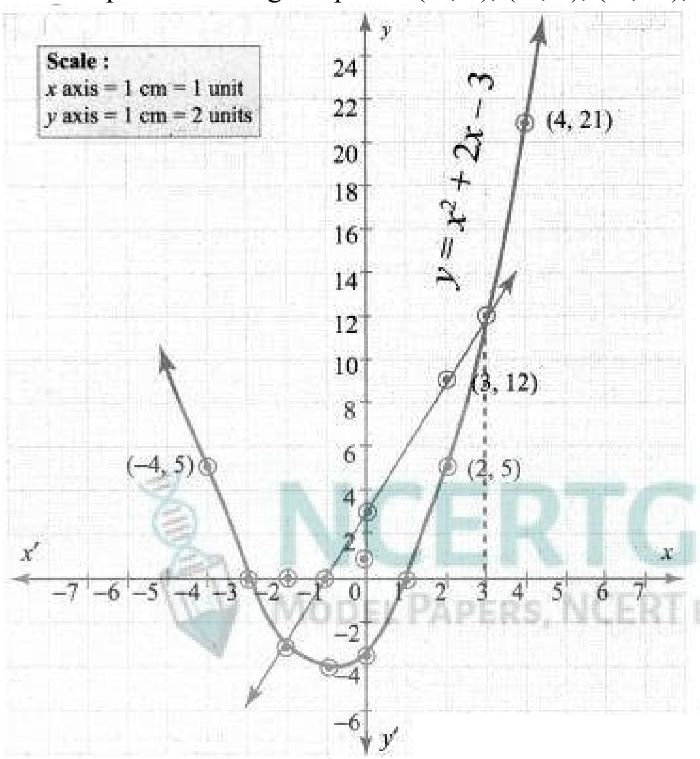
x	-2	0	2
1	1	1	1
$y = x + 1$	-1	1	3

Draw a straight line using the points (-2, -1), (0, 1), (2, 3). The points of intersection of the parabola and the straight line forms the roots of the equation.
The x-coordinates of the points of intersection forms the solution set.
∴ Solution {-1, 3}

Question 8.
Draw the graph of $y = (x - 1)(x + 3)$ and hence solve $x^2 - x - 6 = 0$.
Solution:
$y = (x - 1)(x + 3) = x^2 - x + 3x - 3 = 0$
$y = x^2 + 2x - 3$

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$2x$	–8	–6	–4	–2	0	2	4	6	8
–3	–3	–3	–3	–3	–3	–3	–3	–3	–3
$y = x^2 + 2x - 3$	5	0	–3	–4	–3	0	5	12	21

Draw the parabola using the points (-4, 5), (-3, 0), (-2, -3), (-1,-4), (0, -3), (1, 0), (2, 5), (3, 12), (4, 21)



To solve the equation $x^2 - x - 6 = 0$, subtract $x^2 - x - 6 = 0$ from $y = x^2 - 2x - 3$.

$$\begin{array}{rl} y = & x^2 + 2x - 3 \\ 0 = & x^2 - x - 6 \\ & (-)\ (+)\quad (+) \\ \hline y = & \quad 3x + 3 \text{ is a straight line} \end{array}$$

x	–2	–1	0	2
$3x$	–6	–3	0	6
3	3	3	3	3
$y = 3x + 3$	–3	0	3	9

Plotting the points (-2, -3), (-1, 0), (0, 3), (2, 9), we get a straight line.
The points of intersection of the parabola with the straight line gives the roots of the equation. The co¬ordinates of the points of intersection forms the solution set.
∴ Solution {-2, 3}

Ex 3.16

Question 1.

In the matrix A = $\begin{bmatrix} 8 & 9 & 4 & 3 \\ -1 & \sqrt{7} & \frac{\sqrt{3}}{2} & 5 \\ 1 & 4 & 3 & 0 \\ 6 & 8 & -11 & 1 \end{bmatrix}$

(i) The number of elements
(ii) The order of the matrix
(iii) Write the elements a_{22}, a_{23}, a_{24}, a_{34}, a_{43}, a_{44}

Solution:
(i) 16
(ii) 4 × 4
(iii) $\sqrt{7}$, $\frac{\sqrt{3}}{2}$, 5, 0, -11, 1

Question 2.
If a matrix has 18 elements, what are the possible orders it can have? What if it has 6 elements?
Solution:
1 × 18, 2 × 9, 3 × 6, 6 × 3, 9 × 2, 18 × 1 and 1 × 6, 2 × 3, 3 × 2, 6 × 1

Question 3.
Construct a 3 × 3 matrix whose elements are given by then find the transpose of A.
(i) $a_{ij} = |i - 2j|$
(ii) $a_{ij} = \frac{(i+j)^3}{3}$
Solution:
(i) $a_{ij} = |i - 2j|$
$a_{11} = |1 - 2 \times 1| = |1 - 2| = |-1| = 1$
$a_{12} = |1 - 2 \times 2| = |1 - 4| = |-3| = 3$
$a_{13} = |1 - 2 \times 3| = |1 - 6| = |-5| = 5$

$a_{21} = |2 – 2 \times 1| = |2 – 2| = 0$
$a_{22} = |2 – 2 \times 2| = |2 – 4| = |-2| = 2$
$a_{23} = |2 – 2 \times 3| = |2 – 6| = |-4| = 4$
$a_{31} = |3 – 2 \times 1| = |3 – 2| = |1| = 1$
$a_{32} = |3 – 2 \times 2| = |3 – 4| = |-1| = 1$
$a_{33} = |3 – 2 \times 3| = |3 – 6| = |-3| = 3$

$\therefore \begin{bmatrix} 1 & 3 & 5 \\ 0 & 2 & 4 \\ 1 & 1 & 3 \end{bmatrix}$ is the required 3×3 matrix

(ii) $a_{ij} = \dfrac{(i+j)^3}{3}$

$$a_{11} = \frac{(1+1)^3}{3} = \frac{(2)^3}{3} = \frac{8}{3}$$

$$a_{12} = \frac{(1+2)^3}{3} = \frac{(3)^3}{3} = \frac{27}{3} = 9$$

$$a_{13} = \frac{(1+3)^3}{3} = \frac{(4)^3}{3} = \frac{64}{3}$$

$$a_{21} = \frac{(2+1)^3}{3} = 9$$

$$a_{22} = \frac{(2+2)^3}{3} = \frac{(4)^3}{3} = \frac{64}{3}$$

$$a_{23} = \frac{(2+3)^3}{3} = \frac{(5)^3}{3} = \frac{125}{3}$$

$$a_{31} = \frac{(3+1)^3}{3} = \frac{(4)^3}{3} = \frac{64}{3}$$

$$a_{32} = \frac{(3+2)^3}{3} = \frac{125}{3}$$

$$a_{33} = \frac{(3+3)^3}{3} = \frac{216}{3} = 72$$

$$\begin{bmatrix} \frac{8}{3} & 9 & \frac{64}{3} \\ 9 & \frac{64}{3} & \frac{125}{3} \\ \frac{64}{3} & \frac{125}{3} & 72 \end{bmatrix}$$ is the required 3×3 matrix

Question 4.

If $A = \begin{bmatrix} 5 & 4 & 3 \\ 1 & -7 & 9 \\ 3 & 8 & 2 \end{bmatrix}$ **then find the transpose of A.**

Solution:

If $A = \begin{bmatrix} 5 & 4 & 3 \\ 1 & -7 & 9 \\ 3 & 8 & 2 \end{bmatrix}$

Transpose of A = $A^T = \begin{bmatrix} 5 & 1 & 3 \\ 4 & -7 & 8 \\ 3 & 9 & 2 \end{bmatrix}$

Question 5.

If $A = \begin{bmatrix} \sqrt{7} & -3 \\ -\sqrt{5} & 2 \\ \sqrt{3} & -5 \end{bmatrix}$ **then find the transpose of –A.**

Solution:

$$\text{If } A = \begin{bmatrix} \sqrt{7} & -3 \\ -\sqrt{5} & 2 \\ \sqrt{3} & -5 \end{bmatrix},$$

$$-A = \begin{bmatrix} -\sqrt{7} & 3 \\ \sqrt{5} & -2 \\ -\sqrt{3} & 5 \end{bmatrix}$$

Transpose of $-A = (-A)^T = \begin{bmatrix} -\sqrt{7} & +\sqrt{5} & -\sqrt{3} \\ +3 & -2 & +5 \end{bmatrix}$

Question 6.

If $A = \begin{bmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{bmatrix}$ then verify $(A^T)^T = A$

Solution:

$$\text{If } A = \begin{bmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{bmatrix}, A^T = \begin{bmatrix} 5 & -\sqrt{17} & 8 \\ 2 & 0.7 & 3 \\ 2 & \frac{5}{2} & 1 \end{bmatrix}$$

$$(A^T)^T = \begin{bmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{bmatrix} = A. \therefore \text{ verified}$$

Question 7.
Find the values of x,y and z from the following equations

(i) $\begin{bmatrix} 12 & 3 \\ x & \frac{3}{2} \end{bmatrix} = \begin{bmatrix} y & z \\ 3 & 5 \end{bmatrix}$

(ii) $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

(iii) $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

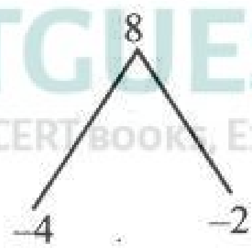
Solution:

(i) $\begin{bmatrix} 12 & 3 \\ x & \frac{3}{2} \end{bmatrix} = \begin{bmatrix} y & z \\ 3 & 5 \end{bmatrix}$

$$x = 3$$
$$y = 12$$
$$z = 3$$

(ii) $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

$$\Rightarrow \quad 5 + z = 5$$
$$z = 0$$
$$x + y = 6$$
$$x = 6 - y$$
$$xy = 8$$
$$(6 - y)y = 8$$
$$\Rightarrow \quad 6y - y^2 - 8 = 0$$
$$y^2 - 6y + 8 = 0$$
$$(y - 4)(y - 2) = 0$$



$y = 4, 2, x = 2, 4$

$\mathbf{x = 2,\ y = 4,\ z = 0}$

(or)

$\mathbf{x = 4,\ y = 2,\ z = 0}$

(iii) $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

$$\Rightarrow \quad x+y+z = 9 \quad \text{...(1)}$$
$$\Rightarrow \quad x+z = 5 \quad \text{...(2)}$$
$$\Rightarrow \quad y+z = 7 \quad \text{...(3)}$$

$$(1)-(2) \Rightarrow \begin{array}{r} \cancel{x}+y+\cancel{z} = 9 \\ \underset{(-)}{\cancel{x}} \quad \underset{(-)}{+}\underset{(-)}{\cancel{z}} = 5 \\ \hline y \quad = 4 \end{array}$$

Sub. $y = 4$ in (3)

$$4+z = 7$$
$$z = 3$$

Sub. $z = 3$ in (2)

$$x+3 = 5$$
$$x = 2$$

$x = 2, \; y = 4, \; z = 3$

Ex 3.17

Question 1.

If A = $\begin{bmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{bmatrix} = \begin{bmatrix} 5 & 7 \\ 3 & 3 \\ 1 & 0 \end{bmatrix}$ then verify that

(i) A + B = B + A

(ii) A + (-A) = (-A) + A = 0

Solution:

$$\text{L.H.S} = A + B = \begin{bmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{bmatrix} + \begin{bmatrix} 5 & 7 \\ 3 & 3 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 16 \\ 6 & 7 \\ 9 & -3 \end{bmatrix} \quad \ldots(1)$$

$$\text{R.H.S} = B + A = \begin{bmatrix} 5 & 7 \\ 3 & 3 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 16 \\ 6 & 7 \\ 9 & -3 \end{bmatrix} \quad \ldots(2)$$

(1) = (2) ⇒ L.H.S = R.H.S. Hence verified.

(ii) $A + (-A) = (-A) + A = 0$

L.H.S = A + (–A)

$$= \begin{bmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{bmatrix} + \begin{bmatrix} -1 & -9 \\ -3 & -4 \\ -8 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \ldots(1)$$

R.H.S = (–A) + A

$$= \begin{bmatrix} -1 & -9 \\ -3 & -4 \\ -8 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \ldots(2)$$

(1) = (2) ⇒ L.H.S. = R.H.S. Hence verified.

Question 2.

If $A = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 3 & -8 \\ 1 & 0 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 9 & 2 \\ -7 & 1 & -1 \end{bmatrix}$ **and**

$C = \begin{bmatrix} 8 & 3 & 4 \\ 1 & -2 & 3 \\ 2 & 4 & -1 \end{bmatrix}$ **then verify that**

$A + (B + C) = (A + B) + C$

Solution:

$$(B + C) = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 9 & 2 \\ -7 & 1 & -1 \end{bmatrix} + \begin{bmatrix} 8 & 3 & 4 \\ 1 & -2 & 3 \\ 2 & 4 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 6 & 8 \\ 2 & 7 & 5 \\ -5 & 5 & -2 \end{bmatrix}$$

$$A + (B + C) = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 3 & -8 \\ 1 & 0 & -4 \end{bmatrix} + \begin{bmatrix} 10 & 6 & 8 \\ 2 & 7 & 5 \\ -5 & 5 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 9 & 9 \\ 4 & 10 & -3 \\ -4 & 5 & -6 \end{bmatrix} \qquad (1)$$

R.H.S. $(A + B) + C$

$$(A + B) = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 3 & -8 \\ 1 & 0 & -4 \end{bmatrix} + \begin{bmatrix} 2 & 3 & 4 \\ 1 & 9 & 2 \\ -7 & 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 6 & 5 \\ 3 & 12 & -6 \\ -6 & 1 & -5 \end{bmatrix} \qquad (1)$$

$$(A+B)+C = \begin{bmatrix} 6 & 6 & 5 \\ 3 & 12 & -6 \\ -6 & 1 & -5 \end{bmatrix} + \begin{bmatrix} 8 & 3 & 4 \\ 1 & -2 & 3 \\ 2 & 4 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 9 & 9 \\ 4 & 10 & -3 \\ -4 & 5 & -6 \end{bmatrix} \quad ...(2)$$

(1) = (2) ⇒ L.H.S. = R.H.S. Hence verified.

Question 3.

Find X and Y if $X + Y = \begin{bmatrix} 7 & 0 \\ 3 & 5 \end{bmatrix}$ and $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$

Solution:

$$X + Y = \begin{bmatrix} 7 & 0 \\ 3 & 5 \end{bmatrix} \quad ...(1)$$

$$X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix} \quad ...(2)$$

$$(1) + (2) \Rightarrow 2x = \begin{bmatrix} 10 & 0 \\ 3 & 9 \end{bmatrix} \Rightarrow x = \frac{1}{2}\begin{bmatrix} 10 & 0 \\ 3 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 0 \\ \frac{3}{2} & \frac{9}{2} \end{bmatrix}$$

$$(1) - (2) \Rightarrow X + Y = \begin{bmatrix} 7 & 0 \\ 3 & 5 \end{bmatrix}$$

$$X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$$

$$(-) \quad (+) \quad (-)$$

$$2Y = \begin{bmatrix} 4 & 0 \\ 3 & 1 \end{bmatrix} \Rightarrow Y = \frac{1}{2}\begin{bmatrix} 4 & 0 \\ 3 & 1 \end{bmatrix}$$

$$\therefore Y = \begin{bmatrix} 2 & 0 \\ \frac{3}{2} & \frac{1}{2} \end{bmatrix}$$

$$X = \begin{bmatrix} 5 & 0 \\ \frac{3}{2} & \frac{9}{2} \end{bmatrix}, y = \begin{bmatrix} 2 & 0 \\ \frac{3}{2} & \frac{1}{2} \end{bmatrix}$$

Question 4.
If A = $\begin{bmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{bmatrix}$, B = $\begin{bmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{bmatrix}$ find the value of
(i) B – 5A
(ii) 3A – 9B
Solution:

$$A = \begin{bmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{bmatrix}, B = \begin{bmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{bmatrix}$$

(i) $$B - 5A = \begin{bmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{bmatrix} - 5\begin{bmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{bmatrix} - \begin{bmatrix} 0 & 20 & 45 \\ 40 & 15 & 35 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & -17 & -37 \\ -39 & -11 & -26 \end{bmatrix}$$

(ii) $$3A - 9B = 3\begin{bmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{bmatrix} - 9\begin{bmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 12 & 27 \\ 24 & 9 & 21 \end{bmatrix} - \begin{bmatrix} 63 & 27 & 72 \\ 9 & 36 & 81 \end{bmatrix}$$

$$= \begin{bmatrix} -63 & -15 & -45 \\ 15 & -27 & -60 \end{bmatrix}$$

Question 5.
Find the values of x, y, z if

(i) $$\begin{pmatrix} x-3 & 3x-z \\ x+y+7 & x+y+z \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 6 \end{pmatrix}$$

(ii) $$\begin{bmatrix} x & y-z & z+3 \end{bmatrix} + \begin{bmatrix} y & 4 & 3 \end{bmatrix}$$

Solution:
(i) $$\begin{pmatrix} x-3 & 3x-z \\ x+y+7 & x+y+z \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 6 \end{pmatrix}$$

$x - 3 = 1 \Rightarrow x = 4$
$3x - z = 0$
$3(4) - z = 0$
$-z = -12 \Rightarrow z = 12$
$x + y + 7 = 1$
$x + y = -6$
$4 + y = -6$
$y = -10$
$x = 4, y = -10, z = 12$

(ii) $\begin{bmatrix} x & y-z & z+3 \end{bmatrix} + \begin{bmatrix} y & 4 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 8 & 16 \end{bmatrix}$
$x + y = 4$ ……………. (1)
$y - z + 4 = 8$ ………….. (2)
$z + 3 + 3 = 16$ ………….. (3)
From (3), we get $z = 10$
From (2), we get $y - 10 + 4 = 8$
From (2), we get $y = 14$
From (1) we get $x + 14 = 4$
$x = -10$
$x = -10, y = 14, z = 10$

Question 6.

Find x and y if $x\begin{pmatrix} 4 \\ -3 \end{pmatrix} + y\begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$.

Solution:

$4x - 2y = 4$ ...(1)
$-3x + 3y = 6$ ...(2)

$$\begin{aligned}
(1) \times -3 \Rightarrow -12x + 6y &= -12 \\
(2) \times 4 \Rightarrow \underset{(+)}{-12x} \underset{(-)}{+ 12y} &\underset{(-)}{=} 24 \\
\hline
-6y &= -36 \\
y &= 6
\end{aligned}$$

Sub. $y = 6$ in (1) $\Rightarrow 4x - 2(6) = 4$
$4x = 16$
$x = 4$

$x = 4, y = 6$

Question 7.
Find the non-zero values of x satisfying the matrix equation

$$x\begin{bmatrix} 2x & 2 \\ 3 & x \end{bmatrix} + 2\begin{bmatrix} 8 & 5x \\ 4 & 4x \end{bmatrix} = 2\begin{bmatrix} x^2+8 & 24 \\ 10 & 6x \end{bmatrix}$$

Solution:

$$\begin{bmatrix} 2x^2 & 2x \\ 3x & x^2 \end{bmatrix} + \begin{bmatrix} 16 & 10x \\ 8 & 8x \end{bmatrix} = \begin{bmatrix} 2x^2+16 & 48 \\ 20 & 12x \end{bmatrix}$$

$$\begin{bmatrix} 2x^2+16 & 12x \\ 3x+8 & x^2+8x \end{bmatrix} = \begin{bmatrix} 2x^2+16 & 48 \\ 20 & 12x \end{bmatrix}$$

$$\Rightarrow 12x = 48 \Rightarrow x = 4$$

Question 8.

Solve for x, y : $\begin{bmatrix} x^2 \\ y^2 \end{bmatrix} + 2\begin{bmatrix} -2x \\ -y \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$

Solution:
$x^2 – 4x = 5$
$y^2 – 2y = 8$
$y^2 – 2y – 8 = 0$
$(y – 4)(y + 2) = 0$
$y = 4, -2$
$x^2 – 4x – 5 = 0$
$(x – 5)(x + 1) = 0$
$x = 5, -1$
$x = -1, 5, y = 4, -2$

Ex 3.18

Question 1.
If A is of order $p \times q$ and B is of order $q \times r$ what is the order of AB and BA?
Solution:
If A is of order $p \times q$ [$\because p \times q \; q \times r = p \times r$]
the order of AB = $p \times r$ [$\because q \times r \; p \times q = r \neq p$]
Product of BA cannot be defined/found as the number of columns in B ≠. The number of rows in A.

Question 2.
If A is of order $p \times q$ and B is of order $q \times r$ what is the order of AB and BA?
Answer:
Order of A = $a \times (a + 3)$
Order of B = $b \times (17 - b)$
Given: Product of AB exist
$a + 3 = b$
$a - b = -3$ ….(1)
Product of BA exist
$17 - b = a$
$-a - b = -17$
$a + b = 17$ ………(2)
$(1) + (2) \Rightarrow 2a = 14$
$a = \frac{14}{2} = 7$
Substitute the value of a = 7 in (1)
$7 - b = -3 \Rightarrow -b = -3 - 7$
$-b = -10 \Rightarrow b = 10$
The value of b = 7 and b = 10

Question 3.
Find the order of the product matrix AB if

	(i)	(ii)	(iii)	(iv)	(v)
Orders of A	3×3	4×3	4×2	4×5	1×1
Orders of B	3×3	3×2	2×2	5×1	1×3

Solution:

(i) A 3×3
B 3×3
Order of AB is 3×3.

(ii) A 4×3
B 3×2
Order of AB is 4×2.

(iii) A 4×2
B 2×2
Order of AB is 4×2.

(iv) A 4×5
B 5×1
Order of AB is 4×1.

(v) A 1×1
B 1×3
Order of AB is 1×3.

Question 4.
If A = $\begin{bmatrix} 2 & 5 \\ 4 & 3 \end{bmatrix}$, B = $\begin{bmatrix} 1 & -3 \\ 2 & 5 \end{bmatrix}$ find AB, BA and check if AB = BA?
Solution:

$$AB = \begin{bmatrix} 2 & 5 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ 2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} (2 \quad 5)\begin{pmatrix} 1 \\ 2 \end{pmatrix} & (2 \quad 5)\begin{pmatrix} -3 \\ 5 \end{pmatrix} \\ (4 \quad 3)\begin{pmatrix} 1 \\ 2 \end{pmatrix} & (4 \quad 3)\begin{pmatrix} -3 \\ 5 \end{pmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} (2+10) & (-6+25) \\ (4+6) & (-12+15) \end{bmatrix} = \begin{bmatrix} 12 & 19 \\ 10 & 3 \end{bmatrix} \quad ...(1)$$

$$BA = \begin{bmatrix} 1 & -3 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 2 & 5 \\ 4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} (1 \quad -3)\begin{pmatrix} 2 \\ 4 \end{pmatrix} & (1 \quad -3)\begin{pmatrix} 5 \\ 3 \end{pmatrix} \\ (2 \quad 5)\begin{pmatrix} 2 \\ 4 \end{pmatrix} & (2 \quad 5)\begin{pmatrix} 5 \\ 3 \end{pmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} (2-12) & (5-9) \\ (4+20) & (10+15) \end{bmatrix} = \begin{bmatrix} -10 & -4 \\ 24 & 25 \end{bmatrix} \quad (2)$$

$(1) \neq (2) \therefore AB \neq BA$

Question 5.

Given that $A = \begin{bmatrix} 1 & 3 \\ 5 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 5 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 3 & 2 \\ -4 & 1 & 3 \end{bmatrix}$ verify that $A(B + C) = AB + AC$.

Solution:

$$A(B+C) = AB + AC.$$

$$L.H.S = A(B+C)$$

$$(B+C) = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 5 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 3 & 2 \\ -4 & 1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 2 & 4 \\ -1 & 6 & 5 \end{pmatrix}$$

$$A(B+C) = \begin{pmatrix} 1 & 3 \\ 5 & -1 \end{pmatrix} \begin{pmatrix} 2 & 2 & 4 \\ -1 & 6 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} (1 \quad 3)\begin{pmatrix} 2 \\ -1 \end{pmatrix} & (1 \quad 3)\begin{pmatrix} 2 \\ 6 \end{pmatrix} & (1 \quad 3)\begin{pmatrix} 4 \\ 5 \end{pmatrix} \\ (5 \quad -1)\begin{pmatrix} 2 \\ -1 \end{pmatrix} & (5 \quad -1)\begin{pmatrix} 2 \\ 6 \end{pmatrix} & (5 \quad -1)\begin{pmatrix} 4 \\ 5 \end{pmatrix} \end{pmatrix}$$

$$= \begin{bmatrix} (2-3) & (2+18) & (4+15) \\ (10+1) & (10-6) & (20-5) \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 20 & 19 \\ 11 & 4 & 15 \end{bmatrix} \qquad ...(1)$$

$$AB = \begin{bmatrix} 1 & 3 \\ 5 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 \\ 3 & 5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} (1 \quad 3)\begin{pmatrix} 1 \\ 3 \end{pmatrix} & (1 \quad 3)\begin{pmatrix} -1 \\ 5 \end{pmatrix} & (1 \quad 3)\begin{pmatrix} 2 \\ 2 \end{pmatrix} \\ (5 \quad -1)\begin{pmatrix} 1 \\ 3 \end{pmatrix} & (5 \quad -1)\begin{pmatrix} -1 \\ 5 \end{pmatrix} & (5 \quad -1)\begin{pmatrix} 2 \\ 2 \end{pmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} (1+9) & (-1+15) & (2+6) \\ (5-3) & (-5-5) & (10-2) \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 14 & 8 \\ 2 & -10 & 8 \end{bmatrix}$$

$$AC = \begin{bmatrix} 1 & 3 \\ 5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ -4 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} (1 \quad 3)\begin{pmatrix} 1 \\ -4 \end{pmatrix} & (1 \quad 3)\begin{pmatrix} 3 \\ 1 \end{pmatrix} & (1 \quad 3)\begin{pmatrix} 2 \\ 3 \end{pmatrix} \\ (5 \quad -1)\begin{pmatrix} 1 \\ -4 \end{pmatrix} & (5 \quad -1)\begin{pmatrix} 3 \\ 1 \end{pmatrix} & (5 \quad -1)\begin{pmatrix} 2 \\ 3 \end{pmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} (1-12) & (3+3) & (2+9) \\ (5+4) & (15-1) & (10-3) \end{bmatrix}$$

$$= \begin{bmatrix} -11 & 6 & 11 \\ 9 & 14 & 7 \end{bmatrix}$$

$$AB + AC = \begin{bmatrix} 10 & 14 & 8 \\ 2 & -10 & 8 \end{bmatrix} + \begin{bmatrix} -11 & 6 & 11 \\ 9 & 14 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 20 & 19 \\ 11 & +4 & 15 \end{bmatrix} \qquad ...(2)$$

(1) = (2) $\Rightarrow$ L.H.S. = R.H.S.

$\therefore$ A(B + C) = AB + AC verified.

Question 6.

Show that the matrices $A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$ satisfy commutative property AB = BA

Solution:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (1-6) & (-2+2) \\ (3-3) & (-6+1) \end{bmatrix} = \begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix} \quad ...(1)$$

$$BA = \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (1-6) & (2-2) \\ (-3+3) & (-6+1) \end{bmatrix} = \begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix} \quad ...(2)$$

$\therefore$ (1) = (2)

$\therefore$ AB= BA verified.

Question 7.

Let $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}, B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}, C = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$ **show**

that

(i) A(BC) = (AB)C

(ii) (A – B)C = (AC – BC)

(iii) $(A- B)^T = A^T – B^T$

Solution:

(i) A(BC) = (AB)C

L.H.S. = A(BC)

$$(BC) = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} (8+0) & (0+0) \\ (2+5) & (0+10) \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 0 \\ 7 & 10 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 8 & 0 \\ 7 & 10 \end{bmatrix} = \begin{bmatrix} (8+14) & (0+20) \\ (8+21) & (0+30) \end{bmatrix}$$

$$= \begin{bmatrix} 22 & 20 \\ 29 & 30 \end{bmatrix} \qquad \text{...(1)}$$

R.H.S = (AB)C

$$AB = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 4 & 0 \\ 1 & 5 \end{pmatrix} = \begin{bmatrix} (4+2) & (0+10) \\ (4+3) & (0+15) \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 10 \\ 7 & 15 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} 6 & 10 \\ 7 & 15 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} (12+10) & (0+20) \\ (14+15) & (0+30) \end{bmatrix}$$

$$= \begin{bmatrix} 22 & 20 \\ 29 & 30 \end{bmatrix} \qquad \text{...(2)}$$

(1) = (2) $\Rightarrow$ L.H.S. = R.H.S.

$\therefore$ A(BC) = (AB)C, verified.

(ii) (A – B)C = AC – BC

L.H.S. = (A – B)C

$$A - B = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ 0 & -2 \end{bmatrix}$$

$$(A - B)C = \begin{bmatrix} -3 & 2 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} (-6+2) & (0+4) \\ (0-2) & (0-4) \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 4 \\ -2 & -4 \end{bmatrix} \quad ...(1)$$

R.H.S = AC– BC

$$AC = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} (2+2) & (0+4) \\ (2+3) & (0+6) \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 4 \\ 5 & 6 \end{bmatrix}$$

$$BC = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} (8+0) & (0+0) \\ (2+5) & (0+10) \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 0 \\ 7 & 10 \end{bmatrix}$$

$$AC - BC = \begin{bmatrix} 4 & 4 \\ 5 & 6 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ 7 & 10 \end{bmatrix} = \begin{bmatrix} -4 & 4 \\ -2 & -4 \end{bmatrix} ...(2)$$

(1) = (2) $\Rightarrow$ LHS = RHS. Hence verified.

(iii) $(A-B)^T = A^T - B^T$

$$\text{L.H.S} = (A-B) = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 2 \\ 0 & -2 \end{bmatrix} \quad ...(2)$$

$$(A-B)^T = \begin{bmatrix} -3 & 0 \\ 2 & -2 \end{bmatrix} \quad ...(1)$$

$$A^T = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}, B^T = \begin{bmatrix} 4 & 1 \\ 0 & 5 \end{bmatrix}$$

$$A^T - B^T = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 1 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 0 \\ 2 & -2 \end{bmatrix} \quad ...(2)$$

(1) = (2). L.H.S. = R.H.S. Hence verified.

Question 8.

If $A = \begin{bmatrix} \cos\theta & 0 \\ 0 & \cos\theta \end{bmatrix}$, $B = \begin{bmatrix} \sin\theta & 0 \\ 0 & \sin\theta \end{bmatrix}$ then show that $A^2 + B^2 = I$.

Solution:

$$\text{L.H.S} = A^2 + B^2$$

$$A^2 = \begin{bmatrix} \cos\theta & 0 \\ 0 & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & 0 \\ 0 & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta & 0 \\ 0 & \cos^2\theta \end{bmatrix}$$

$$B^2 = \begin{pmatrix} \sin\theta & 0 \\ 0 & \sin\theta \end{pmatrix}\begin{pmatrix} \sin\theta & 0 \\ 0 & \sin\theta \end{pmatrix}$$

$$= \begin{pmatrix} \sin^2\theta & 0 \\ 0 & \sin^2\theta \end{pmatrix}$$

$$A^2 + B^2 = \begin{pmatrix} \cos^2\theta & 0 \\ 0 & \cos^2\theta \end{pmatrix} + \begin{pmatrix} \sin^2\theta & 0 \\ 0 & \sin^2\theta \end{pmatrix}$$

$$= \begin{pmatrix} \sin^2\theta + \cos^2\theta & 0 \\ 0 & \sin^2\theta + \cos^2\theta \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I = \text{R.H.S.}$$

Hence proved.

Question 9.

If $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ prove that $AA^T = I$.

Solution:

$$\therefore A^T = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$A \cdot A^T = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$= \begin{pmatrix} \cos^2\theta + \sin^2\theta & -\cos\theta\sin\theta + \cos\theta\sin\theta \\ -\sin\theta\cos\theta + \cos\theta\sin\theta & \sin^2\theta + \cos^2\theta \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

Hence it is proved.

Question 10.

Verify that $A^2 = I$ when $A = \begin{pmatrix} 5 & -4 \\ 6 & -5 \end{pmatrix}$

Solution:

$$A = \begin{pmatrix} 5 & -4 \\ 6 & -5 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 5 & -4 \\ 6 & -5 \end{pmatrix}\begin{pmatrix} 5 & -4 \\ 6 & -5 \end{pmatrix}$$

$$= \begin{pmatrix} (25-24) & (-20+20) \\ (30-30) & (-24+25) \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I.$$

Hence it is proved.

Question 11.

If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ **and** $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ **show that**

$\mathbf{A^2 - (a + d)\,A = (bc - ad)\,I_2}$.

Solution:

L.H.S $= A^2 - (\boldsymbol{a} + \boldsymbol{d})A$

$$A^2 = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a^2 + bc & ad + bd \\ ac + cd & bc + d^2 \end{pmatrix}$$

$$(a + d)\,A = (a + d)\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$= \begin{bmatrix} a^2 + ad & ab + bd \\ ac + cd & ad + d^2 \end{bmatrix}$$

$A^2 - (a + d)A$

$$= \begin{bmatrix} a^2 + bc & ad + bd \\ ac + cd & bc + d^2 \end{bmatrix} - \begin{bmatrix} a^2 + ad & ab + bd \\ ac + cd & ad + d^2 \end{bmatrix}$$

$$= \begin{bmatrix} bc - ad & 0 \\ 0 & bc - ad \end{bmatrix}$$

$$= (bc - ad)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = (bc - ad)\ I_2 = \text{R.H.S.}$$

Hence it is proved.

Question 12.

If $A = \begin{bmatrix} 5 & 2 & 9 \\ 1 & 2 & 8 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 7 \\ 1 & 2 \\ 5 & -1 \end{bmatrix}$ **verify that**

$\mathbf{(AB)^T = B^T A^T}$.

Solution:

$$A = \begin{bmatrix} 5 & 2 & 9 \\ 1 & 2 & 8 \end{bmatrix}, B = \begin{bmatrix} 1 & 7 \\ 1 & 2 \\ 5 & -1 \end{bmatrix}$$

S.T. $(AB)^T = B^T A^T$

L.H.S $= (AB)^T$

$$(AB) = \begin{bmatrix} 5 & 2 & 9 \\ 1 & 2 & 8 \end{bmatrix} \begin{bmatrix} 1 & 7 \\ 1 & 2 \\ 5 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} (5+2+45) & (35+4-9) \\ (1+2+40) & (7+4-8) \end{bmatrix} = \begin{bmatrix} 52 & 30 \\ 43 & 3 \end{bmatrix}$$

$$(AB)^T = \begin{bmatrix} 52 & 43 \\ 30 & 3 \end{bmatrix} \quad \ldots(1)$$

$$B^T = \begin{bmatrix} 1 & 1 & 5 \\ 7 & 2 & -1 \end{bmatrix}, A^T = \begin{bmatrix} 5 & 1 \\ 2 & 2 \\ 9 & 8 \end{bmatrix}$$

$$B^T A^T = \begin{bmatrix} (5+2+45) & (1+2+40) \\ (35+4-9) & (7+4-8) \end{bmatrix}$$

$$= \begin{bmatrix} 52 & 43 \\ 30 & 3 \end{bmatrix} \quad \ldots(2)$$

(1) = (2) $\Rightarrow$ L.H.S. = R.H.S. Verified.

Question 13.

If A = show that $A^2 - 5A + 7I_2 = 0$.

Solution:

$$L.H.S = A^2 - 5A + 7I_2$$

$$A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} (9-1) & (3+2) \\ (-3-2) & (-1+4) \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$5A = 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}$$

$$7I_2 = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$A^2 - 5A + 7I_2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

Hence verified.

Ex 3.19

Multiple choice questions.
Question 1.
A system of three linear equations in three variables is inconsistent if their planes
(1) intersect only at a point
(2) intersect in a line
(3) coincides with each other
(4) do not intersect.
Solution:
(4) do not intersect

Question 2.
The solution of the system $x + y – 3z = – 6$, $-7y + 7z = 7$, $3z = 9$ is …………
(1) $x = 1, y = 2, z = 3$
(2) $x = -1, y = 2, z = 3$
(3) $x = -1, y = -2, z = 3$
(4) $x = 1, y = 2, z = 3$
Answer:
(1) $x = 1, y = 2, z = 3$
Hint.
$x + y – 3x = – 6$ ….(1)
$– 7y + 7z = 7$ ….(2)
$3z = 9$ ….(3)

From (3) we get
$z = \frac{9}{3} = 3$
Substitute the value of z in (2)
$-7y + 7(3) = 7$
$-7y = -14$
Substitute the value of $y = 2$ and $z = 3$ in (1)
$x + 2 – 3(3) = -6$
$x + 2 – 9 = -6$

x = -6 + 7
x = 1
The value of x = 1, y = 2 and z = 3

Question 3.
If (x – 6) is the HCF of $x^2 - 2x - 24$ and $x^2 - kx - 6$ then the value of k is
(1) 3
(2) 5
(3) 6
(4) 8
Solution:
(2) 5

Question 4.
$\frac{3y-3}{y} \div \frac{7y-7}{3y^2}$ **is**

(1) $\frac{9y}{7}$ (2) $\frac{9y^3}{(21y-21)}$

(3) $\frac{21y^2-42y+21}{3y^3}$

(4) $\frac{7(y^2-2y+1)}{y^2}$

Solution:
(1) $\frac{9y}{7}$

$$= \frac{3y-3}{\cancel{y}} \times \frac{3y^{\cancel{2}}}{7y-7} = \frac{3\cancel{(y-1)} \times 3y}{7\cancel{(y-1)}} = \frac{9y}{7}$$

Question 5.
$\mathbf{y}^2 + \frac{1}{y^2}$ is not equal to

(1) $\frac{y^4+1}{y^2}$ (2) $\left(y+\frac{1}{y}\right)^2$

(3) $\left(y-\frac{1}{y}\right)^2+2$ (4) $\left(y+\frac{1}{y}\right)^2-2$

Solution:
(2) $\left(y+\frac{1}{y}\right)^2$
Hint:
$y^2+\frac{1}{y^2} \neq \left[y+\frac{1}{y}\right]^2$

Question 6.
$$\frac{x}{x^2-25}-\frac{8}{x^2+6x+5} \text{ gives}$$
(1) $\frac{x^2-7x+40}{(x+5)(x-5)}$

(2) $\frac{x^2+7x+40}{(x+5)(x-5)(x+1)}$

(3) $\frac{x^2-7x+40}{(x^2-25)(x+1)}$

(4) $\frac{x^2+10}{(x^2-25)(x+1)}$

Solution:
(3) $\frac{x^2-7x+40}{(x+5)(x-5)(x+1)}$
Hint:
$$= \frac{x}{(x+5)(x-5)}-\frac{8}{(x+5)(x+1)}$$
$$= \frac{x(x+1)-8(x-5)}{(x+5)(x-5)(x+1)} = \frac{x^2+x-8x+40}{(x+5)(x-5)(x+1)}$$
$$= \frac{x^2-7x+40}{(x+5)(x-5)(x+1)}$$

Question 7.

The square root of $\frac{256x^8y^4z^{10}}{25x^6y^6z^6}$ is equal to

(1) $\frac{16}{5}\left|\frac{x^2z^4}{y^2}\right|$ (2) $16\left|\frac{y^2}{x^2z^4}\right|$

(3) $\frac{16}{5}\left|\frac{y}{xz^2}\right|$ (4) $\frac{16}{5}\left|\frac{xz^2}{y}\right|$

Solution:
(4) $\frac{16}{5}\left|\frac{xz^2}{y}\right|$
Hint:

$$\frac{16x^4y^2z^5}{5x^3y^3z^3} = \frac{16}{5}\,\frac{xz^2}{y} = \left|\frac{16xz^2}{5y}\right|$$

Question 8.
Which of the following should be added to make $x^4 + 64$ a perfect square ……….
(1) $4x^2$
(2) $16x^2$
(3) $8x^2$
(4) $-8x^2$
Answer:
(2) $16x^2$
Hint.
$x^2 + 64 = (x^2)^2 + 8^2 – 2 \times x^2 \times 8$
$= (x^2 – 8)^2$
$2 \times x^2 \times 8$ must be added
i.e, $16x^2$ must be added

Question 9.
The solution of $(2x – 1)^2 = 9$ is equal to
(1) -1
(2) 2
(3) -1, 2
(4) None of these

Solution:
(3) -1, 2
Hint:
$(2x - 1)^2 = (\pm 3)^2$
$\Rightarrow 2x - 1 = +3$
$2x - 1 = 3, 2x - 1 = -3$
$2x = 4, 2x = -2$
$x = 2, -1$

Question 10.
The values of a and b if $4x^4 - 24x^3 + 76x^2 + ax + b$ is a perfect square are
(1) 100, 120
(2) 10, 12
(3) -120, 100
(4) 12, 10
Solution:
(3) -120, 100
Hint:

$$
\begin{array}{r|l}
 & 2x^2 - 6x + 10 \\
\hline
2x^2 & \cancel{4x^4} - 24x^3 + 76x^2 - ax + b \\
 & \underset{(-)}{\cancel{4x^4}} \\
\hline
4x^2 - 6x & -\cancel{24x^3} + 76x^2 \\
 & \underset{(+)}{-}\cancel{24x^3} \underset{(-)}{+} 36x^2 \\
\hline
4x^2 - 12x + 10 & \cancel{40x^2} + \quad ax + b \\
 & \underset{(-)}{\cancel{40x^2}} \underset{(+)}{-} 120x \underset{(-)}{+} 100 \\
\hline
 & (a + 120)x + (b - 100)
\end{array}
$$

$a = -120, b = 100.$

Question 11.
If the roots of the equation $q^2x^2 + p^2x + r^2 = 0$ are the squares of the roots of the equation $qx^2 + px + r = 0$, then q,p, r are in ______.
(1) A.P
(2) G.P
(3) Both A.P and G.P
(4) none of these
Solution:
(2) G.P

Hint: $q^2x^2 + p^2x + r^2 = 0$
(2) G.P.

Question 12.
Graph of a linear polynomial is a …………..
(1) straight line
(2) circle
(3) parabola
(4) hyperbola
Answer:
(1) straight line

Question 13.
The number of points of intersection of the T quadratic polynomial $x^2 + 4x + 4$ with the X axis.
(1) 0
(2) 1
(3) 0 or 1
(4) 2
Solution:
(2) 1
$(x + 2)^2 = (x + 2)(x + 2)$
$= x = -2, -2 = 1$

Question 14.
For the given matrix $A = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 2 & 4 & 6 & 8 \\ 9 & 11 & 13 & 15 \end{bmatrix}$ the order of the matrix A^T is
(1) 2 × 3
(2) 3 × 2
(3) 3 × 4
(4) 4 × 3
Solution:
(3) 3 × 4
Hint:

$$A^T = \begin{bmatrix} 1 & 2 & 9 \\ 3 & 4 & 11 \\ 5 & 6 & 13 \\ 7 & 6 & 15 \end{bmatrix}_{4 \times 3}$$

Question 15.
If A is a 2 × 3 matrix and B is a 3 × 4 matrix, how many columns does AB have
(1) 3
(2) 4
(3) 2
(4) 5
Solution:
(2) 4
Hint:

$$A = 2 \times 3$$
$$B = 3 \times 4$$
$$AB = \underset{\substack{\downarrow \\ \text{rows}}}{2} \times \underset{\substack{\downarrow \\ \text{columns}}}{4}$$

Question 16.
If a number of columns and rows are not equal in a matrix then it is said to be a …………..
(1) diagonal matrix
(2) rectangular matrix
(3) square matrix
(4) identity matrix
Answer:
(2) rectangular matrix

Question 17.
Transpose of a column matrix is
(1) unit matrix
(2) diagonal matrix
(3) column matrix
(4) row matrix
Solution:
(4) row matrix

Question 18.

Find the matrix X if $2X + \begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 5 & 7 \\ 9 & 5 \end{bmatrix}$

(1) $\begin{pmatrix} -2 & -2 \\ 2 & -1 \end{pmatrix}$ (2) $\begin{pmatrix} 2 & 2 \\ 2 & -1 \end{pmatrix}$

(3) $\begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix}$ (4) $\begin{pmatrix} 2 & 1 \\ 2 & 2 \end{pmatrix}$

Solution:

(2) $\begin{pmatrix} 2 & 2 \\ 2 & -1 \end{pmatrix}$

Hint:

$$2X + \begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 5 & 7 \\ 9 & 5 \end{bmatrix}$$

$$2X = \begin{bmatrix} 5 & 7 \\ 9 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 4 & -2 \end{bmatrix}$$

$$X = \begin{bmatrix} 2 & 2 \\ 2 & -1 \end{bmatrix}$$

Question 19.

Which of the following can be calculated from the given matrices

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

(i) A^2

(ii) B^2

(iii) AB

(iv) BA

(1) (i) and (ii) only

(2) (ii) and (iiii) only

(3) (ii) and (iv) only

(4) all of these

Solution:

(3) (ii) and (iv) only

Hint:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}_{3\times2}, \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times3}$$

(i) 3×2 ✗
3×2

(ii) 3×3 ✔
3×3

(iii) 3×2 ✗
3×3

(iv) 3×3 ✔
3×2

Question 20.

If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 0 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 1 \\ -2 & 5 \end{bmatrix}$ Which of the following statements are correct? (i) $AB + C = \begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix}$ (ii) $BC = \begin{bmatrix} 0 & 1 \\ 2 & -3 \\ -4 & 10 \end{bmatrix}$ (iii) $BA + C = \begin{bmatrix} 2 & 5 \\ 3 & 0 \end{bmatrix}$ (iv) $(AB)C = \begin{bmatrix} -8 & 20 \\ -8 & 13 \end{bmatrix}$

(1) (i) and (ii) only
(2) (ii) and (iii) only
(3) (ii) and (iv) only
(4) all of these

Solution:
(1) (i) and (ii) only

Hint:

(i) $AB + C = \begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix}$

(ii) $BC = \begin{bmatrix} 0 & 1 \\ 2 & -3 \\ -4 & 10 \end{bmatrix}$

Unit Exercise 3

Question 1.
Solve $\frac{1}{3}$ (x + y – 5) = y – z = 2x – 11 = 9 – (x + 2 z).
Solution:
Given

$$\overset{A}{\frac{1}{3}(x+y-5)} = \overset{B}{y-z} = \overset{C}{2x-11} = \overset{D}{9-(x+2z)}$$

From A & B, $\frac{1}{3}(x+y-5) = y-z$

$\Rightarrow x + y - 5 = 3y - 3z \Rightarrow x - 2y + 3z = 5$ ....(1)

From B & C, $y - z = 2x - 11$

$\Rightarrow 2x - y + z = 11$ ....(2)

From C & D, $2x - 11 = 9 - x - 2z$

$\Rightarrow 3x + 2z = 20$ ....(3)

$$\begin{array}{lrcrcrcl}
(1) \rightarrow & x & - & 2y & + & 3z & = & 5 \\
(2) \times 2 \rightarrow & 4x & - & 2y & + & 2z & = & 22 \\
 & (-) & & (+) & & (-) & & (-) \\
\hline
 & -3x & & & + & z & = & -17 \quad ....(4) \\
(3) \rightarrow & 3x & & & + & 2z & = & 20 \\
\hline
\end{array}$$

3z = 3 ⇒ z = 1
(3) becomes, 3x + 2 = 20 ⇒ 3x = 20 – 2 = 18

$x = \frac{18}{3} = 6$

(1) becomes, $6 - 2y + 3(1) = 5 \Rightarrow 9 - 2y = 5$

$\Rightarrow 9 - 5 = 2y \Rightarrow 2y = 4$

$\therefore y = \frac{4}{2} = 2$

$\therefore$ Solution set is $\{6, 2, 1\}$

Question 2.
One hundred and fifty students are admitted to a school. They are distributed over three sections A, B and C. If 6 students are shifted from section A to section C, the sections will have equal number of students. If 4 times of students of section C exceeds the number of students of section A by the number of students in section B, find the number of students in the three sections.
Solution:
Let the students in section A, B, C be a, b, c, respectively.

$$\begin{aligned} a+b+c &= 150 \qquad \ldots(1) \\ a-6 &= c+6 \\ 4c &= a+b \\ a+b-4c &= 0 \qquad \ldots(2) \\ \Rightarrow \quad a+b+c &= 150 \\ \Rightarrow \quad a+b-4c &= 0 \\ (-)\quad(-)\quad(+) & \\ \hline 5c &= 150 \\ c &= 30 \\ \therefore a &= 42 \qquad (\because a = c+12) \\ b &= 150-30-42 = 78 \\ \therefore a &= 42 \\ b &= 78 \\ c &= 30 \end{aligned}$$

Question 3.
In a three-digit number, when the tens and the hundreds digit are interchanged the new number is 54 more than three times the original number. If 198 is added to the number, the digits are reversed. The tens digit exceeds the hundreds digit by twice as that of the tens digit exceeds the unit digit. Find the original number.
Solution:
Let the three digits numbers be 100a +10b + c.

100b + 10a + c = 3(100a + 10b + c) + 54 ………. (1)

100a + 106 + c + 198 = 100c + 106 + a ……… (2)

$(b - a) = 2(b - c)$ ……… (3)
(1) $\Rightarrow 100b + 10a + c = 300a + 30b + 3c + 54$
$\Rightarrow 290a - 70b + 2c = -54$
(2) $\Rightarrow 99a - 99c = -198 \Rightarrow a - c = -2$
$\Rightarrow a = c - 2$
(3) $\Rightarrow a + b - 2c = 0 \Rightarrow a + b = 2c$
$\Rightarrow b = 2c - c + 2$
$\Rightarrow b = c + 2$
Substituting a, b in (1)
$290(c - 2) - 70 (c + 2) + 2c = -54$
$290c - 580 - 70c - 140 + 2c = -54$
$222c = 666 \Rightarrow c = 3$
$a = 1, 6 = 5$
$\therefore$ The number is 153.

Question 4.
Find the least common multiple of
$xy (k^2 + 1) + k(x^2 + y^2)$ and
$xy(k^2 - 1) + k (x^2 - y^2)$
Answer:
$xy (k^2 + 1) + k(x^2 + y^2) = k^2xy + xy + kx^2 + ky^2$
$= (k^2xy + kx^2) + (ky^2 + xy)$
$= kx(ky + x) + y (ky + x)$
$= (ky + x) (kx + y)$
$xy (k^2 - 1) + k(x^2 - y^2) = k^2xy - xy + kx^2 - ky^2$
$= (k^2xy + kx^2) - xy - ky^2$
$= kx(ky + x) -y (ky + x)$
$= (ky + x) (kx - y)$
L.C.M. $= (ky + x) (kx + y) (kx - y)$
$= (ky + x)(k^2x^2 - y^2)$
The least common multiple is
$(ky + x) (k^2x^2 - y^2)$

Question 5.
Find the GCD of the following by division algorithm $2x^4 + 13.x^3 + 21 x^2 + 23x + 7, x^3 + 3x^2 + 3x + 1, x^2 + 2x + 1$.
Solution:
$2x^4 + 13x^3 + 27x^2 + 23x + 7$,
$x^3 + 3x^2 + 3x + 1, x^2 + 2x + 1$.
By division algorithm, first divide

$$
\begin{array}{r|l}
 & x + 1 \\
\cline{2-2}
x^2 + 2x + 1 & x^3 + 3x^2 + 3x + 1 \\
 & x^3 + 2x^2 + x \\
 & (-)\quad(-)\quad(-) \\
\cline{2-2}
 & \quad x^2 + 2x + 1 \\
 & \quad (-)\quad(-)\quad(-) \\
 & \quad x^2 + 2x + 1 \\
\cline{2-2}
 & \qquad 0 \\
\cline{2-2}
\end{array}
$$

$\therefore$ $(x + 1)^2$ is G.C.D of $x^3 + 3x^2 + 3x + 1$ and $x^2 + 2x + 1$.

Next let us divide

$2x^4 + 13x^3 + 27x^2 + 23x + 7$ by $x^2 + 2x + 1$

$$
\begin{array}{r|l}
 & 2x^2 + 9x + 7 \\
\cline{2-2}
x^2 + 2x + 1 & 2x^4 + 13x^3 + 27x^2 + 23x + 7 \\
 & (-)\quad(-)\qquad(-) \\
 & 2x^4 + 4x^3 + 2x^2 \\
\cline{2-2}
 & \quad 9x^3 + 25x^2 + 23x \\
 & \quad (-)\quad(-)\qquad(-) \\
 & \quad 9x^3 + 18x^2 + 9x \\
\cline{2-2}
 & \quad 7x^2 + 14x + 7 \\
 & \quad 7x^2 + 14x + 7 \\
\cline{2-2}
 & \qquad 0 \\
\cline{2-2}
\end{array}
$$

$\therefore$ G.C.D of $2x^4 + 13x^3 + 21x^2 + 23x + 7$, $x^3 + 3x^2 + 3x + 1$, $x^2 + 2x + 1$ is $(x + 1)^2$.

Question 6.

Reduce the given Rational expressions to its lowest form

(i) $\dfrac{x^{3a} - 8}{x^{2a} + 2x^a + 4}$

(ii) $\dfrac{10x^3 - 25x^2 + 4x - 10}{-4 - 10x^2}$

Solution:

(i) $\dfrac{x^{3a}-8}{x^{2a}+2x^{a}+4}$

$$=\frac{(x^a)^3-8}{(x^a)^2+2x^a+4}=\frac{(x^a)^3-2^3}{x^{2a}+2x^a+4}$$

$$=\frac{(x^a-2)(x^{2a}+2x^a+4)}{x^{2a}+2x^a+4}=(x^a-2)$$

(ii) $\dfrac{10x^3-25x^2+4x-10}{-4-10x^2}$

$$=\frac{5x^2(2x-5)+2(2x-5)}{-2(5x^2+2)}=\frac{\cancel{(5x^2+2)}\,(2x-5)}{-2\,\cancel{(5x^2+2)}}$$

$$=\frac{(2x-5)}{-2}=-x+\frac{5}{2}$$

Question 7.

Simplify $\dfrac{\dfrac{1}{p}+\dfrac{1}{q+r}}{\dfrac{1}{p}-\dfrac{1}{q+r}}\times\left[1+\dfrac{q^2+r^2-p^2}{2qr}\right].$

Solution:

$$= \frac{\dfrac{q+r+p}{\cancel{p(q+r)}}}{\dfrac{q+r-p}{\cancel{p(q+r)}}} \times \frac{2qr+q^2+r^2-p^2}{2qr}$$

$$= \frac{(q+r)+p}{(q+r)-p} \times \frac{(q+r)+p}{(q+r)+p} \times \frac{2qr+q^2+r^2-p^2}{2qr}$$

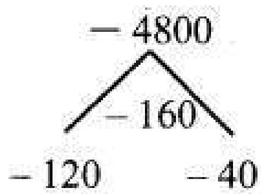
$$= \frac{(q+r+p)^2}{\cancel{(q^2+2qr+r^2-p^2)}} \times \frac{\cancel{(2qr+q^2+r^2-p^2)}}{2qr}$$

$$= \frac{(q+r+p)^2}{2qr}$$

Question 8.
Arul, Ravi and Ram working together can clean a store in 6 hours. Working alone, Ravi takes twice as long to clean the store as Arul does. Ram needs three times as long as Arul does. How long would it take each if they are working alone?

Solution:
Let Aral's speed of working be x
Let Ravi's speed of working be y
Let Ram's speed of working be z
given that they are working together. ,
Let V be the quantum of work, x + y + z = $\frac{w}{6}$ …………. (1)
Also given that Ravi takes twice the time as Aral for finishing the work.

$\therefore \frac{w}{y} = 2 \times \frac{w}{x}$ $\qquad \therefore x = 2y$



$\therefore \; y = \frac{x}{2}$ $\qquad$ (2)

Also Ram takes 3 times the time as Aral for finishing the work.
$\therefore \frac{w}{z} = 3 \times \frac{w}{x}$
$\therefore$ x = 3z $\therefore$ z = $\frac{x}{3}$

Substitute (2) and (3) in (1),

$$x+\frac{x}{2}+\frac{x}{3}=\frac{w}{6}$$

$$\therefore\ 6x+3x+2x=w$$

$$11x=w$$

$$x=\frac{w}{11},\ y=\frac{w}{22},\ z=\frac{w}{33}$$

Working alone time taken as

Arul: $\frac{w}{x}=\frac{w}{w/11}=11$ hrs.

Ravi: $\frac{w}{y}=\frac{w}{w/22}=22$ hrs.

Ram: $\frac{w}{z}=\frac{w}{w/33}=33$ hrs.

Question 9.
Find the square root of $289x^4 - 612x^3 + 970x^2 - 684x + 361$
Solution:

$$\begin{array}{l|l}
 & 17x^2-18x+19 \\ \hline
17x^2 & 289x^4-612x^3+970x^2-684x+361 \\
 & \cancel{289x^4} \\
 & (-) \\ \hline
34x^2-18x & -612x^3+970x^2 \\
 & \quad (+)\qquad (-) \\
 & -\cancel{612x^3}+324x^2 \\ \hline
4x^2-36x+19 & 646x^2-684x+361 \\
 & (-)\qquad (-)\qquad (-) \\
 & \cancel{646x^2}-\cancel{684x}+\cancel{361} \\ \hline
 & 0
\end{array}$$

$$\therefore \sqrt{289x^4-612x^3+970x^2-684x+361}$$

$$=|17x^2-18x+19|$$

Question 10.
Solve $\sqrt{y+1}+\sqrt{2y-5}=3$
Solution:
Squaring both sides

$$\left(\sqrt{y+1}+\sqrt{2y-5}\right)^2 = 3^2$$

$$y+1+2y-5+2\left(\sqrt{y+1}\sqrt{2y-5}\right)=9$$

$$3y-4-9=-2\sqrt{y+1}\sqrt{2y-5}$$

$9y^2 - 78y + 169 = 4(y + 1)(2y - 5)$
$9y^2 - 78y + 169 = 4(2y^2 + 2y - 5y - 5)$
$9y^2 - 78y + 169 = 8y^2 + 8y - 20y - 20$
$9y^2 - 78y + 169 - 8y^2 + 12y + 20 = 0$
$y^2 - 66y + 189 = 0$
$y^2 - 63y - 3y + 189 = 0$
$y(y - 63) - 3(y - 63) = 0$
$(y - 63)(y - 3) = 0$
$y = 63, 3$

Question 11.
A boat takes 1.6 hours longer to go 36 kms up a river than down the river. If the speed of the water current is 4 km per hr, what is the speed of the boat in still water?
Solution:
Let the speed of boat in still water be 'v'

$$\therefore \text{ speed} = \frac{\text{distance}}{\text{time}} \Rightarrow \quad \text{time} = \frac{\text{distance}}{\text{speed}}$$

$$\therefore \frac{36}{v-4} - \frac{36}{v+4} = \frac{96}{60} = \frac{8}{5} \qquad (\because 1.6 \text{ hrs} = \frac{96}{60})$$

$\Rightarrow 36(v + 4) - 36(v - 4) = \frac{8}{5}(v - 4)(v + 4)$
$\Rightarrow 36v + 144 - 36v + 144 = \frac{8}{5}(v^2 - 4v + 4v - 16)$
$\Rightarrow 288 = \frac{8}{5}v^2 - \frac{128}{5} \Rightarrow 8v^2 - 128 = 1440$
$\Rightarrow 8v^2 = 1568 \Rightarrow v^2 = 196$ $v = \pm 14$
$\therefore$ Speed of the boat = 14 km/hr. ($\because$ speed cannot be -ve)

Question 12.
Is it possible to design a rectangular park of perimeter 320 m and area 4800 m^2? If so find its length and breadth.
Solution:
Let the length and breadth of the rectangle be lm and bm
Given 2(1 + b)
⇒ l + b = 160 ………. (1)
Also l b = 4800

$$\therefore \quad b = \frac{4800}{l} \quad ...(2)$$

Substituting (2) in (1) we get

$$l + \frac{4800}{l} = 160$$

$$\Rightarrow l^2 + 4800 = 160l$$

$$\Rightarrow (l-120)(l-40) = 0$$

$$\Rightarrow l = 120 \text{ or } 40$$

When $l = 120$, $b = \frac{4800}{120} = 40$

When $l = 40$, $b = \frac{4800}{40} = 120$

∴ Length and breadth of the rectangular park is 120m and 40 m

Question 13.
At t minutes past 2 pm, the time needed to 3 pm is 3 minutes less than $\frac{t^2}{4}$ Find t.
Solution:
$60 - t = \frac{t^2}{4} - 3$
⇒ $t^2 - 12 = 240 - 4t$
⇒ $t^2 + 4t - 252 = 0$
⇒ $t^2 + 18t - 14t - 252 = 0$
⇒ $t(t + 18) - 14(t + 18) = 0$
⇒ $(t + 18)(t - 14) = 0$
∴ t = 14 or t = -18 is not possible.

Question 14.
The number of seats in a row is equal to the total number of rows in a hall. The total number of seats in the hall will increase by 375 if the number of rows is doubled and the number of seats in each row is reduced by 5. Find the number of rows in the hall at the beginning.

Solution:
Let the no of seats in each row be x

$$(x-5)\underset{\text{twice the no. of rows}}{\underset{\downarrow}{(2x)}} = \underset{\text{initial no. of seats}}{\underset{\downarrow}{x^2}} + 375$$

$\Rightarrow 2x^2 - 10x = x^2 + 375$
$\Rightarrow x^2 - 10x - 375 = 0$
$\Rightarrow x^2 - 25x + 15x - 375 = 0$
$\Rightarrow x(x - 25) + 15(x - 25) = 0$
$\Rightarrow (x - 25)(x + 15) = 0$
$\Rightarrow x = 25, x = -15, x > 0$
$\therefore$ 25 rows are in the hall.

Question 15.
If a and b are the roots of the polynomial $f(x) = x^2 - 2x + 3$, find the polynomial whose roots are
(i) $\alpha + 2, \beta + 2$
(ii) $\frac{\alpha-1}{\alpha+1}, \frac{\beta-1}{\beta+1}$

Solution:

$$f(x) = \frac{1x^2}{a} - \frac{2x}{b} + \frac{3}{c}$$

$$\text{Sum of the roots } (\alpha + \beta) = \frac{-b}{a} = -\frac{(-2)}{1}$$

$$\text{Product of the roots } (\alpha\beta) = \frac{c}{a} = \frac{3}{1} = 3$$

(i) $\alpha + 2, \beta + 2$ are the roots (given)
Sum of the roots $= \alpha + 2 + \beta + 2$
$= \alpha + \beta + 4$
$= 2 + 4 = 6$
Product of the roots $= (\alpha + 2)(\beta + 2)$
$= \alpha\beta + 2\alpha + 2\beta + 4$
$= \alpha\beta + 2(\alpha + \beta) + 4$
$= 3 + 2 \times 2 + 4$
$= 3 + 4 + 4 = 11$
$\therefore$ The required equation $= x^2 - 6x + 11 = 0$.

(ii) $\frac{\alpha-1}{\alpha+1}, \frac{\beta-1}{\beta+1}$

$\Rightarrow$ sum of the roots $= \frac{\alpha-1}{\alpha+1} + \frac{\beta-1}{\beta+1}$

$$\text{Sum} = \frac{(\alpha-1)(\beta+1)+(\beta-1)(\alpha+1)}{(\alpha+1)(\beta+1)}$$

$$= \frac{\alpha\beta \cancel{-\beta} \cancel{+\alpha} -1+\alpha\beta \cancel{-\alpha} \cancel{+\beta} -1}{\alpha\beta+\beta+\alpha+1}$$

$$= \frac{\alpha\beta+\alpha\beta-2}{\alpha\beta+(\alpha+\beta)+1} = \frac{2\alpha\beta-2}{\alpha\beta+(\alpha+\beta)+1}$$

$$= \frac{2(3)-2}{3+2+1} = \frac{\cancel{4}^{2}}{\cancel{6}_{3}} = \frac{2}{3}$$

$$\text{Product} = \frac{\alpha-1}{\alpha+1} \times \frac{\beta-1}{\beta+1} = \frac{(\alpha-1)(\beta-1)}{(\alpha+1)(\beta+1)}$$

$$= \frac{\alpha\beta-\beta-\alpha+1}{\alpha\beta+\beta+\alpha+1} = \frac{\alpha\beta-(\alpha+\beta)+1}{\alpha\beta+(\alpha+\beta)+1}$$

$$= \frac{3-2+1}{3+2+1}$$

$$= \frac{2}{6} = \frac{1}{3}$$

$\therefore$ Required equation $= x^2 - \frac{2}{3}x + \frac{1}{3} = 0$

$\Rightarrow 3x^2 - 2x + 1 = 0$

Question 16.
If -4 is a root of the equation
$x^2 + px - 4 = 0$ and if the equation
$x^2 + px + q = 0$ has equal roots, find the values of p and q.
Answer:
Let $p(x) = x^2 + px - 4$
– 4 is the root of the equation

P(-4) = 0
16 – 4p – 4 = 0
-4p + 12 = 0
-4p = -12
$p = \frac{12}{4} = 3$
The equation $x^2 + px + q = 0$ has equal roots
$x^2 + 3x + q = 0$
Here a = 1, b = 3, c = q
since the roots are real and equal
$b^2 - 4ac = 0$
$3^2 - 4(1)(q) = 0$
9 – 4q = 0
9 = 4q
$q = \frac{9}{4}$
The value of p = 3 and $q = \frac{9}{4}$

Question 17.
Two farmers Senthil and Ravi cultivates three varieties of grains namely rice, wheat and ragi. If the sale (in □) of three varieties of grains by both the farmers in the month of April is given by the matrix.

April sale in ₹

$$A = \begin{array}{c} \begin{array}{ccc} \text{rice} & \text{Wheat} & \text{ragi} \end{array} \\ \begin{bmatrix} 500 & 1000 & 1500 \\ 2500 & 1500 & 500 \end{bmatrix} \end{array} \begin{array}{l} \text{Senthil} \\ \text{Ravi} \end{array}$$ **and the**

May month sale (in □) is exactly twice as that of the April month sale for each variety.
(i) What is the average sales of the months April and May.
(ii) If the sales continues to increase in the same way in the successive months, what will be sales in the month of August?

Solution:

$$A = \begin{bmatrix} 500 & 1000 & 500 \\ 2500 & 1500 & 500 \end{bmatrix}$$

$$\text{May} = 2 \times A = \begin{bmatrix} 1000 & 2000 & 3000 \\ 5000 & 3000 & 1000 \end{bmatrix} = M$$

(i) $$\text{Average} = \frac{A+M}{2} = \begin{bmatrix} 750 & 1500 & 2250 \\ 3750 & 2250 & 750 \end{bmatrix}$$

(ii)
$$\begin{aligned} \text{May} &= 2A \\ \text{June} &= 2 \times \text{May} = 4A \\ \text{July} &= 2 \times \text{June} = 8A \\ \text{August} &= 2 \times \text{July} = 16A \end{aligned}$$

$$\Rightarrow \quad \text{August} = \begin{bmatrix} 8000 & 16000 & 24000 \\ 40000 & 24000 & 8000 \end{bmatrix}$$

Question 18.

If $\cos\theta \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta \begin{bmatrix} x & -\cos\theta \\ \cos\theta & x \end{bmatrix}$
$= I_2$, Find x.

Solution:

$$\text{L.H.S} = \cos\theta \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta \begin{bmatrix} x & -\cos\theta \\ \cos\theta & x \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta & \cos\sin\theta \\ -\sin\theta\cos\theta & \cos^2\theta \end{bmatrix} + \begin{bmatrix} x\sin\theta & -\sin\theta\cos\theta \\ \sin\theta\cos\theta & x\sin\theta \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \cos^2\theta + x\sin\theta & 0 \\ 0 & \cos^2\theta + x\sin\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{given}$$

$$\therefore \cos^2\theta + x\sin\theta = 1$$

$$x\sin\theta = 1 - \cos^2\theta$$

$$x = \frac{\sin^2\theta}{\sin\theta} = \sin\theta$$

Question 19.

Given $A = \begin{bmatrix} p & 0 \\ 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 0 & q \\ 1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}$

and if $BA = C^2$, find p and q.

Solution:

$$A = \begin{bmatrix} p & 0 \\ 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 0 & -q \\ 1 & 0 \end{bmatrix}, C = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}$$

$$BA = C^2 \Rightarrow \begin{bmatrix} 0 & -q \\ 1 & 0 \end{bmatrix}\begin{bmatrix} p & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & -2q \\ p & 0 \end{bmatrix} = \begin{bmatrix} (4-4) & (-4-4) \\ (4+4) & (-4+4) \end{bmatrix}$$

$$\begin{array}{r|l} -2q = -8 & p = 8 \\ q = 4 & q = 4 \end{array}$$

Question 20.

$A = \begin{bmatrix} 3 & 0 \\ 4 & 5 \end{bmatrix}, B = \begin{bmatrix} 6 & 3 \\ 8 & 5 \end{bmatrix}, C = \begin{bmatrix} 3 & 6 \\ 1 & 1 \end{bmatrix}$ find the matrix D, such that $CD - AB = 0$.

Solution:

$$A = \begin{bmatrix} 3 & 0 \\ 4 & 5 \end{bmatrix}, B = \begin{bmatrix} 6 & 3 \\ 8 & 5 \end{bmatrix}, C = \begin{bmatrix} 3 & 6 \\ 1 & 1 \end{bmatrix}$$

$$CD - AB = 0 \Rightarrow CD = AB$$

$$AB = \begin{bmatrix} 3 & 0 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 6 & 3 \\ 8 & 5 \end{bmatrix} = \begin{bmatrix} (18+0) & (9+0) \\ (24+40) & (12+25) \end{bmatrix}$$

$$= \begin{bmatrix} 18 & 9 \\ 64 & 37 \end{bmatrix}$$

$$\Rightarrow AB = CD = \begin{bmatrix} 18 & 9 \\ 64 & 37 \end{bmatrix}.$$

$$\text{Let } D = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$$

$$\begin{bmatrix} 3 & 6 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} 18 & 9 \\ 64 & 37 \end{bmatrix}$$

$$\begin{bmatrix} 3x+6z & 3y+6w \\ x+z & y+w \end{bmatrix} = \begin{bmatrix} 18 & 9 \\ 64 & 37 \end{bmatrix}$$

$3x + 6z = 18$ ...(1)

$x + z = 64$ ...(2)

$$(1) - 6(2) \Rightarrow \begin{array}{r} 3x + \cancel{6z} = 18 \\ 6x + \cancel{6z} = 384 \\ \hline -3x \quad = -366 \\ x \quad = 122 \end{array}$$

Sub. $x = 122$ in (2)

$122 + z = 64$

$z = 64 - 122 = -58$

$3y + 6w = 9$ ...(3)

$y + w = 37$ ...(4)

$$
\begin{array}{rrcl}
(3) - 3(4) \Rightarrow & 3y + 6w & = & 9 \\
& 3y + 3w & = & 111 \\
& \underset{(-)}{} \quad \underset{(-)}{} & & \underset{(-)}{} \\
\hline
& 3w & = & -102 \\
& w & = & -34
\end{array}
$$

Sub. $w = -34$ in (4)

$$
\begin{aligned}
y - 34 &= 37 \\
y &= 37 + 34 = 71
\end{aligned}
$$

$\therefore$ Solutions:

$$
\begin{aligned}
x &= 122 \\
y &= 71 \\
z &= -58 \\
w &= -34
\end{aligned}
\qquad \therefore D = \begin{bmatrix} 122 & 71 \\ -58 & -34 \end{bmatrix}
$$

Additional Questions

Question 1.
Solve the following system of linear equations in three variables. x + y + z = 6; 2x + 3y + 4z = 20; 3x + 2y + 5z = 22
Solution:
x + y + z = 6 ………….. (1)
2x + 3y + 4z = 20 ………… (2)
3x + 2y + 5z = 22 …………(3)

$$\begin{array}{lrl} (1) \times (3) \Rightarrow & 2x + 2y + 2z & = 12 \\ (3) \Rightarrow & 2x + 3y + 4z & = 20 \\ & \underline{\quad (-) \quad (-) \quad (-) \quad} & \underline{(-)} \\ & -y - 2z & = -8 \qquad \ldots(4) \end{array}$$

$$\begin{array}{lrl} (1) + (3) \Rightarrow & 3x + 3y + 2z & = 18 \\ (3) \Rightarrow & 3x + 2y + 5z & = 22 \\ & \underline{\quad (-) \quad (-) \quad (-) \quad} & \underline{(-)} \\ & y - 2z & = -4 \qquad \ldots(5) \end{array}$$

$$\begin{array}{lrl} (4) + (5) \Rightarrow & -y - 2z & = -8 \\ \Rightarrow & y - 2z & = -4 \\ \hline & -4z & = -12 \Rightarrow z = 3 \end{array}$$

Sub. z = 3 in (5) ⇒ y – 2(3) = -4
y = 2
Sub. y = 2, z = 3 in (1), we get
x + 2 + 3 = 6
x = 1
x = 1, y = 2, z = 3

Question 2.
Using quadratic formula solve the following equations.

(i) $p^2x^2 + (p^2 – q^2)\ x – q^2 = 0$
(ii) $9x^2 – 9\ (a + b)x + (2a^2 + 5ab + 2b^2) = 0$
Solution:
(i) $p^2x^2 + (p^2 – q^2)x – q^2 = 0$
Comparing this with $ax^2 + bx + c = 0$, we have
$a = p^2$
$b = p^2 – q^2$
$c = -q^2$
$\Delta = b^2 – 4ac$
$= (p^2 – q^2)-4 \times p^2 \times -q^2$
$= (p^2 – q^2)^2 + 4p^2\ q^2$
$= (p^2 + q^2)^2 > 0$
So, the given equation has real roots given by
$\alpha = \frac{-b-\sqrt{\Delta}}{2a} = \frac{-\left(p^2-q^2\right)+\left(p^2+q^2\right)}{2p^2} = \frac{q^2}{p^2}$
$\beta = \frac{-b-\sqrt{\Delta}}{2a} = \frac{-\left(p^2-q^2\right)-\left(p^2+q^2\right)}{2p^2}$
$= -1$

(ii) $9x^2 – 9(a + b)x + (2a^2 + 5ab + 2b^2) = 0$
Comparing this with $ax^2 + bx + c = 0$.
$a = 9$
$b = -9\ (a + b)$
$c = (2a^2 + 5\ ab + 2b^2)$
$\Delta = B^2 – 4AC$
$\Rightarrow 81\ (a + b)^2 – 36(2a^2 + 5ab + 2b^2)$
$\Rightarrow 9a^2 + 9b^2 – 18ab$
$\Rightarrow 9(a – b)^2 > 0$

∴ the roots are real and given by

$$\alpha = \frac{-B - \sqrt{\Delta}}{2A} = \frac{9(a+b)+3(a-b)}{18}$$

$$= \frac{12a+6b}{18} = \frac{2a+b}{3}$$

$$\beta = \frac{-B - \sqrt{\Delta}}{2A} = \frac{9(a+b)-3(a-b)}{18}$$

$$= \frac{6a+12b}{18} = \frac{a+2b}{3}$$

Question 3.
Find the HCF of $x^3 + x^2 + x + 1$ and $x^4 – 1$.
Answer:
$x^3 + x^2 + x + 1 = x^2 (x + 1) + 1 (x + 1)$
$= (x + 1) (x^2 + 1)$
$x^4-1 = (x^2)^2 – 1$
$= (x^2 + 1) (x^2 – 1)$
$= (x^2 + 1) (x + 1) (x – 1)$
H.C.F. $= (x^2 + 1)(x + 1)$

Question 4.
Prove that the equation $x^2(a^2 + b^2) + 2x(ac + bd) + (c^2 + d^2) = 0$ has no real root if $ad \neq bc$
Solution:
$\Delta = b^2 – 4ac$
$\Rightarrow 4(ac + bd)^2 – 4(a^2 + b^2)(c^2 + d^2)$
$\Rightarrow 4[(ac + bd)^2 – (a^2 + b^2)(c^2 + d^2)]$
$\Rightarrow 4(a^2c^2 + b^2d^2 + 2acbd – a^2c^2b^2c^2 – a^2d^2 – b^2d^2]$
$\Rightarrow 4[2acbd – a^2d^2 – b^2c^2]$
$\Rightarrow 4\ [a^2c^2 + b^2c^2 – 2adbc]$
$\Rightarrow -4[ad – bc]^2$
We have $ad \neq bc$
$\therefore ad – bc > 0$
$\Rightarrow (ad – bc)^2 > 0$

$\Rightarrow$ $-4(ad - bc)^2 < 0 \Rightarrow \Delta < 0$
Hence the given equation has no real roots.

Question 5.
Find the L.C.M of $2(x^3 + x^2 - x - 1)$ and $3(x^3 + 3x^2 - x - 3)$
Answer:
$2[x^3 + x^2 - x - 1] = 2[x^2(x+ 1)- 1 (x + 1)]$
$= 2(x + 1) (x^2 - 1)$
$= 2(x + 1) (x + 1) (x - 1)$
$= 2(x + 1)^2 (x - 1)$
$3[x^3 + 3x^2 - x - 3] = 3[x^2(x + 3) -1 (x + 3)]$
$= 3[(x + 3)(x^2 - 1)]$
$= 3(x + 3)(x + 1) (x - 1)$
L.C.M. $= 6(x + 1)^2(x - 1) (x + 3)$

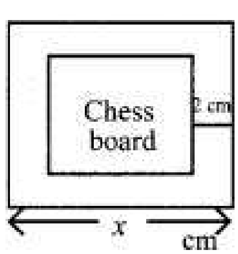
Question 6.
A two digit number is such that the product of its digits is 12. When 36 is added to the number the digits interchange their places. Find the number.
Solution:
Let the ten's digit of the number be x. It is given that the product of the digits is 12.
Unit's digit = $\frac{12}{x}$
Number = $10x + \frac{12}{x}$
If 36 is added to the number the digits interchange their places.

$$\therefore \quad 10x + \frac{12}{x} + 36 = 10 \times \frac{12}{x} + x$$
$$\Rightarrow \quad 10x + \frac{12}{x} + 36 = \frac{120}{x} + x$$
$$\Rightarrow \quad 9x - \frac{108}{x} + 36 = 0$$
$$\Rightarrow 9x^2 - 108 + 36x = 0$$
$$\Rightarrow \quad x^2 + 4x - 12 = 0$$
$$\Rightarrow \quad (x + 6)(x - 2) = 0 \quad (\because (x + 6) \neq 0 \text{ as } x > 0)$$

x = -6, 2.
But a number can never be (-ve). So, x = 2.
The number is $10 \times 2 + \frac{12}{2} = 26$

Question 7.
Seven years ago, Vanin’s age was five times the square of swati’s age. Three years hence Swati’s age will be two fifth of Varun’s age. Find their present ages.
Solution:
Seven years ago, let Swathi’s age be x years.
Seven years ago, let Varun’s age was $5x^2$ years.
Swathi’s present age = x + 7 years
Varun’s present age = $(5x^2 + 7)$ years
3 years hence, we have Swathi’s age = x + 7 + 3 years
= x + 10 years
Varun’s age = $5x^2 + 7 + 3$ years
= $5x^2 + 10$ years
It is given that 3 years hence Swathi’s age will be $\frac{2}{5}$ of Varun’s age.
$\therefore x + 10 = \frac{2}{5}(5x^2 + 10)$
$\Rightarrow x + 10 = 2x^2 + 4$
$\Rightarrow 2x^2 - x - 6 = 0$
$\Rightarrow 2x(x - 2) + 3(x - 2) = 0$
$\Rightarrow (2x + 3)(x - 2) = 0$
$\Rightarrow x - 2 = 0$
$\Rightarrow x = 2$ ($\because 2x + 3 \neq 0$ as $x > 0$)
Hence Swathi’s present age = (2 + 7) years
= 9 years
Varun’s present age = $(5 \times 2^2 + 7)$ years = 27 years

Question 8.
A chess board contains 64 equal squares and the area of each square is 6.25 cm^2. A border round the board is 2 cm wide find its side.
Solution:
Let the length of the side of the chess board be x cm. Then,



Area of 64 squares = $(x - 4)^2$

$(x - 4)^2 = 64 \times 6.25$
$\Rightarrow x^2 - 8x + 16 = 400$
$\Rightarrow x^2 - 8x - 384 = 0$
$\Rightarrow x^2 - 24x + 16x - 384 = 0$
$\Rightarrow (x - 24)(x + 16) = 0$
$\Rightarrow x = 24$ cm.

Question 9.
Find two consecutive natural numbers whose product is 20.
Solution:
Let a natural number be x.
The next number = x + 1
$x (x + 1) = 20$
$x^2 + x - 20 = 0$
$(x + 5)(x - 4) = 0$
$x = -5, 4$
$\therefore x = 4$ ($\because x \neq -5$, x is natural number)
The next number = 4 + 1 = 5
Two consecutive numbers are 4, 5

Question 10.
A two digit number is such that the product of its digits is 18, when 63 is subtracted from the number, the digits interchange their places. Find the number.
Solution:

Let the tens digit be x. Then the units digits = $\frac{18}{x}$

$\therefore$ Number $= 10x + \dfrac{18}{x}$

and number obtained by interchanging the digits

$= 10 \times \dfrac{18}{x} + x$

$$\therefore \left(10x+\frac{18}{x}\right)-\left(10\times\frac{18}{x}+x\right)=63$$

$$\Rightarrow \quad 10x+\frac{18}{x}-\frac{180}{x}-x=0$$

$$\Rightarrow \quad 9x-\frac{162}{x}-63=0$$

$$\Rightarrow \quad 9x^2-63x-162=0$$

$$\Rightarrow \quad x^2-7x-18=0$$

$\Rightarrow (x-9)(x+2)=0 \Rightarrow x = 9, -2$

But a digit can never be (–ve), so $x = 9$.

So, the required number $= 10\times9+\dfrac{18}{9} = 92$.