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MATHEMATICS

Time: 3 Hours *Maximum Marks: 80*

SECTION-A

Objective Type Questions

(10 Q × 1 M = 10 marks)

Q.1. **If** $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$**, then the possible values (s) of 'x' is/are**

(a) 3 **(b)** $\sqrt{3}$ **(c)** $-\sqrt{3}$ **(d)** $\sqrt{3} - \sqrt{3}$

Q.2. **If A, B are non-singular square matrices of the same order, then $(AB^{-1})^{-1}$ =**

(a) $A^{-1}B$ **(b)** $A^{-1}B^{-1}$ **(c)** BA^{-1} **(d)** AB

Q.3. **The degree of the differential equation** $1+\left(\frac{dy}{dx}\right)^2 = x$ **is ....**

Q.4. **If $y = Ae^{5x} + Be^{-5x}$, then $\frac{d^2y}{dx^2}$ is equal to:**

(a) $25y$ **(b)** $5y$ **(c)** $-25y$ **(d)** $15y$

Q.5. **If** $f'(x) = x + \frac{1}{x}$**, then $f(x)$ is**

(a) $x^2 + \log|x| + C$ (b) $\frac{x^2}{x} + \log|x| + C$

(c) $\frac{x}{2} + \log|x| + C$ (d) $\frac{x}{2} - \log|x| + C$

Q.6. **If the radius of the circle is increasing at the rate of 0.5 cm/s, then the rate of increase of its circumference is ....**

Q.7. $\int \frac{e^x(1+x)}{\cos^2(xe^x)} dx$ **is equal to**

(a) $\tan(xe^x) + C$ (b) $\cot(xe^x) + C$ (c) $\cot(e^x) + C$ (d) $\tan|e^x(1+x)| + C$

Q.8. **The value of p for which** $p\left(\hat{i}+\hat{j}+\hat{k}\right)$ **is a unit vector is**

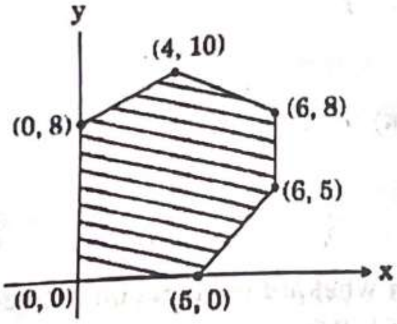
(a) 0 (b) $\frac{1}{\sqrt{3}}$ (c) 1 (d) $\sqrt{3}$

Q.9. The vector equation of XY-plane is

(a) $\vec{r}.\hat{k}=0$ (b) $\vec{r}.\hat{j}=0$ (c) $\vec{r}.\hat{i}=0$

Q.10. The feasible region for an LPP is shown below:

Let $z = 3x - 4y$ be the objective function. Minimum of z occurs at



(a) (0, 0) (b) (0 , 8) (c) (5, 0) (d) (4, 10)

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SECTION - B

Very Short Answer Type Questions

(10Q × 2M = 20 marks)

Q.11. Check if the realation R on the set A = {1, 2, 3, 4, 5, 6} defined as R = {(x, y) : y is divisible by x} is (i) symmetric (ii) transitive

Q.12. Find the value of $\sin^{-1}\left[\sin\left(\frac{13\pi}{7}\right)\right]$

Q.13. Show that the function f defined by $f(x) = (x-1)\, e^x+1$ is an increasing function for all $x > 0$.

Q.14. Find the unit vector perpendicular to each of the vector $\vec{a} = 4\hat{i}+3\hat{j}+\hat{k}$ and $\vec{b} = 2\hat{i}-\hat{j}+2\hat{k}$.

Q.15. Find $|\vec{x}|$ if $(\vec{x}-\vec{a}).(\vec{x}+\vec{a}) = 12$, where $\vec{a}$ is a unit vector.

Q.16. Evaluate the integral $\int(2x+3)\,dx$.

Q.17. Evaluate $\int_0^{2\pi}|\sin x|\,dx$.

Q.18. Compute $P(A\cap B)$, Where P (A) = 0.8, P(B) = 0.5 and P(A/B) = 0.4.

Q.19. If P(A) = 0.25 then find P(not A)

Q.20. If A is symmetric Matric, then show that $A - A'$ is a skew symmetric matrix.

* Evaluate: $\int \frac{e^{\tan^{-1}x}}{1+x^2}dx$

* Evaluate: $\int_0^1 \frac{1}{1+x^2}\,dx$

* Find the values of x, y and z from the equation:

$$\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$$

* Find the order and degree of differential equation:

$$\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right) = 0$$

* If a line has the direction ratios –18, 12, – 4 then what are its direction cosines?

* Define objective function and optimal solution of L.P.P.

* If P(A) = 0.8, P(B) = 0.5, $P\left(\frac{B}{A}\right) = 0.4$, find $P(A \cap B)$.

* If $P(A) = \frac{7}{13}$, $P(B) = \frac{9}{13}$ and $P(A \cap B) = \frac{4}{13}$, find $P\left(\frac{A}{B}\right)$.

* Evaluate: $\int \frac{x^3 - x^2 + x - 1}{x-1}$

* Evaluate: $\int_0^{\pi/4} \tan x\, dx$

* Find *x* and *y*, if: $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

* Find the order and degree of differential equation:

$$\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$$

* Find the direction cosines of the vector $\hat{i} + 2\hat{j} + 3\hat{k}$.

* Define objective function and optimal solution of L.P.P.

* Given the E and F are two events such that P(E) = 0.6, P(F) = 0.3 and $P(E \cap F) = 0{\cdot}2$, find $P\left(\frac{E}{F}\right)$.

* Let E and F be events with $P(E) = \frac{3}{5}$, $P(F) = \frac{3}{10}$ and $P(E\ \ F) = \frac{1}{5}$, Are E and F independent?

* Evaluate: $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

* Find the value of x, y and z from the equation:

$$\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$$

* Find the order and degree of differential equation:

$$\left(\frac{ds}{dt}\right)^4 + 3s\frac{d^2s}{dt^2} = 4$$

* If a line has direction ratios 2, –1, 2, determine its direction cosines.
* Define objective function and optimal solution of L.P.P
* $P(A) = \frac{6}{11}$, $P(B) = \frac{1}{5}$, $P(A \cup B) = \frac{7}{11}$, find $P(A \cup B)$.
* If $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$, find $(A \cap B)$ if A and B are independent events.
* Construct a 2 × 2 matrix, $A = [a_{ij}]$, whose elements are given by:

$$a_{ij} = \frac{(\hat{i} \times 2\hat{j})^2}{2}$$

* Find the value of k so that the function f is continuous at x = 5:

$$f(x) = \begin{cases} kx+1, & if\ x \le 5 \\ 3x-5, & if\ x > 5 \end{cases}$$

* If $y = \frac{e^x}{\sin x}$, find $\frac{dy}{dx}$.
* Find: $\int \frac{3x^2}{x^6+1}dx$
* For given vectors $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$, find the unit vector in the direction of $\vec{a} + \vec{b}$.
* Let A and B be independent events with P(A) = 0.3, P(B) = 0.4. Find P(A|B) and P(B|A).
* Define feasible and infeasible regions in linear programming.
* Define Symmetric and skew symmetric Matrices.
* Evaluate $\int \frac{\cos\sqrt{x}}{\sqrt{x}} \cdot dx$.
* Evaluate $\int_2^3 \frac{1}{x^2-1}dx$.
* Define Order and Degree of a differential Equation.
* Find the projection of $\vec{a}$ on $\vec{b}$ where $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$.
* Define the term Optimization.

* Prove that: $\sin^{-1}x + \cos^{-1}x = \dfrac{\pi}{2}$.
* Find X and Y if:

$$X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} \text{ and } X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

* Find $\dfrac{dy}{dx}$ if $x = a\cos\theta, y = a\sin\theta$.
* Find the rate of change of area of a circle with respect to its radius 'r' when $r = 5$ cm.
* Find the second derivative of $y = x^3 \log x$.
* Find the slope of the tangent to the curve $x = a\sin^3 t$; $y = a\cos^3 t$ at the point where $t = \dfrac{\pi}{2}$.
* Evaluate: $\int \dfrac{\cos x}{1+\cos x}dx$
* Find the direction ratios of the line joining A(2, 3, –4) and C(3, 8, –11).
* Find $|\vec{a}\times\vec{b}|$, if $\vec{a} = 2\hat{i}+\hat{j}+3\hat{k}$ and $\vec{b} = 3\hat{i}+5\hat{j}-2\hat{k}$.
* Find the angle between the vectors $\hat{i}-2\hat{j}+3\hat{k}$ and $3\hat{i}-2\hat{j}+\hat{k}$.
* Given that E and F are events such that P (E) = 0.6, P(F) = 0.3 and P (E $\cap$ F) = 0.2, Find P(E/F) at P (F/E)

* If $A = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$, then find AB and BA.
* Find the value of x if: $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$.
* Find $g \circ f$ and $f \circ g$ if $f(x) = |x|$ and $g(x) = |5x-2|$.
* Solve the differential equation: $\dfrac{dy}{dx} = e^{x+y}$
* If P(A) = 0.8, P(B) = 0.5 and $P\left(\dfrac{B}{A}\right) = 0.4$, find: $P\left(\dfrac{A}{B}\right)$ and $P(A \cup B)$
* Maximize: $Z = 5x + 3y$

  Subject to: $3x + 5y \le 15$.

  $x \ge 0$;

  $y \ge 0$
* $\int x \sin x \, dx$

* Differentiate: $\sqrt{e^{\sqrt{x}}}, x > 0$ w.r.t. x.
* If : $x\begin{bmatrix}2\\3\end{bmatrix}+y\begin{bmatrix}-1\\1\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$ find x and y.
* Evaluate: $\begin{vmatrix}102 & 18 & 36\\1 & 3 & 4\\17 & 3 & 6\end{vmatrix}$
* Find the general solution of: $y \log y\, dx - x\, dy = 0$
* $\int x^2.\log x\, dx$.
* Find $\frac{dy}{dx}$ for $x = a(\theta - \sin\theta)$
  $y = a(1 + \cos\theta)$
* Find the local minima or local maxima (if any) for the function
  $$f(x) = \frac{x}{2}+\frac{2}{x}, x \neq 0$$
* Evaluate: $\int e^x(\sin x + \cos x)dx$
* Evaluate: $\int_2^3 \frac{x\, dx}{x^2+1}$
* Find the general solution of the differential equation:
  $$\frac{dy}{dx} = \frac{x+1}{2-y}$$
* Find the distance of the point (2, 5, -3) from the plane
  $\vec{r}.(6i - 3j + 2k) = 4$
* Given two independent events A and B such that P (A) = 0.3 and P (B) = 0.6. Find P (A and not B).
* Determine graphically the maximum value of $Z = x + 2y$
* Prove that x : $(x > 0)$
  $$2\tan^{-1}\left(\frac{1-x}{1+x}\right) = \tan^{-1} x$$
* Prove that the function $f(x) = 5x - 3$ is continuous at $x = 0$.
* Find the local minima or local maxima (if any) for the function $f(x) = x^3 - 6x^2 + 9x + 15$
* Evaluate: $\int \frac{dx}{e^x + e^{-x}}$
* Evaluate: $\int_0^1 \frac{dx}{1+x^2}$

* Find the angle between two planes $3x - 6y + 2z = 7$ and $2x + 2y - 2z = 5$.
* Evaluate P (A $\cup$ B), if 2P (A) = P (B) = $\frac{5}{13}$ and P (A/B) = $\frac{2}{5}$.
* Prove that $2\tan^{-1}\sqrt{x} = \cos^{-1}\left(\frac{1-x}{1+x}\right)$.
* Find $\frac{dy}{dx}$, if $x = at^2$, $y = at^4$.
* Evaluate: $\int e^x\left(\tan^{-1}x + \frac{1}{1+x^2}\right)dx$
* Find the general solution of the differential equation: $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$
* Find the angle between the line $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ and the plane $10x + 2y - 11z = 3$.
* Compute $P\left(\frac{A}{B}\right)$, if P (B) = 0.5 and P (A $\cap$ B) = 0.32.
* If A = $\begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$ then find AB, BA. Show that AB $\neq$ BA
* Find $\frac{dy}{dx}$, if $\sin^2 y + \cos xy = \pi$
* Find $\frac{dy}{dx}$, if $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right)$; $-1 < x < 1$.
* An edge of a variable cube is increasing at the state of 3 cm/sec. How fast is the volume of cube increasing when the edge is 10 cm long?
* Find $\frac{dy}{dx}$, if $y + \sin y = \cos x$
* Evaluate: $\int \frac{dx}{\cos(x+a)\cos(x+b)}$
* Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}dx$
* Maximise: $z = 3x + 4y$ subject to constraints $x + y \leq 4, x, y \geq 0$
* Let A&B be independent events with P(A) = 0.3 and P(B) = 0.4. Find P(A/B) & P(B/A).

SECTION - C
Short Answer Type Questions
(8Q × 4M = 32 marks)

Q.21. Solve the differnetial equation: $ydx + (x-y^2)\,dy = 0$

Q.22. Evalute: $\int_0^4 |x-1|\,dx$.

Q.23. If $y = a\cos(\log x) + b\sin(\log x)$, show that $x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = 0$.

Q.24. Find the shortest distance between the two lines:

$$\vec{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda\left(\hat{i} - 2\hat{j} + 2\hat{k}\right)$$

and $\vec{r} = -4\hat{i} - 2\hat{k} + \mu\left(3\hat{i} - 2\hat{j} - 2\hat{k}\right)$.

Q.25. If $\vec{a} = \hat{i} - \hat{j} + 7\hat{k}$ **and** $\vec{b} = 5\hat{i} - \hat{j} + \lambda\hat{k}$, **then find the value of λ so that the vectors $\vec{a} + \vec{b}$ are orthogonal.**

Q.26. Solve the following Linear Programming Problem graphically:
Maximize $Z = 400x + 300y$ subject to $x + y \le 200$, $x \le 40$, $x \ge 20$, $y \ge 0$.

Q.27. Show that the function $f: \mathbf{R} \to \mathbf{R}$ defined by $f(x) = \dfrac{x}{x^2+1}, \forall x \in R$ is neither one-one nor onto.

Q.28. The probability distribution of a random variable X, where k is a constant is given below:

$$P(X=x) = \begin{cases} 0.1 & if \quad x = 0 \\ kx^2 & if \quad x = 1 \\ kx, & if \quad x = 2 \text{ or } 3 \\ 0, & \text{otherwise} \end{cases}$$

Determine
(a) the value of k
(b) $P(x \le 2)$

* Show that $f: [-1\ 1] \to \mathrm{R}$ given by $f(x) = \dfrac{x}{x+2}$ is one-one. Find the inverse of the function $f: [-1\ 1] \to$ Range f.

* If $A = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}$ and $B = [1\ \ 3\ \ -6]$ verify that $(AB)' = B'A'$.

* Find the equation of tangent and normal to the curve at the indicated point $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ *at* (0, 5).

* Use differentials, find the approximate value up to 3 places of decimal $(25)^{1/3}$.]

* Integrate the funciton $x \tan^{-1} x$.
* Find the relationship between a and b such that the function f defined by:

$$f(x) = \begin{cases} ax+1 & x \le 3 \\ bx+3 & x > 3 \end{cases}$$ is continuous at $x = 3$.

* If $y = \sin^{-1}x$, show that $(1-x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} = 0$
* Find the angle between two planes:
  $2x + y - 2z = 5$ and $3x - 6y - 2z = 7$ using vector method.
* Solve graphically (L.P.P.):
  Maximize:
  $Z = 4x + y$
  Subject to the constraints:
  $x + y \le 50$
  $3x + y \le 90$
  $x \ge, y \ge 0$
* Consider $f: R \to R$ given by $f(x) = 4x + 3$ show that f is invertible. Find the inverse of f.
* Write in the simplest form:

$$\tan^{-1}\left[\frac{3a^2x - x^3}{a^3 - 3ax^2}\right] a > 0 = \frac{a}{\sqrt{3}} x = \frac{a}{\sqrt{3}}$$

* If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ show that: $A^2 - 5A + 7I = 0$
* Express $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ as the sum of a symmetric and skew symmetric matrix.
* Check whether the relation R in R defined by R = R defined by R = {(a, b): a $\le b^3$ } is reflexive, symmetric or transitive.
* Find the value of $\tan\dfrac{1}{2}\left[\sin^{-1}\dfrac{2x}{1+x^2} + \cos^{-1}\dfrac{1-y^2}{1+y^2}\right] |x| < 1$, $y > 0$ and $xy < 1$.
* Express $\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$ as the sum of symmetric and skew symmetric matrix.
* A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground away from the wall at the rate of 2 cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall.

* Find the general solution of the differential equation: $e^x \tan y\, dx + (1-e^x)\sec^2 y\, dy = 0$

* If $y = (\log x)^{\cos x}$, find $\dfrac{dy}{dx}$.

* Using the property of integrals, evaluate: $\displaystyle\int_0^{\pi/2} \frac{\cos^5 x\, dx}{\sin^5 x + \cos^5 x}$

* If $y = 3\cos(\log x) + 4\sin(\log x)$, show that $x^2y_2 + xy_1 + y = 0$

* Find the angle between the following pair of lines:

$$\vec{r} = \left(2\hat{i} - 5\hat{j} + \hat{k}\right) + \lambda\left(3\hat{i} + 2\hat{j} + 2\hat{k}\right)$$

and $$\vec{r} = \left(7\hat{i} - 6\hat{k}\right) + \mu\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$$

* Prove that if a plane has intercepts a, b, c and is at a distance of p units from the origin, then:

$$\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{p^2}$$

* Determine which of the following binary operations on the set R are associative and which are commutative:

$a * b = 1, \forall\ a, b \in \mathbb{R}$

* Prove that: $\dfrac{9\pi}{8} - \dfrac{9}{4}\sin^{-1}\left(\dfrac{1}{3}\right)$

* If $\tan^{-1}\left(\dfrac{x-1}{x-2}\right) + \tan^{-1}\left(\dfrac{x+1}{x+2}\right)$, then find the value of x.

* Let $f : X \longrightarrow Y$ be invertible function, show that f has unique inverse.

* Write in the simplest form $\tan^{-1}\left(\dfrac{\cos x - \sin x}{\cos x + \sin x}\right)$

* At what point of the curve $y=x^2$ does the tangent make an angle 45° with the *x-axis*.

* Using differentials find the approximate value up to 3 decimal places of $26^{1/3}$.

* Prove that $\displaystyle\int\sqrt{x^2+a^2}\,dx = x\frac{\sqrt{x^2+a^2}}{2}\log\left|x+\sqrt{x^2+a^2}\right| + c$

* Prove that $\displaystyle\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

* If $y = \cos^{-1} x$, Find $\dfrac{d^2y}{dx^2}$ in terms of y alone.

* Solve graphically, Minimize or Maximize $Z = 5x + 10y$.

* Let A and B be sets. Show that $f : A \times B \longrightarrow B \times A$ such that $f(a, b) = (b, a)$ is bijection function.

* Write in the simplest form: $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right) : x \neq 0$

* Evaluate: $\int \frac{\sec^2 x\, dx}{(1+\tan x)(2+\tan x)}$

* Consider $f : R \longrightarrow R$ given by $f(x) = 4x + 3$ show that f is invertible. Find the inverse of f.

* Using determinants, show that the points A $(a, b + c)$, B$(b, c + a)$, C$(c, a + b)$ are collinear?

* Find the relationship between a and b so that the function f defined by $f(x) = \begin{cases} ax+1 & \text{if} \quad x \leq 3 \\ bx+3 & \text{if} \quad x > 3 \end{cases}$ is continous at $x = 3$.

* Prove that the function f is given by $f(x) = \log \cos x$ is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$ and strictly increasing on $\left(\frac{\pi}{2}, \pi\right)$.

* Find both the maximum value and minimum value of $3x^4 - 8x^3 + 12x^2 - 48x + 25$ on the interval $[0,3]$.

* Solve the differential equation $x^2 \frac{dy}{dx} = x^2 - 2y^2 + xy$.

* Show that the lines:

  $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-1}{5}$ and $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$ are coplanar.

* Solve the following L.P.P. graphically:
  Minimise and Maximise: $Z = 3x + 9y$
  Subject toconstraints:

$$x + 3y \leq 60$$
$$x + y \geq 10$$
$$x \leq y$$
$$x, y \geq 0$$

* Consider: $R \rightarrow [4, \infty]$ given by $f(x) = x^2 + 4$, Show that f is the invertible with the inverse of f given by $f^{-1}(y) = \sqrt{y-4}$ where R is the set of all non-negative real numbers.

* Find points of all discountinuity of f where f is defined by $f(x) = \begin{cases} x+1 \text{ if } x \geq 1 \\ x^2+1 \text{ if } x < 1 \end{cases}$

* At what points in the interval $[0, 2\pi]$ does the function six $2x$ attain its maximum value?

* Solve the differential equation: $\frac{dy}{dx} + 2y = \sin x$.

* Using integration find the area of region bounded by the triangle whose vertices are (1, 0); (2, 2) and (3, 1).\

* Minimize $Z = 3x + 2y$

  Subject to the constraints $x + 2y < 10$,

  $3x + y < 15, x \geq 0$ $Y \geq 0$

* Let T be the set of all triangle in a plane with R a relation in T given by R $\{(T_1, T_2) : T_1$ is congruent to $T_2\}$, show that R is an equivalence relation.

* Prove that $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$; $x \in [0, 1]$

* Express the following matrix as the sum of a symmetric and a skew symmetric matrix.

$$A = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$$

* Find the equation of Tangent and normal to curve at the indicated point $x = \cos t$, $y = \sin t$ at $t = \frac{\pi}{4}$.

* Find the intervals in which the following function is strictly increasing or decreasing $-2x^3 - 9x^2 - 12x + 1$.

* Solve the differential equation $\cos^2 x\frac{dy}{dx} + y = \tan x$ $(0 \leq x < \frac{\pi}{2})$

* Using integrating, find the area of the region bounded by the triangle whose vertices are (–1, 0), (1, 3) and (3, 2).

* Find the angle between the line $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ and the plane $10x + 2y - 11z = 3$

* Show that each of the given three vectors is a unit vector

$$\frac{1}{7}\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right), \frac{1}{7}\left(3\hat{i} - 6\hat{j} + 2\hat{k}\right), \frac{1}{7}\left(6\hat{i} + 2\hat{j} - 3\hat{k}\right).$$

  Also, show that they are mutually perpendicular to each other.

* Consider $f : R \longrightarrow R$ given by $f(x) = 4x + 3$. Show that "f" is invertible. Find the inverse of "f".

* Evaluate : $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$.

* Find the values of k, if area of triangle is 4 sq units and vertices are: (–2, 0), (0,4), (0, k).

- Find the value of k, so that the function "f" is continuous at $x = 2$:

$$f(x) = \begin{Bmatrix} kx^2 \text{ if } x \le 2 \\ 2 \text{ if } x > 2 \end{Bmatrix}.$$

- Find the absolute maximum value and the absolute minimum value of the function:

$$f(x) = \sin x + \cos x,\ x \in [0, \pi].$$

- Find $f\,o\,g$ and $f\,o\,g$ if $f(x) = 8x^3$ and $g(x) = x^{1/3}$

- If $\tan^{-1}\dfrac{x-1}{x-2} + \tan^{-1}\dfrac{x-1}{x-2} = \dfrac{\pi}{4}$, then find the value of x.

- Find the value of "k"if area of Δ is 35 sq units an vertices are (2,–6), (5,4) and (K,4) .

- Find the value of "K" , so that the function "f" is continuous at $x = \pi$;

$$f(x) = \begin{cases} kx + 11 & if \quad x \le z \\ \cos x & if \quad x > z \end{cases}$$

- Find the absolute maximum value and the absolute minimum value of the function:-

$$f(x) = 4x - \frac{1}{2}x^2 . x \in \left[-2, \frac{9}{2}\right]$$

- If $f(x) = \dfrac{4x+3}{6x-4}, x \ne \dfrac{2}{3}$ show that $f\,o\,f(x) = x$, for all $x \ne \dfrac{2}{3}$. What is the inverse of f?

- If $\sin\left(\sin^{-1}\dfrac{1}{5} + \cos^{-1} x\right) = 1$, then find the value of x. Evalute definite integrals as limit of sums: $\int_0^5 (x+1)dx$.

- A manufactures produces nuts and bolts. It takes I hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hours on machine B to produce a package of bolts. He earns. a profit of Rs. 17.50 per package on nuts and Rs. 7.00 per package on bolts. How many packages of each should be preduced each day so as to maximise his profit, if he operates his machine for at the most 12 hours a day?

- Consider f : R + $f : R_+ \to [-5, \propto)$ given by $f(x) = 9x^2 + 6x - 5$ show that f is invertible with $f^{-1}(y) = \left[\dfrac{(y+6)-1}{3}\right]$

- For the matrix A and B Verify that $(AB)^1 = B^1 A^1$, where $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$ $B = [-1\ 2\ 1]$

* Evaluate $\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$

* Differentiate the function with respect to x: $\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$

* Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$, $\frac{-\pi}{2} < x < \frac{3\pi}{2}$ in the simplest form.

* Find the general solution for the differential equation: $\frac{dy}{dx}+(\sec x)y=\tan x\left(0 \le x < \frac{x}{2}\right)$

* Evaluate: $\int_{\pi/2}^{\pi} e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx$.

* Evaluate: $\int_0^{\pi} \frac{x\tan x}{\sec x+\tan x}dx$

* If $y = 3e^{2x} + 2e^{3x}$, prove that $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=0$

SECTION - D

Long Answer Type Questions

(3Q × 6M = 18 marks)

Q.29. Solve the following system of equations by matrix method:

$$x-y+2z=7$$
$$2x-y+3z=12$$
$$3x+2y-z=5$$

Or

If $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix}$**, then show that** $A^3 - 4A^2 - 3A + 11I = 0$**. Hence find** A^{-1}**.**

Q.30. Evaluate: $\int \frac{dx}{\sqrt{5-4x-x^2}}$

Or

Find $\int \frac{(x^3+x+1)}{(x^2-1)}dx$

Q.31. If $y = (\tan^{-1}x)^2$**, show that** $(x^2+1)^2y_2+2x(x^2+1)y_1 = 2$

Or

Find $\frac{dy}{dx}$**, if** $x^y, y^x = x^x$**.**

* Solve the system of equations by matrix method:

$$3x - 2y + 3z = 8$$
$$2x + y - z = 1$$
$$4x - 3y + 2z = 4$$

* If $y = x^{\sin x} + (\sin x)^{\cos x}$, find $\dfrac{dy}{dx}$.

* Show that of all the rectangles inscribed in a given fixed circle the square has the maximum area.

* Find $\int \dfrac{3(\sin x - 2)\cos x}{5 - \cos^2 x - 4\sin x} dx$.

* Find the area of the region bounded by the ellipse: $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$

* Find the general solution of the differential equation given by $x\dfrac{dy}{dx} + 2y = x^2 \log x$

* Find the general solution of the differnetial equation: $\dfrac{dy}{dx} = \dfrac{x+1}{2-y}$, $y \neq 2$

* Find the probability distribution of the number of dobulets in three throws of a pair of dice.

* If $y = \left(x + \dfrac{1}{x}\right)^x + x^{\left(1+\frac{1}{x}\right)}$, find $\dfrac{dy}{dx}$.

* Differentiate $\sqrt{\dfrac{(x-3)(x^2+4)}{3x^2+4x+5}}$ w. r. to x.

* Find $\int \dfrac{x^2 + x + 1}{(x+1)^2(x+2)} dx$

* Find the shortest distance between the lines whose vector equations are:

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 3\hat{j} + 2\hat{k})$$

and $\vec{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(2\hat{i} + 3\hat{j} + \hat{k})$

* Solve the system of equations: $2x + 3y + 3z = 5$, $x - 2y + z = -4$, $3x - y - 2z = 3$

* Solve the Linear differential equation $\dfrac{dx}{dx} + \dfrac{y}{x} = x^2$

* Find the equation of the curve passing through the point (0.0) and whose differential Equation is $\dfrac{dx}{dx} = e^x \sin x$.

* Find the vector equation of the line passing through (1,2,3) and parallel to the planes $\vec{r}(\hat{i} - \hat{j} + 2\hat{k})$ and $\vec{r}(3\hat{i} + \hat{j} + \hat{k}) = 6$

* Find the shortest distance between the lines l_1 and l_2 whose vector equation are:
$\vec{r} = \hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = \hat{i} + \hat{j} - \hat{k} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$
* Using properties of determinants, show that

$$\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$$

* Using matrix method, solve the linear system of equations:

$$2x + y + z = 1$$
$$x - 2y - z = \frac{3}{2}$$
$$3y - 5z = 9$$

* Using matrix method, solve the linear system of equations:

$$x - y + 2z = 7$$
$$3x + 4y - 5z = -5$$
$$2x - y + 3z = 12$$

* Let A = $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, show that $(aI + bA)^n = a^nI + na^{n-1}bA$ where I is the matrix of order 2 and $n \in N$.

* For the matrix A = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$ show that $A^3 - 6A^2 + 5A + 11\,I = 0$. Hence, find A^{-1}.
* Let A = $\begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$ and B = $\begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$ verify $(AB)^{-1} = B^{-1}A^{-1}$.
* Differentiate the function $\sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$.
* Find all points of discontinuity of f, where f is defined by: $f(x) = \begin{cases} 2x+3, \text{if } x \le 2 \\ 2x-3, \text{ if } x>2 \end{cases}$.
* If $y = \sin^{-1} x$, show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$
* If $x = \sqrt{a^{\sin^{-1}t}}$, $y = \sqrt{a^{\cos^{-1}t}}$, show that: $\frac{dy}{dx} = \frac{-y}{x}$
* Differentiate $x^{\sin x} + (\sin x)^{\cos x}$.
* If $y = (\tan^{-1} x)^2$ show that $(x^2+1)^2 y_2 + 2x(x^2+1)y_1 = 2$
* Find $\frac{dy}{dx}$, if $x^y + y^x + x^x = a^b$

* Find the relation between 'a' and 'b' so that the function defined by

  $f(x) = ax + 1, x \leq 3 = bx + 3, x > 3$ is continuous at $x = 3$

* Differentiate the function with respect to 'x': $(\sin x)^x + \sin^{-1}\sqrt{x}$

* Find the equation of the tangent and normal to the curve:-

  $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at (1, 3).

* Find the equations of tangent and normal to the curve:-

  $y = x^3$ at (1, 1).

* Differentiate the function with respect to "x": $(\log x)^x + x^{Logx}$.

* Find the equation of the tangent and normal to the curve: $y^2 = 4ax$ at the point (at^2, 2 at).

3. Evalute the following definite integral as limit of sums: $\int_1^4 \left(x^2 - x\right) dx$.

* Evaluate the intergral: $\int_2^8 \frac{\sqrt[3]{x+1}}{\sqrt[3]{x+1}+\sqrt[3]{11-x}}dx$

* Show that: $\int \frac{dx}{\sin^4 x \cos^2 x} = \tan x - 2\cot x \frac{1}{3}\cot^3 x + c$

* Show that: $\int_0^1 x(\tan^{-1} x)^2 dx = \frac{\pi}{4}\left[\frac{\pi}{4} - 1\right] + \frac{1}{2}\log 2$

* Evaluate: $I = \int \frac{\cos x}{(1-\sin x)(2-\sin x)} dx$

* Evaluate: $\int_{-1}^{1} e^x dx$

* Evaluate: $\int_0^{\frac{\pi}{2}} (2\log\sin x - \log\sin 2x)\,dx$

* Evaluate: $\int \frac{(3\sin\phi - 2)\cos\phi}{5 - \cos^2\phi - 4\sin\phi} d\phi$

* Evaluate: $\int_2^3 x^2 dx$ as the limit of a sum.

* Find the general solution of the differential equation: $\cos^2 x \frac{dy}{dx} + y = \tan x$

  $\left(0 \leq x < \frac{\pi}{2}\right)$

* Solve the differential Equations: $\cos^2 x \frac{dy}{dx} = y^2 - 2y^2 + xy$

* Find the general solution of the differential equation: $(1+x^2)dy + 2xy\,dx = \cot x\,dx$

* Find the particulars solution of the differential equation

$$(1+x^2)\frac{dy}{dx}+2xy=\frac{1}{1+x^2},\ \text{when } x=1,\ y=0$$

* Solve the differential equation: $x\frac{dy}{dx}-y+x\sin\left(\frac{y}{x}\right)=0$
* Find the area enclosed by the parabola $4y = 3x^2$ and the line $2y = 3x + 12$.
* By using propereties of definite integralss, evaluate the integral $\int_0^x \frac{x}{a^2\cos^2 x+b^2\sin^2 x}dx$.
* Solve the following differential equation: $(x^2 + xy)\, dy = (x^2 + y^2)\, dx$.
* Evaluate the integral: $\int_{\pi/6}^{\pi/3}\frac{\sin x+\cos x}{\sqrt{\sin 2x}}dx$ by using properties of definite integrals.
* Find the equation of plane through the intersection of planes $3x - y + 2z - 4 = 0$ and $x + y + z - 2 = 0$ and the point (2, 2, 1).
* Find the shortest distance between the lines whose vector equation are:

$$\vec{r}=(1-t)\hat{i}+(t-2)\hat{j}+(3-2t)\hat{k}$$

$$\vec{r}=(S+1)\hat{i}+(2S-1)\hat{j}-(2S+1)\hat{k}$$

* Find the general solution of differential equation: $\frac{xdy}{dx}+2y=x^2\log x$.
* Show that the differential equation is homogeneous and solve it; $(x - y)\, dy - (x + y)\, dx = 0$.
* Find the general solution of the differential equation: $ydx + (x - y^2)dy = 0$.
* Find the general solution of the differential equation: $x\frac{dy}{dx}+y-x+xy\cot x=0\ (x\neq 0)$
* Show that the differential equation is homogeneous and solve it:

$$ydx+x\log\left(\frac{y}{x}\right)dy-2xdy+0$$

* If a unit vector $\vec{a}$ makes angles $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$ and an acute angle θ with $\hat{k}$, then find θ and hence, the components of $\vec{a}$.
* Find the shortest distance between lines:

$$\vec{r}=6\vec{i}+2\hat{j}+2\hat{k}+\lambda(\hat{i}-2\hat{j}+2\hat{k})$$

and $\vec{r}=-4\hat{i}-\hat{k}+\mu(3\hat{i}-2\hat{j}-2\hat{k})$.

* Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of aces.

* Find the probability distribution of number of doublets in three throws of a pair of dice.

If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{b} = -\hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$

are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$, then find the value of λ.

* Find the shortest distance between the lines $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$.

* An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of accidents are 0.01, 0.03 & 0.15 respectively. One of the insured person meets with an accident. What is the probability that he is a scooter driver?

* Show that the vectors $2\hat{i} - \hat{j} + \hat{k}, \hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ form the vertices of a right angled triangle.

* Find the vector and cartesion equations of the plane passing through the intersection of the plane $\vec{\Sigma}.(2\hat{i} + 2\hat{j} + 3\hat{k}) = 7$, $\vec{\Sigma}.(2\hat{i} + 2\hat{j} + 3\hat{k}) = 9$, and through the point (2, 1, 3).

* Find thevalue of p so that the lines $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles.

* If $\vec{a}, \vec{b}$ and $\vec{c}$ are three vectors of equal magnitude and mutually perpendicular to each other. Show that $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to $\vec{a}, \vec{b}$ and $\vec{c}$.